

# Analysis of Performance and Efficiency of Conservation Voltage Optimization Considering Load Model Uncertainty

Zhaoyu Wang<sup>1</sup> and Jianhui Wang<sup>2</sup>

**Abstract:** This paper presents a novel method for evaluating the performance of conservation voltage optimization (CVO). The method investigates the load-to-voltage (LTV) sensitivity. A time-varying exponential load model is developed to represent the load's dependence on voltage and other factors. The model parameters are estimated by applying the recursive least squares (RLS) algorithm. The effects of CVO can be assessed by using the estimated model parameters. The proposed RLS-based algorithm is validated by simulation tests and the Euclidian distance-based comparison method. Field test results also show the accuracy and effectiveness of the presented algorithm. To address the uncertainty and variability of the CVO performance, the Kolmogorov-Smirnov test is used to determine the distribution that represents the CVO effect of each substation. The proposed methodology can assist utilities in selecting target substations to implement voltage optimization. DOI: 10.1061/(ASCE)EY.1943-7897.0000190. © 2014 American Society of Civil Engineers.

**Author keywords:** Conservation voltage optimization; Recursive least squares; Kolmogorov-Smirnov test.

## Introduction

Energy deficit and environmental concerns make energy conservation essential (Chowdhury and Tseng 2007; Lasseter 2007; Frimpong 2008). An effective and economic way to reduce demand and save energy in a distribution system is conservation voltage optimization (CVO) or conservation voltage reduction (CVR). CVR regulates voltage at the substation to operate feeders at the lowest acceptable voltage levels (Begovic et al. 2000). As an important topic in demand responses, CVR has been thoroughly studied and successfully implemented to reduce demand and energy consumption and to increase the margins of system stability in many electric utilities (Kennedy and Fletcher 1991; Wilson 2010).

There are two types of CVRs: long-term energy saving (Beck 2007) and short-term load reduction (Warnock and Kirkpatrick 1986; Dabic et al. 2010). In the long-term CVR, voltage is reduced permanently; in the short-term reduction, voltage is reduced during peak hours. Fig. 1 shows a conceptual reference of CVR. Estimating the outcome of CVR is a major concern for deciding its implementation, selecting suitable feeders to apply voltage reduction, and performing cost/benefit analyses. A conservation voltage reduction factor ( $CVR_f$ ), which is defined as the ratio of the percentage change in energy or demand to the percentage change in average voltage, is the metric most often used to gauge the performance of voltage reduction as a means for load reduction or energy saving (Wang and Wang 2013). Previous tests have demonstrated the various performances of CVR.

The earliest reported CVR test was performed on 15 feeders at American Electric Power System (AEP) in 1973 (Preiss and Warnock 1978). It was found that for 1% voltage reduction, the

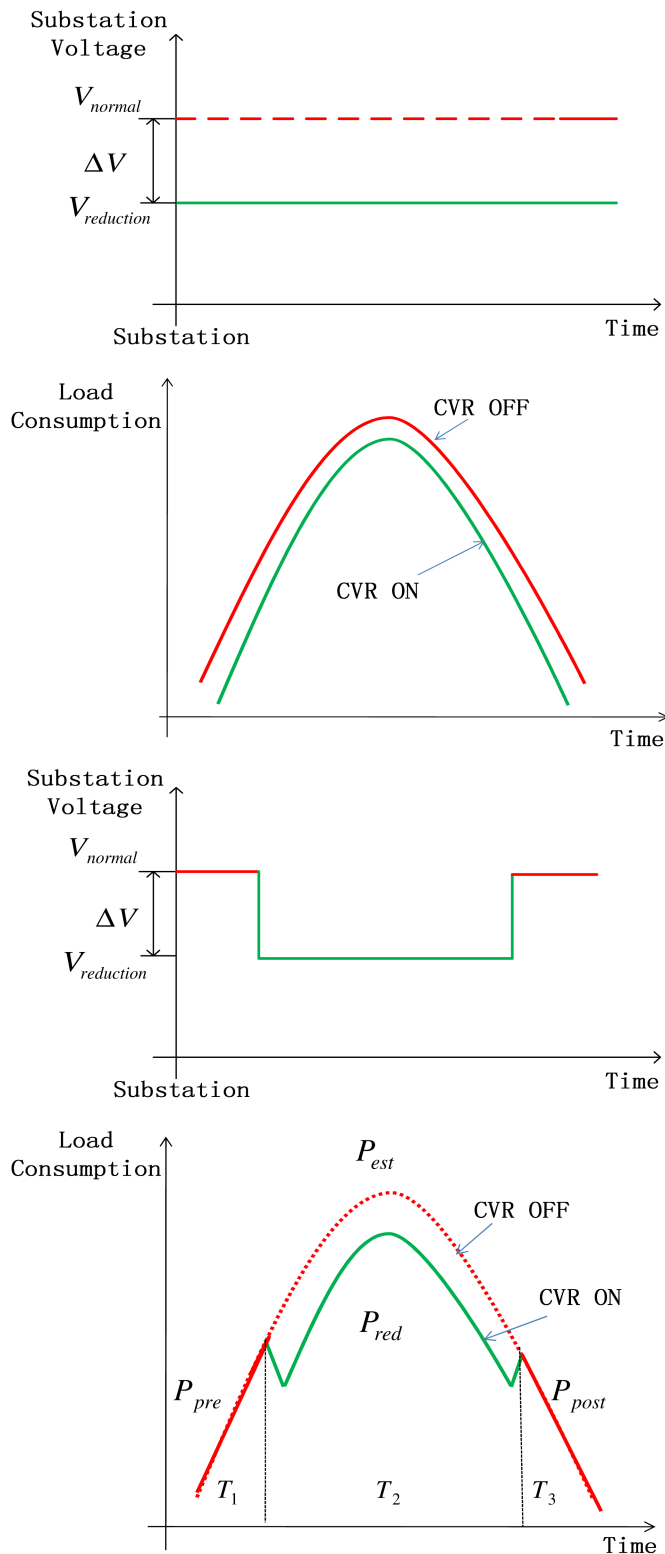
energy savings of residential, commercial, and industrial circuits were 0.61, 0.89, and 0.35%, respectively. In 1984, Northeast Utilities (NU) conducted a field test on five substations of its distribution system. Results show that a 1% reduction in voltage produced an approximately 1% reduction in energy consumption for the tested substations, which supply predominantly residential and commercial loads (Lauria 1987). In 1987, Bonneville Power Administration (BPA) conducted a CVR pilot test and concluded that CVR was an effective energy conservation measure and that potential conservation was available in the BPA service territory up to  $2.37 \times 10^6$  MW · h/year (De Steese et al. 1990). In 1988, Snohomish Public Utility District (PUD) conducted a pilot study of CVR on its 12 distribution feeders. The results of the study confirmed significant energy conservation on those feeders; 1% reduction of voltage yielded 0.6% reduction in electricity consumption (Kennedy and Fletcher 1991). In 1990, BC Hydro launched a program that included load-to-voltage (LTV) dependency studies at a substation on Vancouver Island. The tests showed significant reductions in both active and reactive power (Dwyer et al. 1995). In 2004, the Northwest Energy Efficiency Alliance (NEEA) began its studies on CVR and found a summer CVR factor of 0.67, compared to the winter CVR factor of 0.20 (Short and Mee 2012). Hydro Quebec implemented a CVR pilot project in 2005 and found an overall CVR factor of 0.4 (Lefebvre et al. 2008). In 2009, Dominion Virginia Power began a CVR project with feedback from advanced metering infrastructure (AMI), and found a CVR factor of 0.92 (Peskin et al. 2012). In 2010, Consolidated Edison conducted CVR tests on the New York City network and found CVR factors between 0.5 and 1 for active power and between 1.2 and 2 for reactive power (Diaz-Aguilo et al. 2013). Recent reports show that the deployment of CVR on all distribution feeders of the U.S. may provide a 3.04% reduction in the annual national energy consumption (Schneider et al. 2010). As a part of a scheme for voltage/var optimization (VVO), CVR has been integrated into commercially available products such as PCS UtiliData's AdaptiVolt (Wilson and Bell 2004).

The existing methodologies to calculate the CVR factor can be classified into two categories: control group method (Krupa and Asgerisson 1987; Kennedy and Fletcher 1991; Wilson 2010) and regression method (Lauria 1987; Beck 2007). There are two ways

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**Fig. 1.** Illustration of conservation voltage reduction

to perform the control group method. The first is to select two similar feeders during the same day. “Similar” means that the two feeders should have similar configurations, topologies, load conditions, and load mixes, and are geographically close. Voltage reduction is applied to one feeder while normal voltage is applied to the other feeder at the same time. The second control group method is to perform CVR tests on a feeder and apply normal voltage to the same feeder, but during another day with similar weather

conditions. Both methods compare energy consumptions of the test and nontest groups to calculate CVR factors. However, it is difficult to find a satisfactory control group, because there are no two feeders or two days whose operation conditions are exactly the same. The regression method assumes a linear model for the load, with a linear dependence on voltage and an asymmetric linear relationship with ambient temperature, in addition to a stochastic component representing random load behaviors. This method requires long-term “CVR on/CVR off” tests and simulates the CVR on/off loads for each season based on the assumed load model, recorded load consumption, and weather data. The method can only provide a statistical CVR factor for a certain period, such as one season. It is controversial whether this simple linear model can represent complicated load behaviors. To better estimate the load if there is no voltage reduction during the CVR period, nonparametric load models and support vector regression (SVR) are used in assessing the CVR effect (Wang et al. 2014). However, the basic idea is to estimate load consumptions without CVR.

In general, the existing methods have two primary drawbacks:

- None can calculate the CVR factor for any test feeder during any test period; and
- None can provide credible ways to guide the selection of preferred feeders to implement CVR.

All of the previously mentioned methodologies try to estimate the load if there is no voltage reduction during the CVR period, to calculate the CVR factor. However, this is challenging because the load at normal voltage during the CVR period cannot be measured. This paper proposes a new method to analyze CVR effects in a different manner. One key characteristic, which determines the performance of CVR, is the nature of the load. For example, the CVR factor will increase when the voltage dependence of the load changes from a constant power type to a constant impedance type. Based on this fact, this paper estimates the LTV sensitivity to isolate the change of load consumption owing to voltage from other factors. A time-varying exponential load model (TELM) is used to represent the dependence of the load on voltage and other factors. TELM was proposed based on previous studies of exponential load models and their time-varying parameters (Ohyama et al. 1985; Srinivasan and Lafond 1995). This paper applies the modeling method to analyze CVR effects. Based on measurements from metering devices, the recursive least squares (RLS) algorithm is used to identify the proposed load models. After detecting the LTV, the CVR factor can be calculated through the identified model parameters. The proposed method does not require long-term tests and can provide the CVR factors of any feeder during any test period.

CVR effects are subject to different types of uncertainty, depending on load composition, season, time of the day, weather conditions, and human behavior. For a certain feeder, different performances of CVR may be observed at different times; for a certain period, CVR factors may vary from feeder to feeder. Thus, it is also necessary to develop a solid evaluation technique to address the probabilistic nature of CVR effects. In this study, the Kolmogorov-Smirnov (K-S) goodness-of-fit test (Massey 1951) is used to identify the most suitable probability distributions representing CVR factors of different feeders. The proposed method is validated by *OpenDSS* simulation tests and a developed Euclidean distance-based technique. The presented method is applied to field measurements from a utility company and is shown to be effective. The results can be used to aid utilities in rapidly assessing the CVR benefits of candidate feeders before making financial investments for applying CVR.

The organization of the paper is as follows. The first section introduces the basic concepts of CVR, the time-adaptive exponential

load model, and an RLS filter. Next, a RLS-based CVR factor analysis scheme is proposed. The presented method is verified by simulation tests and applied to field measurements from a utility company. K-S tests are run to identify the most suitable distribution to represent CVR effects. The paper concludes by highlighting the major findings.

## Conservation Voltage Reduction and Load Modeling

To identify LTV and calculate CVR factors, it is necessary to model the load as a function of voltage. A substation is composed of thousands of load components, such as lights, monitors, and motors (Choi et al. 2006). Thus, the substation load model discussed in this paper is actually an aggregated model to represent the overall load behaviors of all downstream load components and associated equipment. For CVR analysis, load should be modeled as a function of voltage. Exponential load model is one of the most frequently used models to represent LTV. An exponential recovery load model (ERLM) can be represented as follows (Karlsson and Hill 1994):

$$T_p \dot{P}_r(t) = -P_r(t) + P_0 \left( \frac{V}{V_0} \right)^{\alpha_{ps}} - P_0 \left( \frac{V}{V_0} \right)^{\alpha_{pr}} \quad (1)$$

$$P_d = P_r + P_0 \left( \frac{V}{V_0} \right)^{\alpha_{pr}} \quad (2)$$

$$T_q \dot{Q}_r(t) = -Q_r(t) + Q_0 \left( \frac{V}{V_0} \right)^{\alpha_{qs}} - Q_0 \left( \frac{V}{V_0} \right)^{\alpha_{qr}} \quad (3)$$

$$Q_d = Q_r + Q_0 \left( \frac{V}{V_0} \right)^{\alpha_{qr}} \quad (4)$$

where  $T_p$  and  $T_q$  are the active and reactive load recovery time constants, respectively,  $P_d$  and  $Q_d$  are the active and reactive load demands, respectively,  $P_r$  and  $Q_r$  are the active and reactive recovery load states, respectively,  $P_0$  and  $Q_0$  are the nominal active and reactive power, respectively,  $\alpha_{ps}$  and  $\alpha_{qs}$  are the steady-state active and reactive load-voltage dependences, respectively,  $\alpha_{pr}$  and  $\alpha_{qr}$  are the transient-state active and reactive load-voltage dependences, respectively.

The ERLM uses an exponential recovery process expressed as an input-output relationship between power and voltage to capture the load restoration characteristics. The steady state model has the following form:

$$P_d + jQ_d = P_0 \left( \frac{V}{V_0} \right)^{\alpha_{ps}} + jQ_0 \left( \frac{V}{V_0} \right)^{\alpha_{qs}} \quad (5)$$

Many papers have claimed that the preceding model accurately represents the statics and dynamics of loads (Begovic and Mills 1995). Obviously, load consumption is always changing with time owing to human behaviors, weather conditions, and continuous switching of different kinds of loads on and off, which means the parameters of the load model are not constant. Even for the same circuit, different load models may be found at different times. Based on the preceding analysis, the TELM can be defined as

$$S = P + jQ = P_0(t) \left[ \frac{V(t)}{V_0} \right]^{\alpha_p(t)} + jQ_0(t) \left[ \frac{V(t)}{V_0} \right]^{\alpha_q(t)} \quad (6)$$

where  $P_0(t)$ ,  $Q_0(t)$ ,  $k_p(t)$ , and  $k_q(t)$  are time-varying model parameters that need to be identified.

A CVR factor is defined as the change in load consumptions related to the change in voltage (Warnock and Kirkpatrick 1986; Dabic et al. 2010; Wang et al. 2014; Wang and Wang 2014):

$$\text{CVR}_f = \frac{\% \text{Load Change}}{\% \text{Voltage Reduction}} = \frac{(P_{\text{CVR-off}} - P_{\text{CVR-on}}) / P_{\text{CVR-off}}}{(V_{\text{CVR-off}} - V_{\text{CVR-on}}) / V_{\text{CVR-off}}} \quad (7)$$

where  $P_{\text{CVR-off}}$  and  $P_{\text{CVR-on}}$  are active load consumption without and with CVR, respectively,  $V_{\text{CVR-off}}$  is the normal voltage without CVR,  $V_{\text{CVR-on}}$  is the reduced voltage with CVR.

Using the time-varying exponential load model in Eq. (6), a CVR factor can be calculated as

$$\text{CVR}_f = \frac{1 - \left( \frac{V_{\text{CVR-on}}}{V_{\text{CVR-off}}} \right)^{\alpha_p(t)}}{1 - \left( \frac{V_{\text{CVR-on}}}{V_{\text{CVR-off}}} \right)} \quad (8)$$

As shown, the CVR factor is determined by  $\alpha_p(t)$ , which denotes the degree of the load's dependence on voltage. The next step is to estimate  $\alpha_p(t)$  during a CVR period. According to the TELM model defined in Eq. (6), two parameters need to be identified:  $P_0(t)$  and  $\alpha_p(t)$ . Because both parameters continuously vary with time, a recursive identification is required.

For calculation of CVR factors requires focus on the active part of Eq. (6). One may assume that the measurements of active power and voltage are available as sampled data at the end of every time interval,  $\Delta t$ . Thus, the sets of available measurements collected in the interval  $[t_0, t_0 + k\Delta t]$  are

$$P(k\Delta t) = [P(t_0), P(t_0 + \Delta t), \dots, P(t_0 + k\Delta t)] \quad (9)$$

$$V(k\Delta t) = [V(t_0), V(t_0 + \Delta t), \dots, V(t_0 + k\Delta t)] \quad (10)$$

The active part of Eq. (6) can be linearized as

$$\ln P(t_k) = \ln P_0(t_k) + \alpha_p(t_k) \ln V(t_k) \quad (11)$$

Eq. (11) can be written as

$$P(k) = \varphi(k)^T \theta(k) \quad (12)$$

where  $P(k) = \ln P(t_k)$ ,  $\varphi(k) = \begin{bmatrix} 1 \\ \ln V(t_k) \end{bmatrix}$ , and  $\theta(k) = \begin{bmatrix} \ln P_0(t_k) \\ \alpha_p(t_k) \end{bmatrix}$ .

One may assume that the errors between measured system outputs and estimated model outputs are  $e = [e_0, e_1, \dots, e_N]^T$ , and  $N$  is the number of measurement points, then

$$P = \varphi^T \theta + e \quad (13)$$

The identification procedure tunes model parameters to solve the following problem:

$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \omega_i \varepsilon_i(\theta) \quad (14)$$

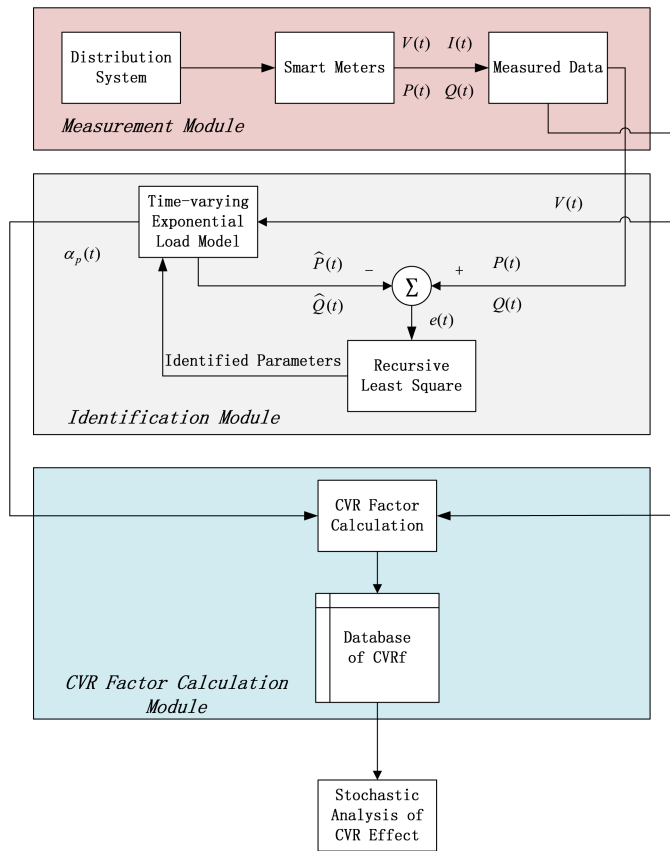
where  $\omega_i$  = weighting factor for the  $i$ th error function  $\varepsilon_i(\theta)$ ,  $\varepsilon_i(\theta) = [P(k) - \hat{P}(k|\theta)]^2$ .

As discussed before, the CVR factor changes with time. To take advantage of the updated information to perform the identification, an RLS algorithm is used. The RLS is given as follows (Begovic and Mills 1995):

$$\hat{\theta}(k+1) = \hat{\theta}(k) + G(k)[P(k) - \varphi^T(k+1)\hat{\theta}(k)] \quad (15)$$

$$G(k) = R(k)\varphi(k+1)[1 + \varphi^T(k+1)R(k)\varphi(k+1)]^{-1} \quad (16)$$

$$R(k+1) = [I - G(k)\varphi^T(k+1)]R(k)/\lambda \quad (17)$$



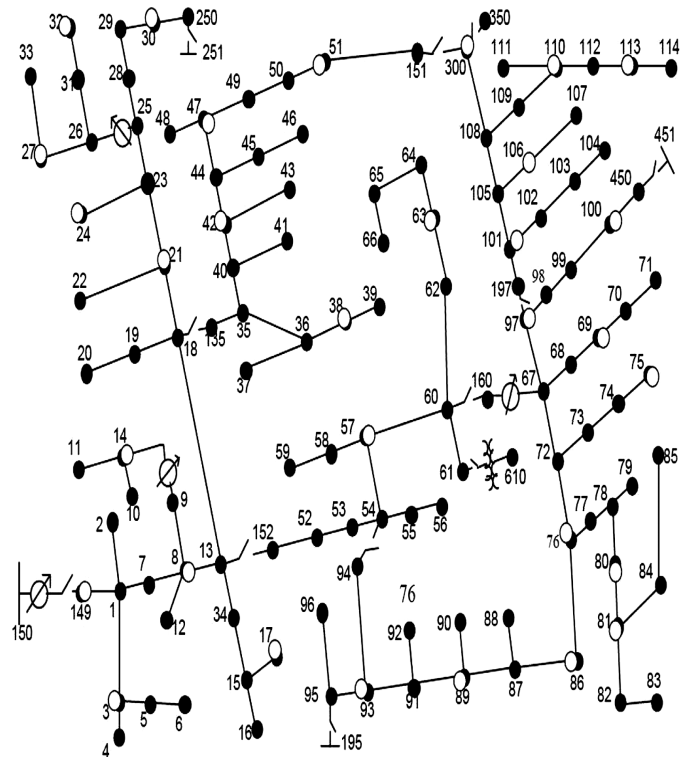
**Fig. 2.** Flowchart of CVR assessment methodology

where  $I$  = identity matrix and  $R$  = initialized as  $R(0) = \text{diag}\{\beta_i\}$ ;  $R$  and  $\beta_i$  are large positive numbers, which amounts to assigning large initial covariances to the unknown parameters,  $\hat{\theta}$ . Given that the CVR factor is time-varying, the algorithm is improved if the present data are given more importance; this is accomplished by introducing a forgetting factor,  $\lambda$ , which applies exponential deweighting,  $\lambda^{N-i}$ , to  $i$  samples of old data in recursion over  $N$  samples. The value of  $\lambda$  should be in the range (0.9–1.0) for the best results.

After  $\alpha_p$  is identified, the CVR factor can be calculated by using Eq. (8). Fig. 2 shows the flowchart of the proposed CVR assessment methodology. Measurement devices are installed at the substation to continuously monitor system operation data, such as voltage and real and reactive power. Measured data from meters are transmitted into the parameter identification module, in which load is modeled as TELM. Because this examination focuses on the active power CVR factor, only the active part of Eq. (6) needs to be identified. The RLS identification algorithm tunes the parameter set  $\theta(k) = [P_0(k), \alpha_p(k)]^T$  to minimize the difference between model output,  $\hat{P}(k)$ , and measured system output,  $P(k)$ . Once the steady stream of load parameter set  $\hat{\theta}(k)$  becomes available from the identification results, the results can be utilized to calculate the CVR factors. The calculated CVR factors are stored in a database for further analysis to determine the statistical law behind CVR effects and aid utilities to select preferable feeders, which will be discussed in the following sections.

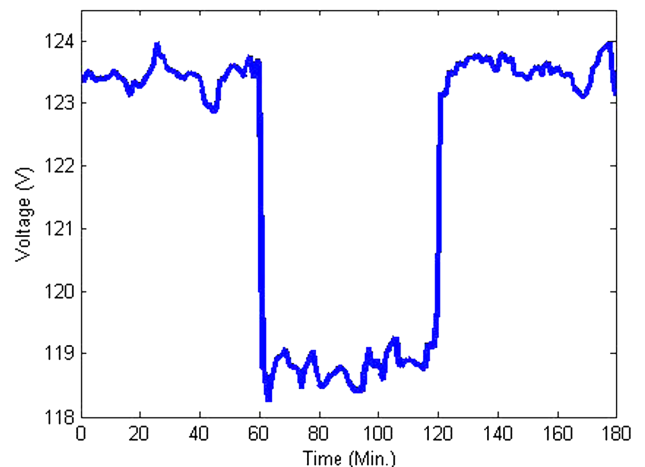
### Simulation Results

To verify the performance of the proposed method in estimating and tracking the LTV, CVR simulation tests are conducted



**Fig. 3.** IEEE 123-node test feeder

by *OpenDSS*. The purposes of the simulation tests are: (1) to test the performance of the proposed method in noise filtering; (2) to test the performance of the proposed method in estimating and tracking the LTV step changes; (3) to test the performance of the proposed method in estimating and tracking the continuous LTV changes. The test is implemented in an Institute of Electrical and Electronics Engineers (IEEE) 123-node distribution test system (Kersting 1991), which is shown in Fig. 3. For simplicity, all load models are set to be the same time-varying exponential model. A 1% Gaussian noise is applied to measurement data. The time step of the simulation is set to be 1 min. Fig. 4 shows the 3-h active power consumption measured at the substation. In the first hour, the system operates at normal voltage level. CVR is applied in the second hour. The voltage is reduced by 4% from the substation transformer. To verify whether the algorithm can track the step changes



**Fig. 4.** Voltage profile in the simulation test

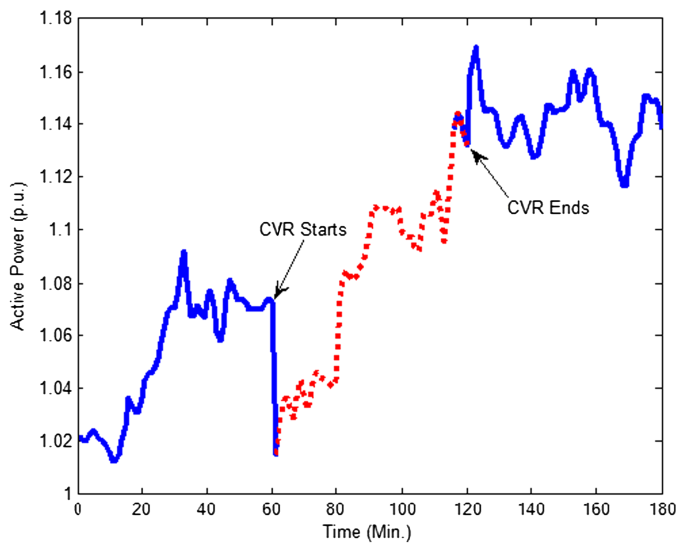


Fig. 5. Active load in the simulation test

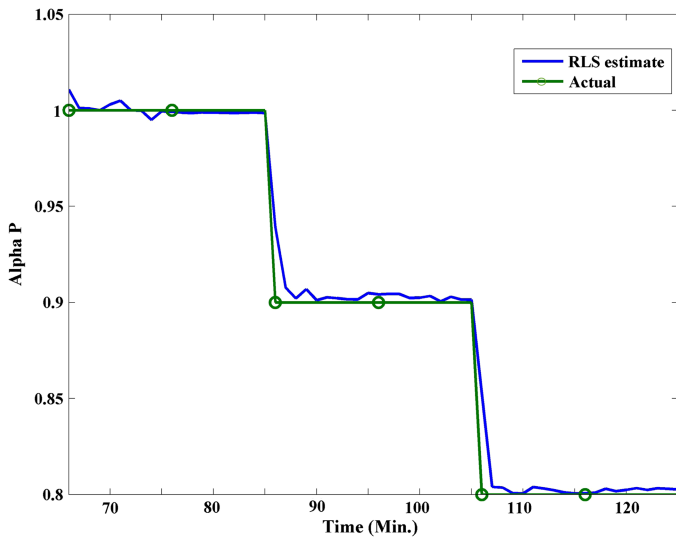


Fig. 6. Actual and estimated  $\alpha_p(t)$

of LTV, the voltage exponent  $\alpha_p(t)$  is changed every 20 min, as shown in Eq. (18). The active power, reactive power, voltage, and current are monitored from the substation transformer in 1-min intervals. Normal voltage is resumed in the third hour:

$$\alpha_p(t) = \begin{cases} 1.0 & \text{for } 60 \leq t \leq 80 \\ 0.9 & \text{for } 80 < t \leq 100 \\ 0.8 & \text{for } 100 < t \leq 120 \end{cases} \quad (18)$$

Fig. 5 shows the active power measured at the substation transformer. The solid line represents the active power consumption at normal voltage level; the dotted line represents the power consumption at reduced voltage level. As shown, the CVR effect tends to be smaller because  $\alpha_p(t)$  decreases during the test period.

The proposed RLS-based method is used to estimate  $\alpha_p(t)$  during the CVR period (60–120 min). The forgetting factor of RLS is set to be 0.95. Fig. 6 shows the identification results. The dotted line represents the actual values of  $\alpha_p(t)$ , which change every 20 min. The solid line represents the RLS identification results.

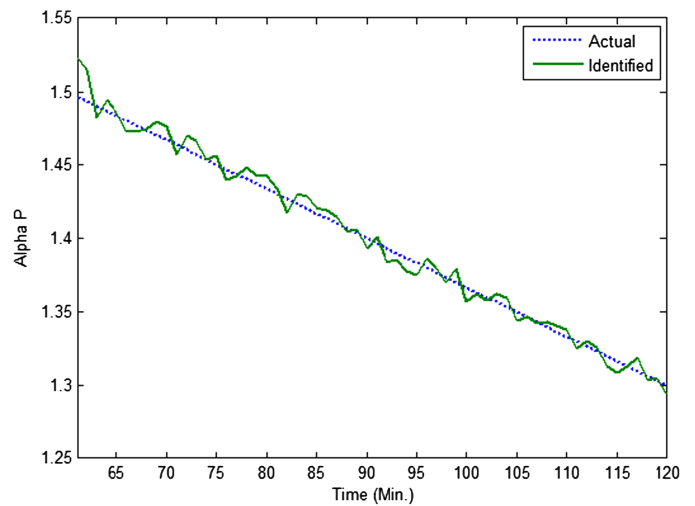


Fig. 7. Actual and estimated  $\alpha_p(t)$

As shown, the estimated  $\alpha_p(t)$  is similar to the actual one, which shows that RLS can accurately identify and track the step changes of the LTV sensitivities. After  $\alpha_p(t)$  is identified, the CVR factor can be calculated by using Eq. (8).

To further test the performance of the proposed method in tracking the continuous changes of LTV, the voltage exponent  $\alpha_p(t)$  is changed continuously from 1.5 to 1.3 during the 1-h period. The identification results are shown in Fig. 7. The dotted line represents the actual values of  $\alpha_p(t)$ , which change continuously. The solid line represents the RLS identification results. The results show that RLS can accurately track the continuous changes of the LTV sensitivities.

## Field Test Results

There are two ways to implement CVR: short-term reduction and long-term reduction. For the short-term reduction, the lower voltage is applied only during peak hours to achieve peak demand reduction; for the long-term reduction, the lower voltage is applied during the whole day. The field tests introduced in this paper are short-term tests; however, it is clear that the proposed technique can be used to quantify the outcomes of both kinds of CVR.

The time-varying exponential load model and RLS algorithm for the estimation of CVR factors have been thoroughly tested by using the CVR test data of a utility company. The utility company is currently conducting a CVR pilot program on five sample substations within its distribution system. The utility uses capacitor, regulator, and LTC controls to limit load losses and provides quality voltage control for all customers, whether they are close to the substation or near the end of the line. This method also provides the capability to reduce the peak kW demand by implementing CVR. Measurement devices, are installed at the substations. The meters can trend kW, kVAR, voltage, and current of the test circuits at 1-min intervals. PQube can detect missing data (labeled as “0”) and bad data (labeled as “F”). Data are transmitted daily via wireless communications. All data labeled as “0” and “F” are removed from the data set.

## Verification of the Proposed RLS Methodology

In the previous section, the proposed RLS method is verified by using simulations. However, the actual LTV is unknown in practice.

To validate the proposed method with the field test data, a Euclidian distance–based comparison method is developed. The challenge in calculating CVR factors is that the load without voltage reduction during the CVR period cannot be measured. The basic idea of the Euclidian distance–based comparison method is to select a load profile from all nontest days so that the profile can approximate the load on the test day if there is no CVR. As shown in Fig. 1, load and voltage profiles of a test day can be divided into three parts: pre-CVR ( $T_1$ ), CVR ( $T_2$ ), and post-CVR ( $T_3$ ). A Euclidian distance–based index for a nontest day,  $k$ , is defined in Eq. (19):

$$\begin{aligned}\varepsilon_{pk} &= \sum_{i \in T_1, T_3}^N \frac{\sqrt{(P_i - P_{ik})^2}}{\max(P_{ik}) \cdot N} \times 100\% \\ \varepsilon_{vk} &= \sum_{i \in T_1, T_3}^N \frac{\sqrt{(V_i - V_{ik})^2}}{\max(V_{ik}) \cdot N} \times 100\%\end{aligned}\quad (19)$$

where  $P_i$  is the load consumption at time  $i$  of the test day,  $P_{ik}$  is the load consumption at time  $i$  of  $k$ th nontest day,  $V_i$  is the voltage at time  $i$  of the test day,  $V_{ik}$  is the voltage at time  $i$  of  $k$ th nontest day,  $T_1$ ,  $T_2$  and  $T_3$  are the pre-CVR, CVR and post-CVR period, respectively.

Thus,  $\varepsilon_p$  and  $\varepsilon_v$  can be used to select a nontest day on which the load and voltage profiles are the most similar to the current profiles under estimation. The CVR factor can be calculated by comparing load and voltage differences of the nontest and test days, as shown in Eq. (20):

$$\text{CVR}_f = \frac{(P_{\text{ed}} - P_{\text{cvr-on}})/P_{\text{ed}}}{(V_{\text{ed}} - V_{\text{cvr-on}})/V_{\text{ed}}}\quad (20)$$

where  $P_{\text{ed}}$  and  $V_{\text{ed}}$  are load and voltage profile, respectively, selected by the Euclidian distance–based method. The calculated CVR factor is compared with that estimated by the proposed RLS-based method for validation.

### Estimation of CVR Factor for an Example Substation

CVR test data of Substation 3 on a summer day are selected as an example. Fig. 8 shows the 24 h (1,440 min) active power and voltage profiles. The solid lines represent load and voltage profiles of the summer test day. CVR starts at 12:35 (755 min) and ends at 17:00 (1,020 min). Power differences,  $\varepsilon_p$ , and voltage differences,  $\varepsilon_v$ , of all nontest days in the same month are calculated and the results are shown in Fig. 9. It is clear that Date 18 should be selected as the control group because the power and voltage differences are the smallest on this day. The dotted lines in Fig. 8 show the load and voltage profiles of the control group.

The active load of the CVR test day shown in Fig. 8 is represented by the time-varying exponential load model. The proposed RLS-based algorithm is applied to estimate the load's sensitivity to voltage and the CVR factor is calculated from  $\alpha(t)$ . The solid line in Fig. 10 shows the CVR factor during the test period (755–1,020 min). As shown, the CVR factor is relatively large at the beginning of the test and continuously decreases during the test period. Because the CVR factor varies from circuit to circuit and always changes with time, recursive estimation is necessary to accurately evaluate CVR effects and benefits; it takes “time” into account. The dotted line in Fig. 10 shows the CVR factor calculated by the Euclidian distance–based comparison method. The proposed RLS-based algorithm can be verified if the two calculated CVR factors are similar.

### Comprehensive Results

Fig. 11 shows a box plot of comprehensive results of Substation 1 in summer (June to August) and winter (December to February). The box plot shows the maximum value, minimum value, median, upper quartile (75% quartile), and lower quartile (25% quartile) of the data. As shown in Fig. 11, the maximum value of the CVR factor in winter is 1.16, the minimum value is 0.78, and values that are larger than 1.16 or smaller than 0.78 are identified as outliers. The upper quartile of the winter CVR factor is 1.02, which means that 75% of the calculated CVR factors are lower than this value.

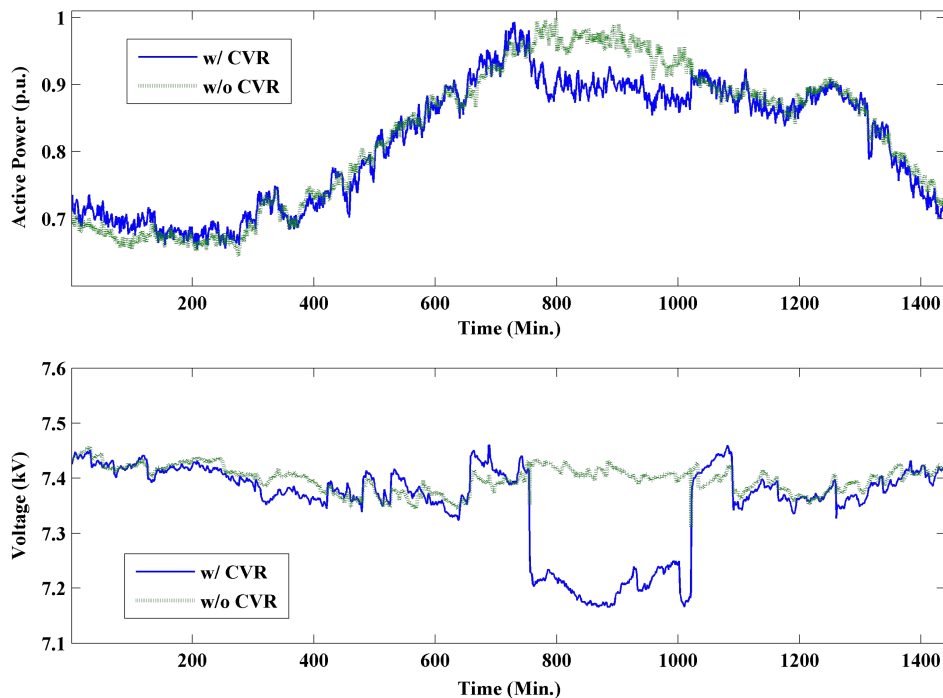
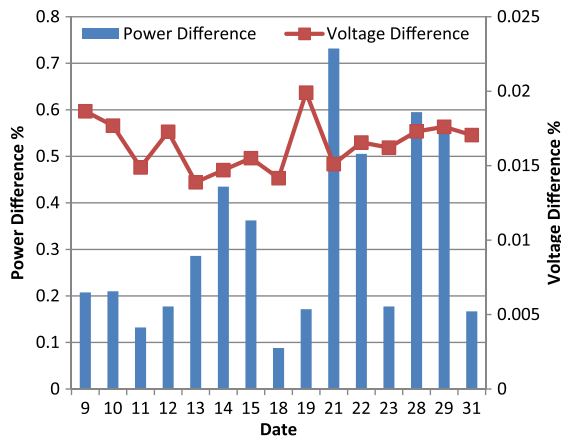
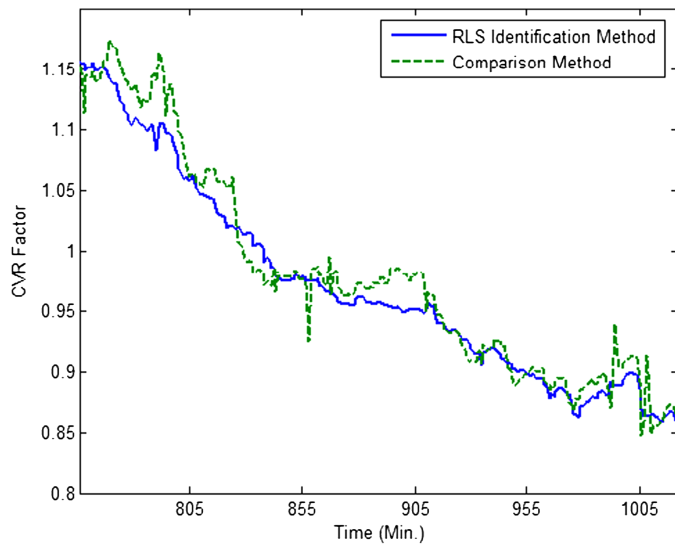


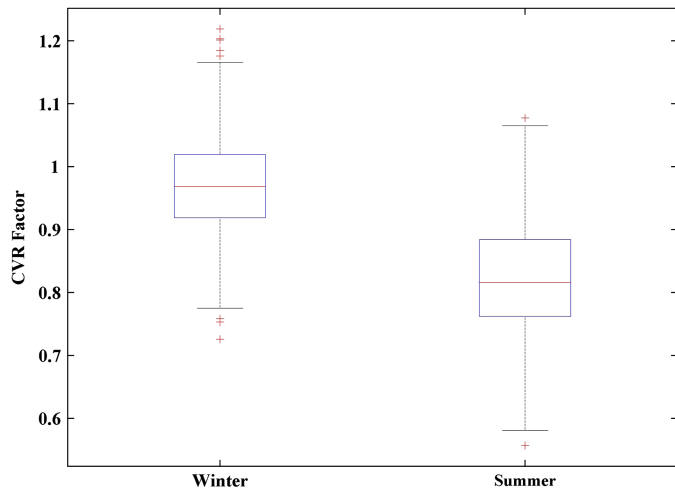
Fig. 8. Load and voltage profiles on test day (with CVR) and nontest day (without CVR)



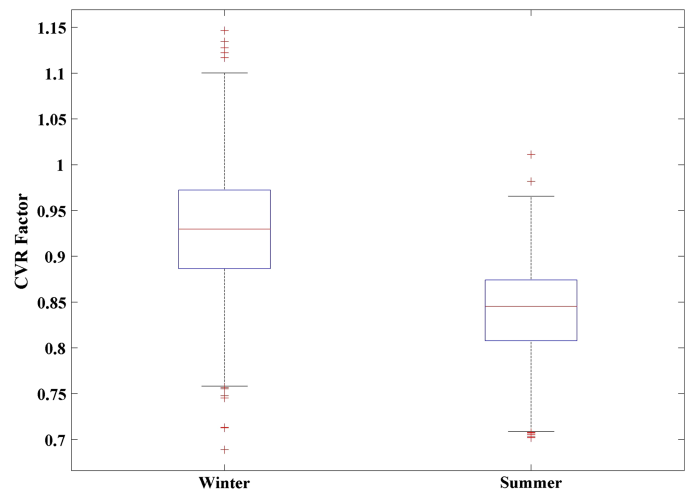
**Fig. 9.** Calculated  $\varepsilon_p$  and  $\varepsilon_v$  on all nontest days in the same winter month



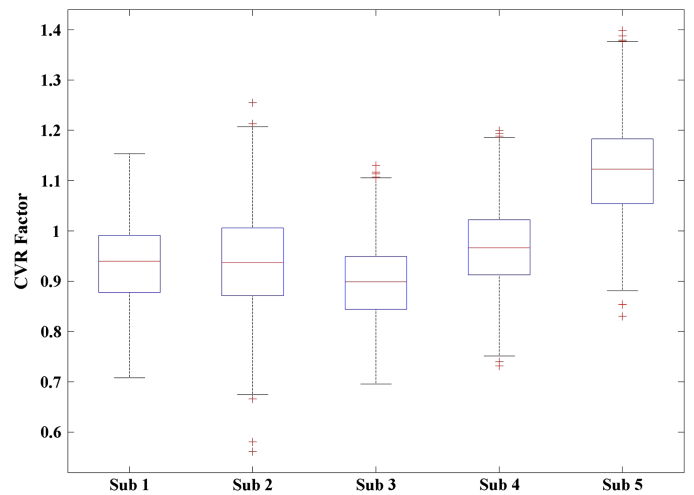
**Fig. 10.** CVR factors of Substation 3 on a summer day



**Fig. 11.** CVR factors of Substation 1



**Fig. 12.** CVR factors of Substation 2



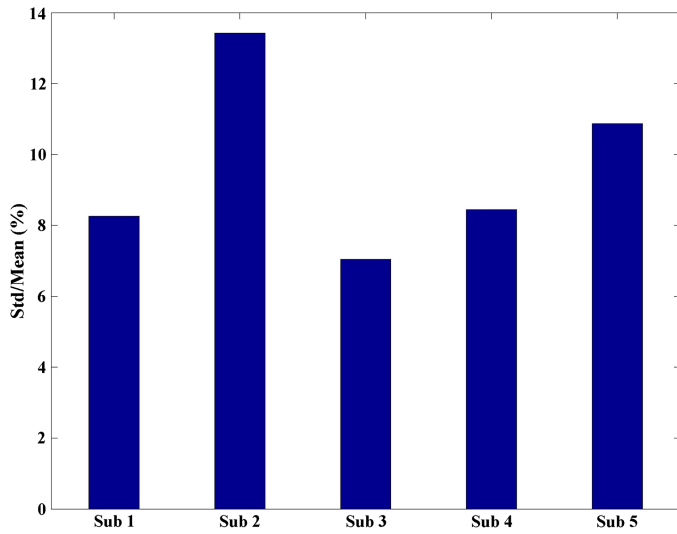
**Fig. 13.** CVR factors of all substations on all test days

Fig. 12 shows the CVR factors of Substation 2. As shown, CVR effects are different from one substation to another, which is a result of the various load behaviors of different substations. CVR factors also change with seasons; it is clear that CVR factors in winter are higher than those in summer, which might be attributable to the large amount of resistive heating loads in winter.

Fig. 13 shows a box plot of CVR factors on all test days from January 2011 to December 2011. Substation 5 has the largest median CVR factor and Substation 3 has the smallest. Substation 2 has the smallest minimum CVR factor and Substation 5 has the largest maximum CVR factor. Because CVR factors may vary greatly from one day to another, preferred CVR feeders cannot be selected by comparing only the statistical median or mean value of CVR factors of each feeder. Fig. 14 shows the SD to mean ratio of CVR factors of all substations shown in Fig. 13. As shown, the fluctuations of CVR factors are relatively large and different from substation to substation. This increases the difficulty of selecting target CVR feeders.

### Stochastic Analysis of CVR Effect

Because of the variability in CVR factors, the CVR effect of each feeder cannot be evaluated deterministically, but can be determined



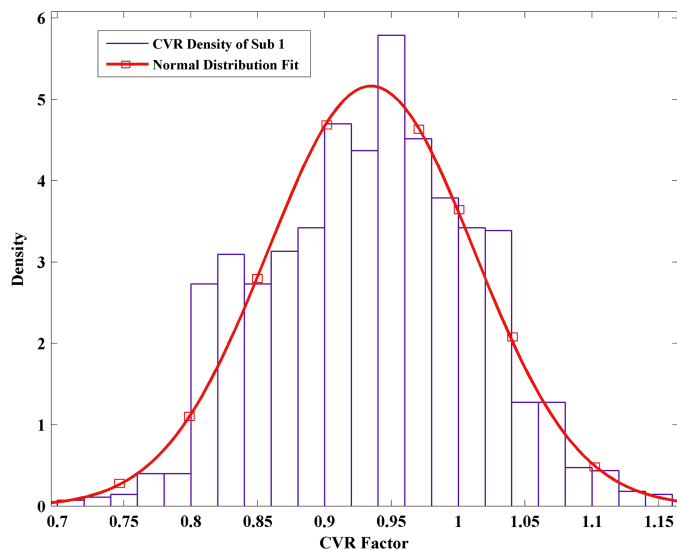
**Fig. 14.** SD to mean ratio of CVR factors of all feeders on all test days

probabilistically. The target CVR feeders can be selected by comparing their probabilistic CVR performances.

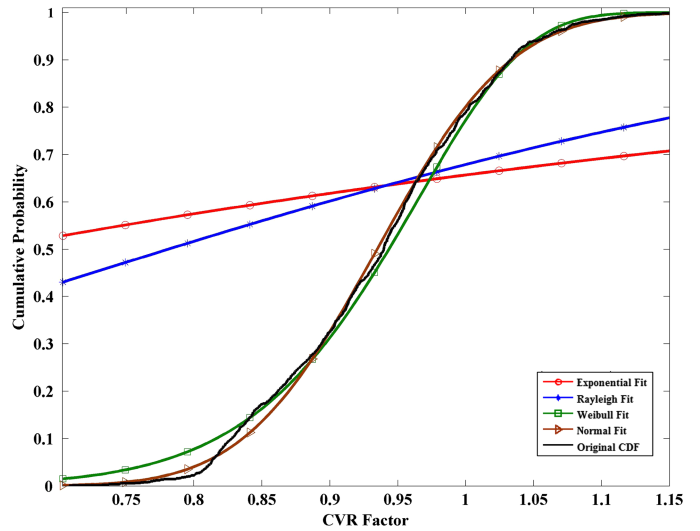
Fig. 15 shows the density of CVR factors of Substation 1. To identify the most suitable probability distribution for CVR factors of each feeder, the K-S goodness-of-fit test has been conducted. The K-S test computes the test error,  $\psi$ , which is the maximum vertical distance between a sample cumulative distribution function (CDF) and a fitted CDF. This error is compared to a critical value,  $\psi_{crit}$ , and any probability distribution fit that satisfies  $\psi \leq \psi_{crit}$  can be accepted. Fig. 16 shows the differences between the CDF of CVR factors of Substation 1 and various other CDFs (normal, gamma, Weibull, Rayleigh, and exponential). It is clear that the normal distribution, as defined in Eq. (21), exhibits the most promising goodness-of-fit:

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (21)$$

where  $\mu$  = mean and  $\sigma$  = SD. Table 1 shows the K-S test errors with normal distribution and the maximum likelihood estimates of



**Fig. 15.** Density of CVR factors of Substation 1



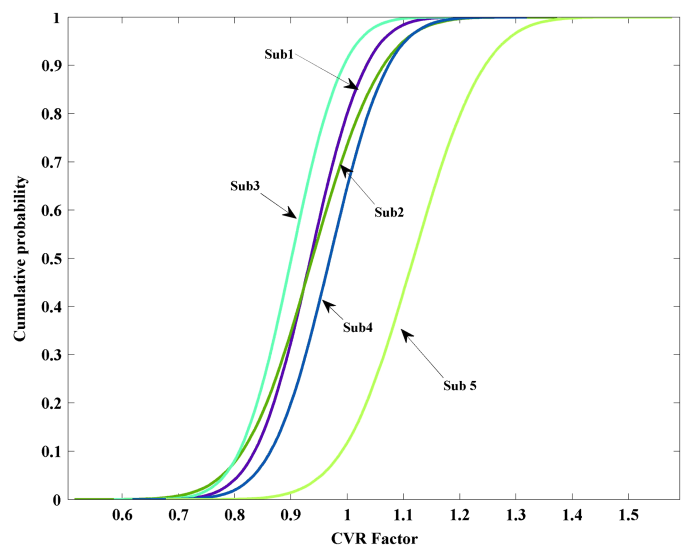
**Fig. 16.** Density of CVR factors of Substation 1

parameters:  $\psi_{crit} = 0.0365$  for the normal distribution fit with a 5% level of significance.

Fig. 17 shows the CDF chart of CVR effects of all feeders. The CDF gives the probability that the variable  $CVR_f$  takes a value less than or equal to some specified value  $CVR_{f-spc}$ . Table 2 summarizes the CVR performances of all feeders. It shows the percentiles, which represent the certainty level of achieving a CVR factor below a particular threshold. In the table,  $CVR_{fmax}$  and  $CVR_{fmin}$  represent maximum and minimum CVR factors at different percentile levels, respectively. For all five test substations, there is zero chance that any of the feeder exhibits a CVR factor less than 0.5157.

**Table 1.** K-S Test Errors and Estimated Parameters

Substation	$\psi$	$\mu$	$\sigma$
1	0.0310	0.9348	0.0773
2	0.0271	0.9386	0.0977
3	0.0273	0.9010	0.0723
4	0.0287	0.9689	0.0818
5	0.0296	1.1174	0.0916



**Fig. 17.** CDF of CVR factors



**Table 2.** Summary of CVR Effects

Percentiles	0%	25%	50%	75%	100%
$CVR_{f\max}$	0.6780	1.0510	1.1151	1.1761	1.4828
$CVR_{f\min}$	0.5157	0.8513	0.9001	0.9497	1.1871

If there are no intersections among the CDF curves, the CDF on the far right of the CDF chart offers the best opportunity for achieving the highest CVR factor at every confidence level, and this substation is the best CVR candidate. In this example, Substation 5 is the best candidate and Substation 3 exhibits the worst CVR performance. If the CDF curves intersect, the best substation is the one that provides the highest CVR factor with the predefined certainty level. For example, if the certainty level is defined to be 90%, then it is clear that Substation 2 is a better candidate than Substation 1.

## Conclusion

CVR works on the principle that load is sensitive to voltage. This paper presents an RLS-based algorithm for identifying LTV sensitivity to calculate CVR factors. A time-varying exponential load model is developed to represent the load's dependences on voltage and other factors. By recursively identifying the LTV, the time-variant CVR effect can be obtained. Simulation tests show that the proposed method can accurately estimate LTV. The accuracy of the proposed RLS-based algorithm is validated by the Euclidian distance-based comparison method. The presented algorithm has been applied to field test data from a utility company, and shown to be an effective and reliable tool for the assessment of CVR effects. It is known from practical test results that CVR performance varies from time to time and from circuits to circuits. When selecting the preferred CVR substations, the variety of CVR effects is taken into account. The K-S test has found that the CVR effects of a substation can be represented by a normal distribution.

Compared with previous techniques to evaluate CVR effects, the proposed method has several notable advantages: first, it can provide CVR factors of any test feeder during any test period; second, it estimates CVR factors by detecting LTV, which does not depend on the selection of control group or assumption of a simple linear relationship between a load and its impact factors; third, it can reveal the time-variant nature of CVR factor; fourth, it does not require a long-term CVR test; finally, it considers the stochastic nature of CVR effects when selecting target CVR substations. The proposed technique can be used to analyze the CVR effects of candidate feeders before making the financial investment of applying CVR.

## Notation

The following symbols are used in this paper:

- $P_{\text{cvr-off}}$  = active load consumption without CVR;
- $P_{\text{cvr-on}}$  = active load consumption with CVR;
- $P_d$  = real load power demand;
- $P_{\text{est}}$  = estimated active load consumption without voltage reduction during CVR period;
- $P_i$  = load consumption at time  $i$  of the test day;
- $P_{ik}$  = load consumption at time  $i$  of  $k$ th nontest day;
- $P_r$  = recovery load state for real power;
- $P_{\text{red}}$  = active load consumption during CVR;
- $P_{\text{pre}}$  = active load consumption before CVR;
- $P_{\text{post}}$  = active load consumption after CVR;
- $P_0$  = nominal real power;

- $P_0(t)$  = time-varying real load component;
- $Q_d$  = reactive load power demand;
- $Q_r$  = recovery load state for reactive power;
- $Q_0$  = nominal reactive power;
- $Q_0(t)$  = time-varying reactive load component;
- $T_p$  = real load recovery time constant;
- $T_q$  = reactive load recovery time constant;
- $T_1$  = pre-CVR period;
- $T_2$  = CVR period;
- $T_3$  = post-CVR period;
- $V_{\text{cvr-off}}$  = normal voltage without CVR;
- $V_{\text{cvr-on}}$  = reduced voltage with CVR;
- $V_i$  = voltage at time  $i$  of the test day;
- $V_{ik}$  = voltage at time  $i$  of  $k$ th nontest day;
- $V_0$  = nominal voltage;
- $\alpha_{ps}$  = steady state real load-voltage dependences;
- $\alpha_{pt}$  = transient state real load-voltage dependences;
- $\alpha_{qs}$  = steady state reactive load-voltage dependences;
- $\alpha_{qt}$  = transient state reactive load-voltage dependences;
- $\alpha_p(t)$  = time varying real load-voltage dependences;
- $\alpha_q(t)$  = time varying reactive load-voltage dependences;
- $\varepsilon_{pk}$  = Euclidian distance-based index for active load of day  $k$ ;  
and
- $\varepsilon_{vk}$  = Euclidian distance-based index for voltage of day  $k$ .

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