Robust Optimization for Transmission Expansion Planning: Minimax Cost vs. Minimax Regret

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***Abstract*—Due to the long planning horizon, transmission expansion planning is typically subjected to a lot of uncertainties including load growth, renewable energy penetration, policy changes, etc. In addition, deregulation of the power industry and pressure from climate change introduced new sources of uncertainties on the generation side of the system. Generation expansion and retirement become highly uncertain as well. Some of the uncertainties do not have probability distributions, making it difficult to use stochastic programming. Techniques like robust optimization that do not require a probability distribution became desirable. To address these challenges, we study two optimization criteria for the transmission expansion planning problem under the robust optimization paradigm, where the maximum cost and maximum regret of the expansion plan over all uncertainties are minimized, respectively. With these models, our objective is to make planning decisions that are robust against all scenarios. We use a two-layer algorithm to solve the resulting tri-level optimization problems. Then, in our case studies, we compare the performance of the minimax cost approach and the minimax regret approach under different characterizations of uncertainties.**

***Index Terms*—Generation retirement, load growth, minimax cost, minimax regret, robust optimization, transmission expansion planning.**

NOMENCLATURE

Sets and Indices:

$\mathcal{U}$	Polyhedron uncertainty set of demand and new generation capacity profile.
$\mathcal{I}$	Set of nodes.
$\mathcal{L}$	Set of existing transmission lines.
$\mathcal{N}$	Set of candidate transmission lines.
$\mathcal{T}$	Set of years in the planning horizon.
$\mathcal{M}$	Set of load blocks.
$\mathcal{K}$	Set of technology types.

Parameters:

$P_{i,k,t}$	Capacity of existing generator of technology k at node i at time t.
$c^{\mathrm{T}}_{ij,t}$	Cost of building the new transmission line ij at time t.
$c^{\mathrm{L}}_{i,t}$	Cost of load curtailment at node i at time t.
$c^{\mathrm{P}}_{i,k,t}$	Cost of power production of technology k at node i.
$f^{\mathrm{C}}_{k,t,m}$	Average capacity factor of generation technology k at year t load block m.
$F^{\max}_{ij}$	Maximum power flow on transmission line ij.
B_{ij}	Susceptance of transmission line ij.
M	Big constant used to linearize the power flow constraint.
$\bar{d}_{i,t,m}$	Average amount of demand at year t load block m at node i.
$\bar{P}^{\mathrm{N}}_{i,k,t}$	Average amount of generation expansion of technology k at node i at time t.
$\theta_{\min}$	Lower bound of voltage angles.
$\theta_{\max}$	Upper bound of voltage angles.
λ	Market interest rate (inflation included).
$P^{\mathrm{N,min}}_{i,k,t}$	Minimum amount of new generation at a node
$P^{\mathrm{N,max}}_{i,k,t}$	Maximum amount of new generation at a node.
$P^{\mathrm{N,min}}$	Lower bound on the total amount of generation at all the nodes.
$P^{\mathrm{N,max}}$	Upper bound on the total amount of generation at all the nodes.

Decision Variables:

x_{ij}	Binary variables indicating whether a transmission line is built.
$P^{\mathrm{N}}_{i,k,t}$	Amount of new generation capacity of technology k at node i at time t.
$P^{\mathrm{N}}_{i,k,t}$	Negative in the case of power plant retirement.

Manuscript received November 05, 2013; revised February 07, 2014; accepted March 21, 2014. Date of publication April 08, 2014; date of current version October 16, 2014. B. Chen and L. Wang are supported in part by the Power Systems Engineering Research Center and the Electric Power Research Center at Iowa State University. The work of J. Wang and part of the work by B. Chen was supported by the U.S. Department of Energy Office of Electricity Delivery and Energy Reliability. Without implication, all errors are the authors'. Paper no. TPWRS-01416-2013.

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Digital Object Identifier 10.1109/TPWRS.2014.2313841

$f_{ij,t,m}$ Power flow from node i to node j at year t load block m.

$p_{i,k,t,m}$ Power production of technology k at node i at year t load block m.

$r_{i,t,m}$ Amount of load shedding at node i at year t load block m.

$\theta_{i,t,m}$ Voltage angle at node i at year t load block m.

$d_{i,t,m}$ Demand at year t load block m at node i.

I. Introduction

Transmission expansion planning is very challenging due to various uncertainties planners need to consider. Besides typical high-frequency uncertainties including demand variations and renewable energy intermittency faced by system operators in short-term scheduling, planners also need to take into consideration low-frequency uncertain events like policy changes, technological advancements, natural disasters, etc. [1]. Such uncertainties cannot be characterized by probability distributions. For example, to cope with challenges of climate change, the power generation industry is facing increasing pressure to reduce greenhouse gas emissions. In addition, a large amount of coal plants are anticipated to retire in response to government regulations and fuel price changes [2], replaced partially by gas-fired plants. Many of such retirements could be announced on relative short notice and unexpected by the system operator. Moreover, after the deregulation of the power system, strategic behavior of generation companies in generation expansion becomes an uncertain factor as well.

Currently, the most common practices in dealing with uncertainties in optimization include stochastic programming and robust optimization. In stochastic programming, scenarios are generated based on a certain probability distribution of the uncertain data. The weighted sum of the total cost under different scenarios is usually optimized. Stochastic programming has been successfully applied to power system capacity expansion planning problems. In [3]–[6], uncertainties including load prediction inaccuracies, transmission line and generator outages, generation and transmission line capacity factors are considered. All of those uncertainties can be described by probability distributions and can be effectively modeled with stochastic programming techniques. However, most of those works only focus on uncertainties in the operation phase and do not address uncertainties in the planning phase. The reason is that it is difficult to obtain probability distributions of the non-random [7] or epistemic uncertainties caused by factors including policy changes, investment behavior of market players, etc. In this paper, besides load uncertainty, we also take into consideration uncertain generation expansion behavior of generating companies and coal power plants retirement.

As an alternative tool to address uncertainties, robust optimization [8]–[10] can avoid some of the difficulties arising from stochastic programming approaches. With robust optimization, uncertainty is described by parametric sets, which contain an infinite number of scenarios. Such uncertainty sets can be constructed with information like the lower and upper bounds of a random variable, which are much easier to derive than probability distributions. This approach can identify a set of decisions that is robust under the worst-case scenario contained in the uncertainty sets, which is desirable in planning problems where reliability is important. The conservativeness of the solution can be adjusted by changing the uncertainty sets [11], depending on how much uncertainty is desired to capture. Robust optimization has been applied to many problems in power systems. In [12], robust unit commitment with the $n-k$ security criterion is studied. The problem is then reformulated into a single-level problem with the help of dual variables. In [13], [14], the two-stage robust unit commitment problems under uncertainty are studied. Both papers propose to use Bender's decomposition to solve the problem. A similar model is used in [15] where demand response is considered. A robust minimax regret model is proposed in [16] to solve the unit commitment problem under uncertainty. However, all those works study operation problems. In long term power system planning problems where more uncertainties need to be considered, application of robust optimization is limited. In [17], a robust transmission expansion planning model is proposed considering load and generation uncertainty. Bender's decomposition is also used.

In this paper, we propose two robust optimization models to address two main sources of uncertainty: load and generation expansion behavior of generating companies. Two criteria, minimax cost (MMC) and minimax regret (MMR), are used as the objective of our models. The MMC criterion has been used widely in robust optimization applications [13]. The MMR criterion is considered in [16] for the unit commitment problem. In comparison with the MMC criterion, it is concluded that MMR outperforms MMC for certain unit commitment problems. However, the same conclusion may not apply to transmission expansion planning problems due to the different structures of such problems. In [18], regret is considered as one of the objectives in a multi-objective optimization framework. It is applied to handle non-random uncertainties in [19] and [20]. Both criteria use the performance of a decision under the worst possible scenario as the objective for optimization, but their main difference is how the "worst scenario" is defined. The MMC criterion focuses on the cost associated with a decision under a scenario, so the scenario that results in the highest cost is identified as the worst scenario. On the other hand, the MMR criterion defines the worst scenario as the one that leads to the highest regret for the decision maker. For a given decision d^0 and a given scenario s^0, the regret is the highest potential cost savings had the decision maker known that scenario s^0 would occur and made a decision accordingly. More rigorously

$$R(d^0, s^0) = C(d^0, s^0) - \min_{d \in \mathcal{D}} C(d, s^0)$$

where $C(d^0, s^0)$ is the cost associated with d^0 and s^0, $\mathcal{D}$ is the set of all feasible decisions, and $R(d^0, s^0)$ is the regret associated with d^0 and s^0. Using these notations, the MMC and MMR criteria can be respectively formulated as

$$\min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} C(d, s) \right\} \text{ and } \min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} R(d, s) \right\} = \min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} \left\{ C(d, s) - \min_{d' \in \mathcal{D}} C(d', s) \right\} \right\}.$$

We use two simple examples in Tables I and II to demonstrate the differences between MMC and MMR. In the first example,

TABLE I
MOTIVATING EXAMPLE FOR THE MINIMAX REGRET MODEL

Cost / Regret	Decision D_1	Decision D_2
Scenario S_1	\$9 / \$1	\$8 / \$0
Scenario S_2	\$2 / \$0	\$7 / \$5

TABLE II
MOTIVATING EXAMPLE FOR THE MINIMAX COST MODEL

Cost / Regret	Decision D_3	Decision D_4	Decision D_5
Scenario S_3	\$4 / \$0	\$16 / \$12	\$18 / \$14
Scenario S_4	\$40 / \$34	\$19 / \$13	\$6 / \$0

under MMC, D_2 is the optimal decision because its worst scenario cost, \$8, is lower than that of D_1, \$9. Under MMR, D_1 is the optimal decision because its worst scenario regret, \$1, is lower than that of D_2, \$5. The argument for MMR is that since scenario S_1 is a "bad" scenario anyway because D_1 and D_2 both lead to higher costs in S_1 than S_2, the difference between the costs associated with the two decisions, which is the regret, may provide more information for decision making than the absolute value of the cost itself. In the second example, decision D_3 is obviously a bad choice because of its high cost in scenario S_4. Decision D_5 will be selected under MMC because its worst cost, \$18, is lower than that of D_3, \$40, and D_4, \$19. Under MMR, decision D_4 will be selected since its worst-case regret is \$13 while the regret of D_5 is \$14. We argue the MMC solution D_5 is better in this example because it is only slightly worse than the MMR decision D_4 in terms of regret in scenario S_3 only because of the existence of decision D_3, which cannot be selected anyway, but has a much lower cost in scenario S_4.

From the previous two examples, we can see that there are no clear cut answers as to which criterion is superior. Each of them has advantages and disadvantages. The examples above can shed some light on which criterion may perform better in what situations. In the first example, both decisions perform better in one scenario and worse in the other. In this case, it makes sense to use MMR as the criterion because the MMC criterion is too conservative and does not consider non-extreme scenarios. On the other hand, in the second example, there exists a very risky decision that performs well under one scenario and extremely poorly under the other. In a planning problem where risks should be controlled, such decisions are usually not desirable, but they may affect the maximum regret of other decisions and distort the final decision.

The two-stage structure of our robust optimization models can capture both the planning and operation stages of the transmission expansion planning problem very well. They can be formulated as special cases of trilevel optimization problems. However, due to their non-linear, non-convex structure, they are very difficult to solve. In previous researches [13], [14], the authors use Bender's decomposition to reformulate the problem into a master problem and a bilinear subproblem, which is then solved with outer approximation. However, the outer approximation approach cannot handle the binary variables in the subproblem when the MMR criterion is used. In [16], statistical upper bounds are used to complement the outer approximation approach. We propose a two-layer algorithm where we decompose our problem into a master problem and a bilevel subproblem. The master problem is updated with a branch and cut type procedure, where new constraints and variables are iteratively generated and then solved as a mixed-integer program. This algorithm is a special case of the bilevel optimization algorithm [21]. Similar algorithms are proposed in [15] and [22]. It works faster than the traditional Bender's decomposition approach with the use of primal information instead of dual variables. The subproblem is a mixed-integer bilevel optimization problem, which is more difficult to solve. In [23], the difficulty of solving a bilevel linear optimization program is discussed and several heuristics are proposed. We use the Karush-Kuhn-Tucker (KKT) conditions [24] to reformulate the bilevel problem into a single level problem with complementarity constraints, which is then reformulated into a mixed-integer programming problem [25].

The contribution of this paper can be summarized as follows. Firstly, we propose two robust optimization models that use two criteria to assess decisions. Both load uncertainties and generation expansion uncertainties are considered. Then, effective algorithms are proposed to solve the resulting trilevel optimization problems. Finally, in our case studies, the two models and their corresponding algorithms are tested with a modified IEEE 118-bus test system. We then analyze the results and compare the performances of the expansion plans under different scenarios.

The rest of the paper is structured as follows. Section III presents the mathematical formulation of the transmission expansion planning problems. In Section IV, we present the trilevel optimization algorithm. Numerical results are presented in Section V. Finally, Section VI concludes this paper.

II. MODEL FORMULATION

Transmission expansion planning problems are usually modeled as two-stage problems to account for the long planning horizon, where the expansion planning decisions are made in the first stage when there is limited information on uncertain parameters and the operational decisions are made in the second stage after uncertainty realizations are observed. In this section, we first present the deterministic model, and then introduce two robust optimization models under the MMC and MMR criteria.

A. Deterministic Model

In the deterministic model, consideration of uncertainty is avoided by assuming perfect information for all parameters. For example, in the following deterministic model, the load is fixed as $\bar{d}$ and the new generation capacity is fixed as $\bar{P}^{\mathrm{N}}$:

$$\min \sum_{ij,t} c^{\mathrm{T}}_{ij,t} x_{ij} + \sum_{i,k,t,m} (1+\lambda)^t \left(c^{\mathrm{P}}_{i,k,t} p_{i,k,t,m} + c^{\mathrm{L}}_{i,t} r_{i,t,m} \right) \quad (1)$$

$$\text{s.t.} \sum_k p_{i,k,t,m} + \sum_j f_{ji,t,m} - \sum_j f_{ij,t,m} = \bar{d}_{i,t,m} - r_{i,t,m}, \quad \forall\, i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \quad (2)$$

$$f_{ij,t,m} - B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}) - (1 - x_{ij})M \le 0, \quad \forall\, ij \in \mathcal{N} \quad (3)$$

$$B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}) - f_{ij,t,m} - (1 - x_{ij})M \le 0, \quad \forall\, ij \in \mathcal{N} \quad (4)$$

$$f_{ij,t,m} = B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}), \forall\, ij \in \mathcal{L} \quad (5)$$
$$f_{ij,t,m} \leq F_{ij}^{\max} x_{ij}, \forall ij \in \mathcal{N} \quad (6)$$
$$- f_{ij,t,m} \leq F_{ij}^{\max} x_{ij}, \forall ij \in \mathcal{N} \quad (7)$$
$$f_{ij,t,m} \leq F_{ij}^{\max}, \forall ij \in \mathcal{L} \quad (8)$$
$$- f_{ij,t,m} \leq F_{ij}^{\max}, \forall ij \in \mathcal{L} \quad (9)$$
$$p_{i,k,t,m} \leq f_{k,t,m}^{\mathrm{C}} \left(P_{i,k,t} + \bar{P}_{i,k,t}^{\mathrm{N}}\right), \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M}, k \in \mathcal{K} \quad (10)$$
$$\theta_{\min} \leq \theta_{i,t,m} \leq \theta_{\max}, \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \quad (11)$$
$$x \text{ binary}. \quad (12)$$

The objective function (1) is the transmission capital investment cost and total operational cost (including cost of power production and load shedding) over the planning horizon. This model is a static model, in which the total operational cost over the planning horizon is estimated by extrapolating from $|\mathcal{T}|$ years. A similar approach has been used by several other related studies [4], [6], [26]. Constraint (2) requires that the net influx at a node should be equal to the net outflow. Constraints (3) and (4) are equivalent to the equation $f_{ij,t,m} = x_{ij} B_{ij}(\theta_{i,t,m} - \theta_{j,t,m})$, which is nonlinear and complicates the model. We introduce the constant M to linearize this equation [26]. When $x_{ij} = 1$, then the two constraints are reduced to $f_{ij,t,m} = B_{ij}(\theta_{i,t,m} - \theta_{j,t,m})$, where the value of M does not matter. When $x_{ij} = 0$, then we need to make sure that M is large enough so that no additional constraints are imposed. On the other hand, if M is too large, it may cause computational difficulties. In our experiments, we set it to be ten times the largest value of $F_{ij}^{\max}$. Equation (5) calculates the power flow on existing transmission lines. Constraints (6)–(9) dictate that the power flow on transmission lines does not exceed their limits. Constraint (10) specifies the generation capacity on each node. Constraint (11) limits the range of phase angles at a node.

To facilitate algorithmic development and simplify the notations, we abstract the deterministic model as follows:

$$\min_{x,z} \quad c^\top x + b^\top z \quad (13)$$
$$\text{s.t.} \quad Ax + C_1 z \leq g_1 \quad (14)$$
$$B\bar{y} + C_2 z \leq g_2 \quad (15)$$
$$Jz = \bar{d}. \quad (16)$$

In this more concise abstract formulation, we use x to represent the binary variable indicating whether or not a transmission line should be built, y to represent the amount of new generation and z to represent operational variables including power production, phase angles, power flow and load curtailment. Vectors c and b represent coefficients of variables in the objective function. Matrices A, B, C_1, C_2, J are the coefficients of variables in the constraints. Vectors g_1, g_2 are the right-hand-side parameters in the constraints. Constraint (14) corresponds to (3)–(11). Constraint (15) corresponds to (10). Constraint (16) corresponds to (2).

B. MMC Model

In the two-stage MMC model, given the first-stage decisions, the second-stage problem is commonly known as the recourse problem [27], where the optimal operation decisions are identified. The feasible set of the recourse problem is defined as follows:

$$\mathcal{Z}(x,d,y) = \{z : C_1 z \leq g_1 - Ax, C_2 z \leq g_2 - By, Jz = d\}.$$

The uncertainty set is defined as

$$\mathcal{U} = \{(d,y) : Q_1 d \leq q_1, Q_2 y \leq q_2\}.$$

The matrices Q_1, Q_2 are the coefficients of d and y in the uncertainty set. Vectors q_1, q_2 are the right-hand-side parameters. They can contain information including the lower and upper bounds of the uncertain parameters, the lower and upper bounds of the linear combination of the uncertain parameters, etc. Such information can be obtained from historical data or statistical tests on historical data. In this paper we consider both the uncertainty caused by load forecast and the uncertainty caused by future generation expansion. In addition, other types of uncertainties can be easily plugged into the model without affecting the algorithm.

The MMC model can be formulated as

$$\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \min_{z\in\mathcal{Z}(x,d,y)} b^\top z \right\}. \quad (17)$$

C. MMR Model

Unlike the MMC model, the MMR model aims to minimize the worst-case regret under all possible scenarios. Before presenting the MMR model, we first define the feasible set of the perfect information solution $\mathcal{G}(d,y)$ and the perfect information cost $G(d,y)$ as follows:

$$\mathcal{G}(d,y) = \{(\hat{x},\hat{z}) : A\hat{x} + C_1\hat{z} \leq g_1, C_2\hat{z} \leq g_2 - By, J\hat{z} = d\}.$$
$$G(d,y) = \min_{\hat{x},\hat{z}\in\mathcal{G}(d,y)} \left\{c^\top \hat{x} + b^\top \hat{z}\right\}.$$

We can see that $G(d,y)$ is only dependent on the uncertain parameters (d,y) and can only be known after the uncertainty realizations are observed. We call it the perfect information solution because $G(d,y)$ can only be achieved if perfect information about the uncertainties is available.

Then we can define the MMR model as follows:

$$\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - G(d,y) \right\} \right\}. \quad (18)$$

Comparing the MMC model and MMR model side by side, their similarities are very noticeable. The difference between them lies in their definition of the "worst-case scenario". With the MMC criterion, the worst-case scenario is defined as the scenario with the highest cost, while the MMR criterion defines the worst-case scenario where regret is the highest.

To shed more light on which criterion is more appropriate under different situations, we can classify scenarios into two categories: regretful vs. regretless. We use x^{C} to denote the MMC solution and x^{R} to denote the MMR solution. $\mathcal{R}(x,s)$ is the regret of decision x under scenario s. If $\mathcal{R}(x^{\mathrm{C}},s) \geq \mathcal{R}(x^{\mathrm{R}},s)$, then we call scenario s a regretful scenario for decision x^{C}. Otherwise, we say it is regretless. When MMR is used, regret is redistributed among the scenarios. The regretful ones become less regretful and the costs in the regretless scenarios increase. As an uncertainty set consists of both regretful scenarios and regretless scenarios, it is unpractical to predict accurately which type of scenario will occur in the future. However, the classification of scenarios compares and illustrates the advantages of the MMR and MMC approaches for decision-makers to choose the criterion more appropriately for their specific problem. For example, if they are more confident that regretful scenarios will occur, they can choose the MMR criterion. Otherwise, they may choose the MMC criterion.

III. Algorithm Development

In this section, we develop a new trilevel optimization algorithm to solve the two robust optimization problems, which we decompose into two levels: the master problem and the sub-problem. We first present the algorithm for the MMC model. This algorithm is then modified for the MMR model. Since we use a cutting plane procedure that does not require duality information, we can reformulate the sub-problem as a mixed-integer linear programming problem. In [16], the worst-case scenarios are identified via statistical upper bounds with Monte Carlo simulation. In contrast, our algorithm provides a theoretical global optimality guarantee to find the worst-case scenarios as the entire problem is solved as a mixed-integer linear programming problem after reformulating the sub-problem.

A. Master Problem for the MMC Model

The master problem is designed to provide a relaxation of the MMC model (17), in which the search for the worst-case scenario is restricted to be within a given finite set of scenarios, $\mathbf{\Omega^C} = \{(d^i, y^i), \forall i = 1, \ldots, |\mathbf{\Omega^C}|\}$, rather than the complete set of scenarios, $\mathcal{U}$. As such, the master problem yields a lower bound of the MMC model (17). We denote the master problem as $\mathbf{M}^{\mathrm{C}}(\mathbf{\Omega^C})$, and it is formulated as the following single level mixed integer linear program:

$$\min_{x,\xi,z^i} \quad c^\top x + \xi \tag{19}$$

$$\text{s.t.} \quad \xi \geq b^\top z^i \quad \forall i = 1, \ldots, |\mathbf{\Omega^C}| \tag{20}$$

$$Ax + C_1 z^i \leq g_1 \quad \forall i = 1, \ldots, |\mathbf{\Omega^C}| \tag{21}$$

$$By^i + C_2 z^i \leq g_2 \quad \forall i = 1, \ldots, |\mathbf{\Omega^C}| \tag{22}$$

$$Jz^i = d^i \quad \forall i = 1, \ldots, |\mathbf{\Omega^C}| \tag{23}$$

$$x \text{ binary}. \tag{24}$$

B. Subproblem for the MMC Model

The subproblem is defined as the MMC model (17) with a given first-stage decision, x. As such, the subproblem yields an upper bound of the MMC model (17). We denote the sub-problem as $\mathbf{S}^{\mathrm{C}}(x)$, and it is formulated as the following bilevel linear program:

$$\max_{(d,y)\in\mathcal{U}} \min_{z\in\mathcal{Z}(x,d,y)} b^\top z. \tag{25}$$

This model can be further reformulated as the following linear program with complementarity constraints (LPCC):

$$\max_{d,y,z,\alpha,\beta,\gamma} \quad b^\top z \tag{26}$$

$$\text{s.t.} \quad Q_1 d \leq q_1 \tag{27}$$

$$Q_2 y \leq q_2 \tag{28}$$

$$0 \leq g_1 - Ax - C_1 z \perp \alpha \geq 0 \tag{29}$$

$$0 \leq g_2 - By - C_2 z \perp \beta \geq 0 \tag{30}$$

$$Jz = d \tag{31}$$

$$C_1^\top \alpha + C_2^\top \beta + J^\top \gamma + b = 0. \tag{32}$$

LPCC problems can be solved by several algorithms ([25] Branch-and-Bound, Bender's, Big-M). The big-M approach [25] was found to be one of the most computationally efficient in our computational experiments. This approach reformulates (26)–(32) as the following mixed-integer linear program (MILP):

$$\max_{d,y,z,\alpha,\beta,\gamma,\omega} \quad b^\top z \tag{33}$$

$$\text{s.t.} \quad \text{Constraints } (27), (28), (31), (32) \tag{34}$$

$$0 \leq g1 - Ax - C_1 z \leq M\omega_1 \tag{35}$$

$$0 \leq \alpha \leq M(1 - \omega_1) \tag{36}$$

$$0 \leq g_2 - By - C_2 z \leq M\omega_2 \tag{37}$$

$$0 \leq \beta \leq M(1 - \omega_2). \tag{38}$$

Here, M is a sufficiently large constant (big-M) and ω_1 and ω_2 are auxiliary binary variables that are introduced to enforce the complementarity conditions in (29) and (30).

C. Algorithm for the MMC Model

The proposed algorithm for the MMC model, which we call $\mathrm{Alg}^{\mathrm{MMC}}$, is an iterative one, in which the master problem is solved to provide an increasing series of lower bound solutions, and then the subproblem is solved to provide a series of decreasing upper bound solutions using the solution from the master problem, x, as an input. The input for the master problem, $\mathbf{\Omega^C}$, is iteratively enriched by the solutions from the subproblem until the gap between the lower and upper bounds falls below a tolerance, ϵ. Detailed steps of this algorithm are described as follows:

$\mathbf{Alg^{MMC}}(\boldsymbol{c, b, A, B, C_1, C_2, g_1, g_2, J, Q_1, q_1, Q_2, q_2})$

Step 0: Initialization. Create $\mathbf{\Omega^C}$ that contains at least one selected scenario. Set $LB = -\infty$, $UB = \infty$, and $k = 1$. Go to Step 1.
Step 1: Update $k \leftarrow k + 1$. Solve the master problem $\mathbf{M}^{\mathrm{C}}(\mathbf{\Omega^C})$ and let (x^k, ξ^k) denote its optimal solution. Update the lower bound as $LB \leftarrow c^\top x^k + \xi^k$ and go to Step 2.
Step 2: Solve the sub-problem $\mathbf{S}^{\mathrm{C}}(x^k)$ and let (d^k, y^k, z^k) denote its optimal solution. Update $\mathbf{\Omega^C} \leftarrow \mathbf{\Omega^C} \cup \{(d^k, y^k)\}$, and $UB \leftarrow c^\top x^k + b^\top z^k$.
Step 3: If $UB - LB > \epsilon$, go to Step 1; otherwise return (x^k, d^k, y^k, z^k) as the optimal solution to (17) and LB as the optimal value.

D. Algorithm for the MMR Model

The MMR model can be solved using a similar algorithmic framework to Alg^{MMC} after the following simplifying yet equivalent reformulation:

$$\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - G(d,y) \right\}\right\}$$
$$= \min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - \min_{(\hat{x},\hat{z})\in\mathcal{G}(d,y)} \left\{ c^\top \hat{x} + b^\top \hat{z} \right\}\right\}\right\}$$
$$= \min_{x \text{ binary}} \left\{ c^\top x + \max_{\substack{(\hat{x},\hat{z})\in\mathcal{G}(d,y)\\ (d,y)\in\mathcal{U}}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \right\}\right\}.$$

To solve this reformulation of the MMR model, which is structurally similar to the MMC model (17), only slight modifications to the master and sub-problems are required. The master problem is defined for a different set of input scenarios, $\mathbf{\Omega^R} = \{(d^i, y^i, \hat{x}^i, \hat{z}^i), \forall i = 1, \dots, |\mathbf{\Omega^R}|\}$, in which the two additional variables, $\hat{x}^i$ and $\hat{z}^i$, represent the optimal investment and recourse decisions with hindsight of the uncertainty realization (d^i, y^i). We denote the master problem as $\mathbf{M}^{\text{R}}(\mathbf{\Omega^R})$, and it is formulated as the following single level mixed integer linear program:

$$\min_{x,\xi,z^i} \quad c^\top x + \xi \tag{39}$$
$$\text{s.t. } \xi \ge b^\top z^i - (c^\top \hat{x}^i + b^\top \hat{z}^i) \quad \forall i = 1, \dots, |\mathbf{\Omega^R}| \tag{40}$$
$$Ax + C_1 z^i \le g_1 \quad \forall i = 1, \dots, |\mathbf{\Omega^R}| \tag{41}$$
$$By^i + C_2 z^i \le g_2 \quad \forall i = 1, \dots, |\mathbf{\Omega^R}| \tag{42}$$
$$Jz^i = d^i \quad \forall i = 1, \dots, |\mathbf{\Omega^R}| \tag{43}$$
$$x \text{ binary.} \tag{44}$$

We denote the subproblem as $\mathbf{S}^{\text{R}}(x)$, and it is formulated as the following bilevel linear program:

$$\max_{(d,y)\in\mathcal{U};(\hat{x},\hat{z})\in\mathcal{G}(d,y)} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \right\}$$

which can be solved using the same big-M approach with the following MILP:

$$\max_{d,y,z,\hat{x},\hat{z},\alpha,\beta,\gamma,\omega} \quad b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \tag{45}$$
$$\text{s.t.} \quad \text{Constraints (34)–(38)} \tag{46}$$
$$A\hat{x} + C_1 \hat{z} \le g_1 \tag{47}$$
$$By + C_2 \hat{z} \le g_2 \tag{48}$$
$$J\hat{z} = d \tag{49}$$
$$\hat{x} \text{ binary.} \tag{50}$$

With the new definitions of master and sub-problems, the same algorithm Alg^{MMC} can be used to solve the MMR model with the following minor modification to Step 2, besides the apparent need to change the superscript "C" to "R":

Step 2: Solve the sub-problem $\mathbf{S}^{\text{R}}(x^k)$ and let $(d^k, y^k, z^k, \hat{x}^k, \hat{z}^k)$ denote its optimal solution. Update $\mathbf{\Omega^R} \leftarrow \mathbf{\Omega^R} \cup \{(d^k, y^k, \hat{x}^k, \hat{z}^k)\}$, and $UB \leftarrow c^\top x^k + b^\top z^k - (c^\top \hat{x}^k + b^\top \hat{z}^k)$.

TABLE III
GENERATION PARAMETERS

Bus NO.	Type	Current Capacity (MW)	Fuel Cost ($/MWh)	Min New Capacity (MW)	Max New Capacity (MW)
25	Wind	1,000	0.0	2,000	3,000
31	Wind	500	0.0	150	400
32	Coal	300	43.0	–160	–80
36	Wind	1,100	0.0	800	1,500
49	Wind	1,000	0.0	1,200	2,000
61	Coal	400	28.2	–200	–70
65	Coal	1,500	52.7	–800	–600
66	Coal	300	28.3	–150	–100
76	Gas	200	31.0	80	100
85	Gas	300	63.9	150	200
87	Wind	900	0.0	1,300	2,500
89	Gas	150	49.1	100	150
92	Gas	200	32.0	200	250
99	Coal	300	27.5	–150	–80
113	Gas	200	65.0	200	300
Total		8,350		6,000	8,500

TABLE IV
CANDIDATE LINE PARAMETERS

From	To	Susceptance (Ω^{-1})	Transmission Capacity (MW)	Construction Cost ($m)
25	4	30	390	40.60
25	18	30	390	32.48
25	115	30	390	40.60
32	6	30	390	50.75
36	34	30	390	28.42
36	77	30	390	44.66
70	25	30	390	97.44
86	82	30	390	36.54
87	106	30	390	62.93
87	108	30	390	52.78

IV. CASE STUDY

In this section, we present numerical experiments of our model and algorithm on an IEEE 118-bus test system, which consists of 186 transmission lines, 5 wind farms, 5 coal plants, 5 gas plants, and 33 loads. The network data is available in [28].

We consider 10 candidate lines. The operation costs are calculated based on the data of 4 load blocks. We consider a planning horizon of 20 years, with the operation cost extrapolated from the cost of year 1. Then the operation cost is assumed to increase at the same rate each year. The characteristics of generation and candidate lines are summarized in Tables III and IV, respectively. In our case study, generation capacity data in the system is set to be able to satisfy all demand levels if there is no network congestion.

Uncertainty in future generation capacity consists of two parts, expansion and retirement. For wind and natural gas fired plants, we set the lower and upper bounds for new capacity. For coal plants, the range of reduced capacity is also provided. We use negative capacity to depict coal retirement. In addition to bounds on individual plants, we also set the lower bound and upper bound on total new generation capacity to control the randomness of the uncertainty set. The mean of our demand $(\bar{d})$ and the capacity factor are modified based on the real data

TABLE V
NUMBER OF ITERATIONS FOR EACH INSTANCE

	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
MMC	3	3	2	2
MMR	3	6	2	3

from WECC [29]. We generate four instances by changing the uncertainty sets. Their definitions are listed as follows:

$$\mathcal{U}_1 = \Big\{0.95\bar{d}_{i,t,m} \leq d_{i,t,m} \leq 1.05\bar{d}_{i,t,m} \tag{51}$$

$$P^{\rm N,min}_{i,k,t} \leq P^{\rm N}_{i,k,t} \leq P^{\rm N,max}_{i,k,t} \tag{52}$$

$$P^{\rm N,min} \leq \sum_{i,k,t} P^{\rm N}_{i,k,t} \leq P^{\rm N,max}\Big\} \tag{53}$$

$$\mathcal{U}_2 = \{0.85\bar{d}_{i,t,m} \leq d_{i,t,m} \leq 1.15\bar{d}_{i,t,m} \tag{54}$$

$$\text{Equations (52)–(53)}\} \tag{55}$$

$$\mathcal{U}_3 = \Big\{0.95\bar{d}_{i,t,m} \leq d_{i,t,m} \leq 1.05\bar{d}_{i,t,m} \tag{56}$$

$$\left[1.25 - 0.5\mathrm{sgn}\left(P^{\rm N,min}_{i,k,t}\right)\right] P^{\rm N,min}_{i,k,t} \leq P^{\rm N}_{i,k,t} \leq \left[0.75 + 0.5\mathrm{sgn}\left(P^{\rm N,max}_{i,k,t}\right)\right] P^{\rm N,max}_{i,k,t} \tag{57}$$

$$0.75P^{\rm N,min} \leq \sum_{i,k,t} P^{\rm N}_{i,k,t} \leq 1.15P^{\rm N,max}\Big\} \tag{58}$$

$$\mathcal{U}_4 = \{0.85\bar{d}_{i,t,m} \leq d_{i,t,m} \leq 1.15\bar{d}_{i,t,m} \tag{59}$$

$$\text{Equations (57)–(58)}\} \tag{60}$$

where $(P^{\rm N,min}_{i,k,t}, P^{\rm N,max}_{i,k,t})$ and $(P^{\rm N,min}, P^{\rm N,max})$ are listed in the last two columns in Table III, with $(P^{\rm N,min}, P^{\rm N,max})$ in the last row. The load curtailment cost is set as 2000\$/MWh. The interest rate is set to be 0.1. The experiment is implemented on a computer with Intel Core i5 3.30 GHz with 4 GB memory and CPLEX 12.5. The computation time of each instance is around 9 hours. The numbers of iterations for solving each instance are summarized in Table V.

The transmission expansion plans, investment costs and objective values of each criterion under the four uncertainty sets are summarized in Tables VI and VII. We then compare the performances of the MMC solution and the MMR solution under various scenarios in Tables VIII and IX, where we use D^c, D^r, and D^d to denote the optimal MMC solution, the optimal MMR solution, and the optimal deterministic solution. The lower cost and regret between D^c and D^r are highlighted. The deterministic solution is derived by setting the mean demand as the load levels and the median of new capacity as the future expansion plans. The scenarios are generated by our algorithms when solving the MMR problems and MMC problems. Each scenario corresponds to an optimal solution to a sub-problem at an iteration of our algorithm and is the worst-case scenario for the first-stage solution obtained at the same iteration. Scenarios $S_1 - S_3$ and $S_6 - S_{10}$ are generated by solving the MMR problems. Scenarios S_4, S_5, S_{11}, and S_{12} are generated when solving the MMC problems. Those scenarios typically have very high costs or regrets, thus are representative of bad scenarios that robust optimization tries to hedge against. The investment and operational costs of both the MMC and MMR decisions under the above scenarios are summarized in Table X.

From Tables VI and VII, we can see that as the uncertainty in demand increases, although the numerical value of the maximum regret and worst-case cost increases, the change in the transmission expansion plan is not very substantial. It means many of the candidate lines are necessary regardless of the demand levels with our unchanged depiction of generation capacity uncertainties. The reason is that those candidate lines

TABLE VI
EXPANSION PLAN OF THE MMC APPROACH

Lines (from bus, to bus)	Uncertainty Set			
	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
Candidate Line $(25,4)$	1	0	1	1
Candidate Line $(25,18)$	1	0	1	1
Candidate Line $(25,115)$	0	1	0	0
Candidate Line $(32,6)$	0	1	0	0
Candidate Line $(36,34)$	1	1	1	1
Candidate Line $(36,77)$	1	1	1	1
Candidate Line $(70,25)$	0	1	0	0
Candidate Line $(86,82)$	1	1	0	0
Candidate Line $(87,106)$	1	1	1	1
Candidate Line $(87,108)$	1	1	1	1
Investment Cost (\$m)	298	414	262	262
Maximum Cost (\$m)	1,233	1,960	1,927	2,802

TABLE VII
EXPANSION PLAN OF THE MMR APPROACH

Lines (from bus, to bus)	Uncertainty Set			
	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
Candidate Line $(25,4)$	1	1	1	1
Candidate Line $(25,18)$	1	1	1	1
Candidate Line $(25,115)$	1	0	1	0
Candidate Line $(32,6)$	0	0	0	0
Candidate Line $(36,34)$	1	1	1	1
Candidate Line $(36,77)$	1	1	1	1
Candidate Line $(70,25)$	0	1	0	1
Candidate Line $(86,82)$	1	1	1	1
Candidate Line $(87,106)$	1	1	1	1
Candidate Line $(87,108)$	1	1	1	1
Investment Cost (\$m)	339	395	339	395
Maximum Regret (\$m)	89	234	110	235

TABLE VIII
COMPARISON OF THE MMC, MMR, AND DETERMINISTIC SOLUTIONS FOR UNCERTAINTY SET $\mathcal{U}_1$

Cost/Regret (\$m)	D^c	D^r	D^d
Scenario S_1	1,137 / 117	**1,109** / **89**	1,085 / 65
Scenario S_2	**1,039** / **0**	1,080 / 41	1,263 / 224
Scenario S_3	**803** / **0**	844 / 41	1,125 / 422
Scenario S_4	**1,126** / **0**	1,167 / 41	1,325 / 199
Scenario S_5	**1,233** / **27**	1,272 / 66	1,367 / 161

TABLE IX
COMPARISON OF THE MMC, MMR, AND DETERMINISTIC SOLUTIONS FOR UNCERTAINTY SET $\mathcal{U}_2$

Cost/Regret ($m)	D^c	D^r	D^d
Scenario S_6	1,730 / 170	**1,597 / 37**	1,861 / 301
Scenario S_7	1,375 / 133	**1,354 / 112**	1,519 / 277
Scenario S_8	1,309 / 497	**1,046 / 234**	836 / 24
Scenario S_9	1,398 / 288	**1,266 / 156**	1,473 / 363
Scenario S_{10}	776 / 246	**701 / 171**	666 / 136
Scenario S_{11}	1,959 / 201	**1,862 / 104**	2,082 / 324
Scenario S_{12}	**1,960 / 31**	1,962 / 33	2,173 / 244

TABLE X
INVESTMENT AND OPERATIONAL COSTS FOR SCENARIOS ($M)

	D^c	D^r		D^c	D^r
Invest $S_1 - S_5$	298	339	Invest $S_6 - S_{12}$	414	395
Operation S_1	839	770	Operation S_6	1,316	1,202
Operation S_2	741	741	Operation S_7	961	959
Operation S_3	505	505	Operation S_8	895	651
Operation S_4	828	828	Operation S_9	984	871
Operation S_5	935	933	Operation S_{10}	362	306
			Operation S_{11}	1,545	1,467
			Operation S_{12}	1,546	1,567

connect regions with very high locational marginal price differences due to the presence of large amount of wind energy. On the other hand, when uncertainty in generation expansion is increased, although the total cost also increases, fewer lines are actually built with the MMC criterion. That is because in the worst-case scenarios, the system contains less renewable energy capacity and the differences in the locational marginal prices between the otherwise connected regions are not substantial enough to justify new transmission lines. When the MMR criterion is used, however, the final expansion plan does not seem to be sensitive to the change of uncertainty in generation expansion. One possible explanation is that since the MMR criterion does not make decisions only based on the boundary scenarios, it is less sensitive to the changes in uncertainty sets.

From the more detailed comparisons of results from the MMC and MMR approaches in Tables VIII and IX, we can gain more insights about which criterion is more appropriate under different situations. Both robust optimization solutions outperform the deterministic solution under most of the scenarios. When the uncertainty set is $\mathcal{U}_1$, the MMC solution outperforms the MMR solution under most of the listed scenarios. When the uncertainty set is $\mathcal{U}_2$, on the other hand, the MMR solution has a lower total cost under more listed scenarios. The solutions of the cases when the uncertainty sets are $\mathcal{U}_3$ and $\mathcal{U}_4$ yield similar results. According to our definition at the end of Section III, scenarios $S_2 - S_5$ in Table VIII and scenario S_{12} in Table IX are regretless scenarios for the MMC decision, while scenarios S_1 and $S_6 - S_{11}$ are regretful ones. Which criterion should be used depends on the decision-maker's perception on the uncertainty sets. In typical regretless scenarios for MMC decisions, there usually exists high demand and low renewable energy penetration. If decision-makers care more about such scenarios or believe they are more likely, then MMC should be used. Otherwise, choosing MMR might be better. Both criteria provide good upper bounds for the total costs under scenarios contained in an uncertainty set. The MMC criterion provides a smaller upper bound with higher average costs while the costs of MMR decisions are lower on average but have higher variability.

From the above results, it is obvious that both the future generation expansion behavior of generation companies and demand uncertainty play important roles in transmission expansion planning. In addition, we can also conclude that both criteria have their merits and can yield relatively reliable expansion plans that guarantee zero curtailment for uncertainty realizations contained in the uncertainty sets. However, depending on the characteristics of uncertainty sets and the preference of decision-makers, they may outperform each other under different situations. Thus, a comparative analysis of the MMC and MMR criteria can shed more light on better utilization of both approaches.

V. CONCLUSION

In this paper, we propose two robust optimization models for the transmission expansion planning problem under uncertainty, where we take into consideration both the high-frequency uncertainty caused by load forecast errors and the low-frequency uncertainty caused by future generation expansion and retirement. We use two criteria: minimax cost and minimax regret, and compare their performances. The uncertain parameters are described by a polyhedral uncertainty set. With this approach, we can derive an expansion plan that is robust under all scenarios. The resulting models can be formulated as trilevel mixed-integer problems. We use a branch and cut type mechanism to decompose the problem into a master problem and a subproblem. The subproblem generates scenarios and returns them to the master problem to cut off sub-optimal solutions. The bilevel mixed-integer subproblem is reformulated into a single level mixed-integer-programming problem with the KKT conditions to obtain the global optimal solution. Our model and algorithm are then tested on an IEEE 118-bus system, where we compare the results of our MMR and MMC models and analyze their differences. We conclude that the MMR and MMC criteria may outperform each other depending on the uncertainty set and decision-maker's preference. Interesting topics for future research include developing effective heuristics and parallel computing mechanisms to speed up our algorithms and implementing our algorithms on high performance machines to test larger systems with more candidate lines.

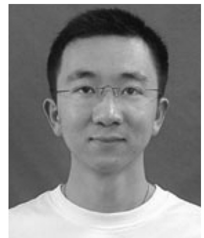
ACKNOWLEDGMENT

The authors thank Prof. A. Conejo for his comments and related discussions which have helped improve the quality of the paper.

REFERENCES

[1] S. Ryan, J. McCalley, and D. Woodruff, Long Term Resource Planning for Electric Power Systems Under Uncertainty, Iowa State Univ., 2011, Tech. Rep.
[2] M. Celebi, F. Graves, G. Bathla, and L. Bressan, "Potential coal plant retirements under emerging environmental regulations," *The Brattle Group, December*, vol. 8, 2010.

[3] J. H. Roh, M. Shahidehpour, and L. Wu, "Market-based generation and transmission planning with uncertainties," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1587–1598, Aug. 2009.
[4] P. Jirutitijaroen and C. Singh, "Reliability constrained multi-area adequacy planning using stochastic programming with sample-average approximations," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 504–513, May 2008.
[5] J. Á. López, K. Ponnambalam, and V. H. Quintana, "Generation and transmission expansion under risk using stochastic programming," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1369–1378, Aug. 2007.
[6] L. P. Garcés, A. J. Conejo, R. García-Bertrand, and R. Romero, "A bilevel approach to transmission expansion planning within a market environment," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1513–1522, Aug. 2009.
[7] M. O. Buygi, G. Balzer, H. M. Shanechi, and M. Shahidehpour, "Market-based transmission expansion planning," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 2060–2067, Nov. 2004.
[8] A. Ben-Tal and A. Nemirovski, "Robust solutions of uncertain linear programs," *Oper. Res. Lett.*, vol. 25, no. 1, pp. 1–13, 1999.
[9] L. El Ghaoui and H. Lebret, "Robust solutions to least-squares problems with uncertain data," *SIAM J. Matrix Anal. Applicat.*, vol. 18, no. 4, pp. 1035–1064, 1997.
[10] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows," *Math. Program.*, vol. 98, no. 1–3, pp. 49–71, 2003.
[11] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35–53, 2004.
[12] A. Street, F. Oliveira, and J. Arroyo, "Contingency-constrained unit commitment with security criterion: A robust optimization approach," *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1581–1590, Aug. 2011.
[13] D. Bertsimas, E. Litvinov, X. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, Feb. 2013.
[14] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 800–810, May 2012.
[15] L. Zhao and B. Zeng, "Robust unit commitment problem with demand response and wind energy," in *Proc. 2012 IEEE Power and Energy Society General Meeting*, 2012, pp. 1–8.
[16] R. Jiang, J. Wang, M. Zhang, and Y. Guan, "Two-stage minimax regret robust unit commitment," *IEEE Trans. Power Syst.*, to be published.
[17] R. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4558–4567, Nov. 2013.
[18] P. Maghouli, S. H. Hosseini, M. O. Buygi, and M. Shahidehpour, "A scenario-based multi-objective model for multi-stage transmission expansion planning," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 470–478, Feb. 2011.
[19] T. De la Torre, J. W. Feltes, T. G. San Roman, and H. M. Merrill, "Deregulation, privatization, and competition: Transmission planning under uncertainty," *IEEE Trans. Power Syst.*, vol. 14, no. 2, pp. 460–465, May 1999.
[20] J. M. Arroyo, N. Alguacil, and M. Carrión, "A risk-based approach for transmission network expansion planning under deliberate outages," *IEEE Trans. Power Syst.*, vol. 25, no. 3, pp. 1759–1766, Aug. 2010.
[21] P. Xu and L. Wang, "An exact algorithm for the bilevel mixed integer linear programming problem under three simplifying assumptions," *Comput. Oper. Res.*, 2013.
[22] B. Zeng and L. Zhao, "Solving two-stage robust optimization problems using a column-and-constraint generation method," *Oper. Res. Lett.*, 2013.
[23] J. T. Moore and J. F. Bard, "The mixed integer linear bilevel programming problem," *Oper. Res.*, pp. 911–921, 1990.
[24] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[25] J. Hu, J. E. Mitchell, J.-S. Pang, K. P. Bennett, and G. Kunapuli, "On the global solution of linear programs with linear complementarity constraints," *SIAM J. Optimiz.*, vol. 19, no. 1, pp. 445–471, 2008.
[26] S. Jin and S. Ryan, "Capacity expansion in the integrated supply network for an electricity market," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 2275–2284, Nov. 2011.
[27] P. Kall and S. W. Wallace, *Stochastic Programming*. New York, NY, USA: Wiley, 1994.
[28] "IEEE 118-bus, 54-unit, 24-hour system," [Online]. Available: http://motor.ece.iit.edu/data/JEAS_IEEE118.doc
[29] J. Price and J. Goodin, "Reduced network modeling of wecc as a market design prototype," in *Proc. 2011 IEEE Power and Energy Society General Meeting*, 2011, pp. 1–6.



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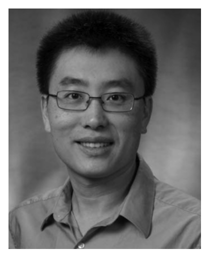

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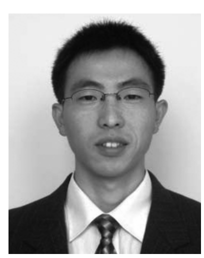

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