PMU Uncertainty Quantification in Voltage Stability Analysis

Chen Chen, Jianhui Wang, *Senior Member, IEEE*, Zhengshuo Li, Hongbin Sun, *Senior Member, IEEE*, and Zhaoyu Wang

***Abstract*—This letter presents an uncertainty quantification method for phasor measurement units (PMUs) in voltage stability assessment. The effect of local phasor measurement uncertainty on the Thevenin equivalent impedance used for voltage stability analysis is quantified analytically. The results can be used to specify the requirements for PMU uncertainty in voltage stability assessment.**

***Index Terms*—Phasor measurement unit, recursive least square, uncertainty, voltage stability.**

I. Introduction

PHASOR measurement units (PMUs), as a powerful tool for wide-area monitoring systems (WAMSs), have proved their value in a variety of power system applications (e.g., state estimation, oscillation detection and control, and voltage stability analysis). As one of the highly cited approaches for real-time power system voltage stability analysis, the proximity of a power system to voltage collapse can be estimated by checking the Thevenin equivalent impedance and the local load impedance using voltage and incident current phasor measurements by PMUs at local buses [1].

However, the uncertainties associated with PMU data, due to errors caused by various sources (e.g., measurements, data processing procedures, communications, or even malicious manipulations), affect the performance of this PMU application.[1] The uncertainty quantification for the Thevenin equivalent impedance and the local load impedance caused by PMU uncertainties is critical to correctly monitor the voltage stability of power systems, which has not been sufficiently discussed in the literature.

In this letter, we analytically derive the uncertainty quantification of these two impedances caused by inaccurate phasor measurements, based on the framework of the Voltage Instability Predictor (VIP) method proposed by Begovic *et al.* in [1] and [4]. This work can provide guidance to specify the requirements for phasor measurement uncertainty in this PMU application. To the best of our knowledge, our work is the first analytical discussion on quantifying PMU uncertainty's effect on voltage stability assessment in power systems.

Manuscript received May 01, 2014; revised July 15, 2014; accepted August 19, 2014. This work was supported by the U.S. Department of Energy Office of Electricity Delivery and Energy Reliability. The work of Z. Li and H. Sun was supported by the 973 Program (2013CB228203) and NSFC (51025725, 51321005) of China. Paper no. PESL-00062-2014.

C. Chen and J. Wang are with the Decision and Information Sciences Division, Argonne National Laboratory, Argonne, IL 60049 USA (e-mail: morningchen@anl.gov; jianhui.wang@anl.gov).

Z. Li and H. Sun are with the Department of Electrical Engineering, State Key Laboratory of Power Systems, Tsinghua University, Beijing 10084, China (e-mail: shuozhengli@sina.com; shb@mail.tsinghua.edu.cn).

Z. Wang is with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: zhaoyuwang@gatech.edu).

Digital Object Identifier 10.1109/TPWRS.2014.2357217

[1]Although the *IEEE Standard C37.118-2011* specifies the general total vector error (TVE) requirement for PMU measurements [2], the impact of PMU uncertainty on a specific application depends on how phasor measurement data is utilized in that application (e.g., see [3]).

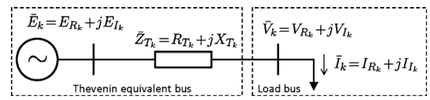

Fig. 1. Local bus and the Thevenin equivalent.

II. Voltage Stability Assessment With PMU Uncertainty

Fig. 1 shows the two-bus equivalent of the power system for voltage stability analysis [1], in which $\bar{E}_k$ and $\bar{Z}_{T_k}$ stand for the Thevenin equivalent voltage source and impedance at time step k, respectively. $\bar{V}_k$ and $\bar{I}_k$ are the voltage and current phasors measured at the load bus by the PMU at k, based on which the impedance at the bus, $\bar{Z}_{L_k} = R_{L_k} + jX_{L_k}$, can be computed as $\bar{Z}_{L_k} = \bar{V}_k / \bar{I}_k$.[2] The voltage collapses at the bus when the load impedance is equal to the Thevenin equivalent impedance (i.e., $|\bar{Z}_{T_k}| = |\bar{Z}_{L_k}|$) corresponding to the maximal power transfer to the load. Thus, the characteristic of the Thevenin equivalent impedance over time can be used to identify the voltage instability [1].

To compute the Thevenin equivalent impedance, we observe $\bar{V}_k = \bar{E}_k - \bar{Z}_{T_k} \cdot \bar{I}_k$ according to the Ohm's law. Separating the imaginary and real parts, we get two linear equations with four unknown variables, which can be written in a matrix form as $\boldsymbol{y}_k = \mathbf{H}_k^{\mathrm{T}} \boldsymbol{x}_k$, where $\boldsymbol{y}_k = [V_{R_k} V_{I_k}]^{\mathrm{T}}$, $\boldsymbol{x}_k = [E_{R_k} E_{I_k} R_{T_k} X_{T_k}]^{\mathrm{T}}$, $\mathbf{H}_k^{\mathrm{T}} = \left[\begin{smallmatrix} 1 & 0 & -I_{R_k} & I_{I_k} \\ 0 & 1 & -I_{I_k} & -I_{R_k} \end{smallmatrix}\right]$. The recursive least square (RLS) method is applied to compute the unknown vector $\boldsymbol{x}_k$ iteratively (see [4, Appendix]) as

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \mathbf{G}_k \left(\boldsymbol{y}_k - \mathbf{H}_k^{\mathrm{T}} \boldsymbol{x}_{k-1} \right) \quad (1)$$

$$\mathbf{G}_k = \mathbf{P}_{k-1} \mathbf{H}_k \left(\lambda \mathbf{I} + \mathbf{H}_k^{\mathrm{T}} \mathbf{P}_{k-1} \mathbf{H}_k \right)^{-1} \quad (2)$$

$$\mathbf{P}_k = \left[\left(\mathbf{I} - \mathbf{G}_k \mathbf{H}_k^{\mathrm{T}} \right) \mathbf{P}_{k-1} \right] / \lambda \quad (3)$$

where $\lambda < 1$ is the forgetting factor assigning larger weight to the recent measurements, and $\mathbf{I}$ denotes the identity matrix. The matrices $\mathbf{G}_k$ and $\mathbf{P}_k$ with proper sizes are iteration variables to include the new measurements in the update of $\boldsymbol{x}_k$. The Thevenin impedance can be obtained from $\boldsymbol{x}_k$ accordingly.

The uncertainties of voltage and current phasor measurements by the PMU at the load bus affect the computation of the Thevenin equivalent impedance ($\bar{Z}_{T_k}$) via (1)–(3) and the load impedance ($\bar{Z}_{L_k}$), which are the key elements in voltage stability analysis. Hence, uncertainty quantification for these impedances is critical for correctly gauging and monitoring the voltage stability status of the power system.

Let $\sigma_{V_{Rk}}, \sigma_{V_{Ik}}, \sigma_{I_{Rk}}, \sigma_{I_{Ik}}$ denote the uncertainties of real and imaginary parts of measured voltage and current phasors at time step k. Let $\sigma_{\boldsymbol{x}_k} = [\sigma_{E_{Rk}}, \sigma_{E_{Ik}}, \sigma_{R_{Tk}}, \sigma_{X_{Tk}}]^{\mathrm{T}}$ denote the uncertainties of the unknown vector $\boldsymbol{x}$. Let $\mathcal{M}_K = \{V_{R_k}, V_{I_k}, I_{R_k}, I_{I_k} | k = 1, 2 \ldots, K\}$ denote the set of the phasor measurements (real and imaginary parts) up to the time step K. With linearization approximation, the uncertainties of the unknown variables up to time step K,

[2]Note that Fig. 1 models the constant-power load; however, this method can also account for the voltage-sensitive load (i.e., ZIP load model) by applying [4, eq. (11)] to compute the Thevenin impedance seen by the constant-power load.

denoted by $\sigma_{\boldsymbol{x}_K}$, in terms of the measurement uncertainties can be expressed as

$$\sigma_{\boldsymbol{x}_K} = \sqrt{\sum_{m \in \mathcal{M}_K} \left(\frac{\partial \boldsymbol{x}_K}{\partial m} \right)^2 \sigma_m^2} \tag{4}$$

and uncertainty of the Thevenin impedance can be obtained accordingly. We see that the total number of partial derivatives to compute $\sigma_{\boldsymbol{x}_K}$ is $4 \cdot K$. These values can be computed along with the iterations in the RLS method to calculate the Thevenin impedance. Algorithm 1 shows the analytical quantification method of the Thevenin impedance uncertainty.

Algorithm 1: Computation method of the Thevenin equivalent uncertainty iteratively

Initialization: $\boldsymbol{x}_0 = [1, 0, 0, 0]^{\mathrm{T}}$, $\mathbf{P}_0 = a \cdot (\mathbf{I})_4$, $a > 0$.
for all $k = 1, 2, \ldots K$ **do**
  Update $\mathbf{H}_k$, $\boldsymbol{y}_k$ with $\{V_{R_k}, V_{I_k}, I_{R_k}, I_{I_k}\}$
  Compute $\mathbf{S}_k = (\lambda \mathbf{I} + \mathbf{H}_k^{\mathrm{T}} \mathbf{P}_{k-1} \mathbf{H}_k)^{-1}$.
  Update $\mathbf{G}_k = \mathbf{P}_{k-1} \mathbf{H}_k \mathbf{S}_k$.
  for all $m_k \in \{V_{R_k}, V_{I_k}, I_{R_k}, I_{I_k}\}$ **do**
    Compute partial derivatives to measurements at k

$$\frac{\partial \mathbf{G}_k}{\partial m_k} = \mathbf{P}_{k-1} \left[\frac{\partial \mathbf{H}_k}{\partial m_k} \mathbf{S}_k - \mathbf{H}_k \mathbf{S}_k \times \left(\frac{\partial \mathbf{H}_k^{\mathrm{T}}}{\partial m_k} \mathbf{P}_{k-1} \mathbf{H}_k + \mathbf{H}_k^{\mathrm{T}} \mathbf{P}_{k-1} \frac{\partial \mathbf{H}_k}{\partial m_k} \right) \mathbf{S}_k \right]$$

$$\frac{\partial \mathbf{P}_k}{\partial m_k} = - \left(\frac{\partial \mathbf{G}_k}{\partial m_k} \mathbf{H}_k^{\mathrm{T}} + \mathbf{G}_k \frac{\partial \mathbf{H}_k^{\mathrm{T}}}{\partial m_k} \right) \frac{\mathbf{P}_{k-1}}{\lambda}$$

$$\frac{\partial \boldsymbol{x}_k}{\partial m_k} = \frac{\partial \mathbf{G}_k}{\partial m_k} \left(\boldsymbol{y}_k - \mathbf{H}_k^{\mathrm{T}} \boldsymbol{x}_{k-1} \right) + \mathbf{G}_k \left(\frac{\partial \boldsymbol{y}_k}{\partial m_k} - \frac{\partial \mathbf{H}_k^{\mathrm{T}}}{\partial m_k} \boldsymbol{x}_{k-1} \right)$$

  end for
  for all $t = 1, 2, \ldots, k-1$ **do**
    for all $m_{k-t} \in \{V_{R_{k-t}}, V_{I_{k-t}}, I_{R_{k-t}}, I_{I_{k-t}}\}$ **do**
      Compute partial derivatives to previous measurements

$$\frac{\partial \mathbf{G}_k}{\partial m_{k-t}} = \left(\mathbf{I} - \mathbf{P}_{k-1} \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^{\mathrm{T}} \right) \frac{\partial \mathbf{P}_{k-1}}{\partial m_{k-t}} \mathbf{H}_k \mathbf{S}_k$$

$$\frac{\partial \mathbf{P}_k}{\partial m_{k-t}} = \frac{\mathbf{I} - \mathbf{G}_k \mathbf{H}_k^{\mathrm{T}}}{\lambda} \frac{\partial \mathbf{P}_{k-1}}{\partial m_{k-t}} - \frac{\partial \mathbf{G}_k}{\partial m_{k-t}} \mathbf{H}_k^{\mathrm{T}} \frac{\mathbf{P}_{k-1}}{\lambda}$$

$$\frac{\partial \boldsymbol{x}_k}{\partial m_{k-t}} = \left(\mathbf{I} - \mathbf{G}_k \mathbf{H}_k^{\mathrm{T}} \right) \frac{\partial \boldsymbol{x}_{k-1}}{\partial m_{k-t}} + \frac{\partial \mathbf{G}_k}{\partial m_{k-t}} \left(\boldsymbol{y}_k - \mathbf{H}_k^{\mathrm{T}} \boldsymbol{x}_{k-1} \right)$$

    end for
  end for
  Save $(\partial \mathbf{P}_k)/(\partial m_{k-t})$, $(\partial \boldsymbol{x}_k)/(\partial m_{k-t})$, $t = 0, 1, \ldots, k-1$ for next iteration
  Update $\mathbf{P}_k$ and $\boldsymbol{x}_k$

$$\mathbf{P}_k = \frac{\left(\mathbf{I} - \mathbf{G}_k \mathbf{H}_k^{\mathrm{T}} \right) \mathbf{P}_{k-1}}{\lambda}$$

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \mathbf{G}_k \left(\boldsymbol{y}_k - \mathbf{H}_k^{\mathrm{T}} \boldsymbol{x}_{k-1} \right).$$

end for
Compute the uncertainty level at the step K using (4)

$$\sigma_{\boldsymbol{x}_K} = \sqrt{\sum_{\beta \in \{R,I\}} \sum_{j=1}^{K} \left(\frac{\partial \boldsymbol{x}_K}{\partial I_{\beta_j}} \right)^2 \sigma_{I_{\beta_j}}^2 + \sum_{\beta \in \{R,I\}} \sum_{j=1}^{K} \left(\frac{\partial \boldsymbol{x}_K}{\partial V_{\beta_j}} \right)^2 \sigma_{V_{\beta_j}}^2}$$

For the load impedance $\bar{Z}_{L_k} = R_{L_k} + jX_{L_k} = \bar{V}_k / \bar{I}_k$, the uncertainties of R_{L_k} and X_{L_k} can be expressed similarly in terms of phasor measurement uncertainties via (4). With the uncertainty levels of these two impedances quantified, the accuracy of voltage stability assessment (e.g., VIP framework in [1]) can be further analyzed. This can help specify the requirements of phasor measurement uncertainty in this PMU application.

TABLE I
COMPUTATION RESULTS OF UNCERTAINTY OF $|Z_T|$

PMU Uncertainty					
	V	0.5%	0.5%	1%	1%
	I	0.5%	1%	0.5%	1%
$\sigma_{\lvert Z_T \rvert}(\%)$	ALG. 1	1.072%	1.672%	1.734%	2.155%
	MC	1.085%	1.695%	1.751%	2.183%

III. NUMERICAL RESULTS

The proposed uncertainty quantification method in voltage stability analysis is tested on the IEEE 39-bus system.[3] The continuation power flow (CPF) module in PSAT is used to simulate the underlying power system to obtain the voltage and current phasor measurements. In the CPF computation, the demand at each load bus is increased stepwisely until the bifurcation point (i.e., the voltage collapse point) is achieved. For each steady state, 150 samples of phasor measurements are used to compute the Thevenin equivalent impedance with the RLS method.

We compute the uncertainty levels of the Thevenin impedance at the voltage collapse point using the proposed Algorithm 1, given different uncertainty levels of voltage and current (V and I) phasor measurements, as shown in Table I. To validate the analytical results, Monte Carlo (MC) simulations are applied to calculate the uncertainty level of the Thevenin impedance accordingly. Specifically, the current and voltage phasor measurements are distorted by additive white Gaussian noise with the variation indicated by the PMU uncertainty levels in the simulation with $N = 1000$ trials. The uncertainty levels of the Thevenin impedances are computed as the standard deviation (in %) with respect to the true values over the N trials. We see that the Thevenin impedance uncertainty quantified by Algorithm 1 matches the simulation results, with relative errors within 2% of the simulation results.

IV. CONCLUSION

In this letter, the uncertainty levels of the Thevenin impedance and the load impedance, in estimation of the proximity of a power system towards voltage collapse, are analytically derived in terms of uncertainties from voltage and current phasor measurements. The results can be further utilized to specify the accuracy requirements for PMU measurements in voltage stability assessment.

REFERENCES

[1] K. Vu, M. Begovic, D. Novosel, and M. Saha, "Use of local measurements to estimate voltage-stability margin," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1029–1034, Aug. 1999.
[2] *Power System Relaying Committee, IEEE Standard for Synchrophasor for Power Systems*, IEEE PES, C37.118-2011 Std., Dec. 2011.
[3] S. Chakrabarti and E. Kyriakides, "PMU measurement uncertainty considerations in WLS state estimation," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1062–1071, May 2009.
[4] B. Milosevic and M. Begovic, "Voltage-stability protection and control using a wide-area network of phasor measurements," *IEEE Trans. Power Syst.*, vol. 18, no. 1, pp. 121–127, Feb. 2003.

[3]Note that voltage stability of a power system depends on the voltage stability indices at several critical buses with PMUs, and the system voltage collapses when one of them approaches the collapse point (e.g., see [4, Section II]). Here we only consider one local bus to show the uncertainty analysis.