Robust Voltage Instability Predictor

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***Abstract*—This letter presents a robust voltage instability predictor using local noisy PMU measurements. A robust recursive least squares estimation is proposed to mitigate the impacts of gross PMU measurement errors on estimating the bus impedance and the Thevenin equivalent impedance used for voltage stability analysis. Numerical results on the IEEE 39-bus system validate the effectiveness and robustness of the proposed method.**

***Index Terms*—Bad data, PMU measurements, robust estimation, voltage stability.**

I. Introduction

MEASUREMENT-based voltage instability predictor (VIP) is important for the online monitoring of power system operation. Calculating the Thevenin equivalence (TE) from measurement data is the fundamental step for existing VIP approaches [1]–[3]. In the existing VIP study, PMU errors are generally considered to be small and modeled as Gaussian noise. However, when impulsive noise, faulty synchronization and/or even cyber attacks occur, the errors of PMU measurements can be quite significant, which are named as gross errors in the literature. The existing VIP methods using recursive least squares (RLS) [1]–[3] is vulnerable to this types of bad measurements or gross errors, resulting in incorrect Thevenin impedance estimations and erroneous stability monitoring alarms to control centers.

In this letter, we propose a robust RLS estimator using the robust M-estimator and the projection statistics to mitigate the effects of gross errors and random Gaussian noise of PMU measurements on estimating the TE impedance used for voltage stability analysis.

II. Proposed Robust Recursive Least Square Estimator for VIP Study

A. Problem Statement

Fig. 1 presents the two-bus equivalent of a power system seen from a local load bus for voltage stability analysis, where $\bar{E}_k$ and $\bar{Z}_{T_k}$ are the Thevenin equivalent voltage source and impedance at time instant k, respectively. The relationship between the equivalent voltage source

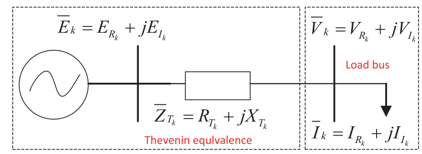

Fig. 1. TE seen from a local load.

and the voltage and current phasor measured by PMU at the local load bus is given by

$$\bar{V}_k = \bar{E}_k - \bar{Z}_{T_k} \cdot \bar{I}_k. \tag{1}$$

By separating the imaginary and real parts, we obtain a system of two linear equations with four unknowns in the following form

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{\varepsilon}_k, \tag{2}$$

where

$$\boldsymbol{y}_k = \begin{bmatrix} V_{R_k} \; V_{I_k} \end{bmatrix}^T; \boldsymbol{H}_k = \begin{bmatrix} 1 & 0 & -I_{R_k} & I_{I_k} \\ 0 & 1 & -I_{I_k} & -I_{R_k} \end{bmatrix};$$

$$\boldsymbol{x}_k = \begin{bmatrix} E_{R_k} \, E_{I_k} \, R_{T_k} \, X_{T_k} \end{bmatrix}^T$$

is the unknown state vector to be estimated; $\boldsymbol{\varepsilon}_k$ quantifies the model uncertainties and the measurement error, and is assumed to be white Gaussian noise. In the literature, it is found that the bus voltage collapses when the load impedance is equal to the Thevenin equivalent impedance ($|\bar{Z}_{T_k}| = |\bar{Z}_{L_k}|$) [1], where $\bar{Z}_{T_k}$ is estimated by using the RLS method and $\bar{Z}_{L_k} = \bar{V}_k / \bar{I}_k$. However, once bad data appears in PMU measurements due to impulsive measurement noise, cyber attacks, etc, the estimated $\bar{Z}_{T_k}$ by RLS is unreliable and the calculation of $\bar{Z}_{L_k}$ is incorrect. Another problem is that the estimated $\bar{Z}_{T_k}$ and $\bar{Z}_{L_k}$ may be equal due to the effects of impulsive measurement noise or well organized cyber attacks, which indicates the instability of the system when it is still stable. This phenomenon can be seen later in the simulation results.

B. Proposed Robust Stability Monitoring Method

To mitigate the impacts of bad PMU measurements, we propose a robust RLS estimator using the robust M-estimator and the projection statistic (PS) algorithm with the following optimization objective function

$$J(\boldsymbol{x}_k) = \sum_{i=1}^{m} \varpi_i^2 \rho(r_{S_i}), \tag{3}$$

where ϖ_i is calculated by applying the PS algorithm [4] to matrix $\hat{\boldsymbol{H}}_{\boldsymbol{k}}$ and $\hat{\boldsymbol{H}}_k = \begin{bmatrix} \boldsymbol{H}_k^T \; \boldsymbol{I} \end{bmatrix}^T$ with a dimension of 6×4 to handle the outliers in current phasor, where $\boldsymbol{I}$ is an 4×4 identity matrix; $r_{S_i} = r_k(i)/s\varpi_i$ is the standardized residual; $r_k(i) = z_k(i) - \boldsymbol{a_i}^T \hat{\boldsymbol{x}}_{\boldsymbol{k}}$ is the residual, where $\boldsymbol{z_k} = [\boldsymbol{y_k}^T \; \boldsymbol{x}_{k-1}^T]^T$ with a dimension of 6×1, and $\boldsymbol{x}_{k-1}$ with dimension 4×1 is the estimated state at the previous time instant; $\boldsymbol{a}_i^T$ is the ith column of matrix $\hat{\boldsymbol{H}}_k$; $s = 1.4826 \cdot b_m \cdot median_i |r_k|$ is the robust scale estimate, $median_i |r_k|$ represents the median of

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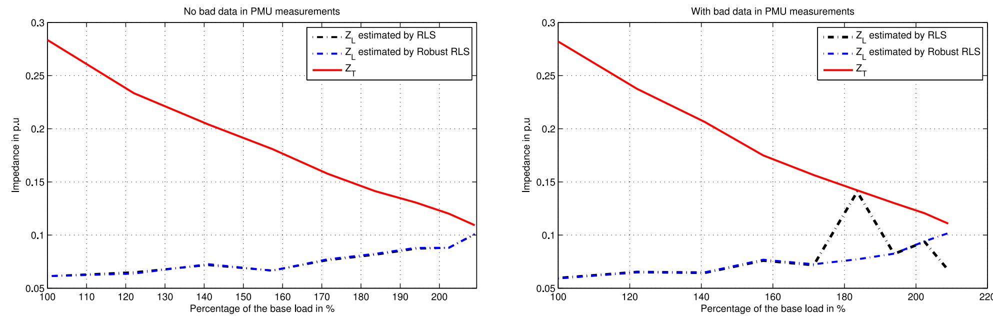

(a) (b)

Fig. 2. Comparison results for stability monitoring using the RLS and proposed method. (a) Without bad PMU measurements. (b) With bad PMU measurements.

the absolute residual r_k; b_m is a correction factor for unbiasedness at the Gaussian distribution; $\rho(\cdot)$ is the Huber nonlinear function of r_{S_i}

$$\rho\left(r_{S_i}\right) = \begin{cases} \dfrac{1}{2} r_{S_i}^2, & \text{for } \left|r_{S_i}\right| < c \\ c\left|r_{S_i}\right| - c^2/2, & \text{elsewhere,} \end{cases} \tag{4}$$

where the parameter is set as $c = 1.5$ with high efficiency at Gaussian noise [4]. To minimize (3), one takes its partial derivative and sets it equal to zero, yielding

$$\frac{\partial J\left(\boldsymbol{x}_k\right)}{\partial \boldsymbol{x}_k} = \sum_{i=1}^{m} -\frac{\varpi_i \boldsymbol{a}_i^T}{s} \psi\left(r_{S_i}\right) = \boldsymbol{0}, \tag{5}$$

where $\psi\left(r_{S_i}\right) = \partial\rho\left(r_{S_i}\right)/\partial r_{S_i}$. Then, we first divide and multiply the standardized residual r_{S_i} to both sides of (5) and rewrite it in a matrix form, yielding

$$\hat{\boldsymbol{H}}_k{}^T \boldsymbol{Q} \hat{\boldsymbol{R}}_{\boldsymbol{k}}{}^{-1}\left(\boldsymbol{z}_k - \hat{\boldsymbol{H}}_k \hat{\boldsymbol{x}}_{\boldsymbol{k}}\right) = \boldsymbol{0}, \tag{6}$$

where $\boldsymbol{Q} = \operatorname{diag}(q(r_{S_i}))$ with a dimension of 6×6; $q\left(r_{S_i}\right) = \psi\left(r_{S_i}\right)/r_{S_i}$; $\widehat{\boldsymbol{R}}_k = \begin{bmatrix} \lambda \boldsymbol{I} & \\ & \boldsymbol{P}_{k-1} \end{bmatrix}$ with a dimension of 6×6. By using the iterative reweighted least square algorithm, the converged state estimation is expressed as

$$\widehat{\boldsymbol{x}}_k = \left(\hat{\boldsymbol{H}}_k{}^T \boldsymbol{Q} \hat{\boldsymbol{R}}_k^{-1} \hat{\boldsymbol{H}}_k\right)^{-1} \hat{\boldsymbol{H}}_k{}^T \boldsymbol{Q} \hat{\boldsymbol{R}}_k^{-1} \boldsymbol{z}_k, \tag{7}$$

and its covariance matrix is updated as

$$\boldsymbol{P}_k = \left(\hat{\boldsymbol{H}}_k{}^T \boldsymbol{Q} \hat{\boldsymbol{R}}_k^{-1} \hat{\boldsymbol{H}}_k\right)^{-1}. \tag{8}$$

After the estimation, the voltage phasor is calculated by using the robust estimated states. Followed by that, the impedance of the load is estimated. Finally, the system stability is decided by checking whether $\left|\bar{Z}_{T_k}\right|$ and $\left|\bar{Z}_{L_k}\right|$ are equal or not. If there are no bad data in the measurements, the proposed robust RLS method can be shown to be equivalent to the conventional RLS by using the matrix inversion Lemma and simple algebraic substitutions. Otherwise, the bad data are greatly downweighted by the projection statistic algorithm and suppressed by the Huber estimator.

III. Numerical Results

The effectiveness and robustness of the proposed method is tested on IEEE 39-bus system. The continuation power flow in power system analysis toolbox [6] is used to simulate the voltage and current phasor measurements, where Gaussian noise with zero mean and standard deviation 0.02 is used to simulate the PMU measurement error [5]. To simulate voltage collapse, the demand at each of the load bus is gradually increased (e.g., 1% of base load) until the power-flow equations become unsolvable. For each steady state, 150 samples of phasor measurements are used to compute the Thevenin equivalent impedance by the widely used RLS method from [1]–[3] and proposed method; the results for the load at bus 21 is used as an example for illustration. Errors on the order 15 times of the standard deviations of the voltage magnitudes and angles are added to the voltage phasor measurements to simulate the outliers.

Fig. 2 shows the comparison results for the RLS and the proposed method, where bad PMU measurements occur when the percentage of the based load reaches 183% and 203%, respectively. It can be observed that when no bad PMU measurements occur, the results of RLS and the proposed robust RLS are the same, which validate the effectiveness of the proposed method. However, when bad PMU measurements occur, the results for RLS method are not reliable, which can be classified into two cases: 1) The RLS incorrectly indicates the instability of the system voltage while it is stable (see the point at 183% load base); 2) The RLS fails to indicate the instability of the system voltage while it is unstable (see the point at 203% load base). On the other hand, the proposed robust RLS can suppress the effects of the bad PMU measurements and correctly assess the voltage stability, which validates its robustness.

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