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Robust Time-Varying Load Modeling for Conservation Voltage Reduction Assessment

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***Abstract*—Due to the increasing penetration of intermittent renewable energy and highly stochastic load behavior, it is challenging to effectively assess Conservation Voltage Reduction (CVR) in power distribution systems. This paper proposes a robust time-varying load modeling technique to accurately identify load-to-voltage (LTV) dependence, yielding an improved CVR assessment scheme. In particular, we propose a robust recursive least squares (RLS) approach to estimate time-varying parameters of a ZIP load model at the substation level. Based on the identified load model, we are able to effectively evaluate LTV and analyze CVR factors. We propose a RLS with variable forgetting factors to capture the variations of model parameters under different situations, including continuous and sudden changes of parameters. To further enable RLS to suppress bad or missing measurements, we advocate to use the Huber M-estimator with a convex cost function. Finally, the robust RLS is solved by an iteratively reweighted technique. We demonstrate the effectiveness and the robustness of the proposed method using both simulations and field tests.**

***Index Terms*—Conservation voltage reduction, robust estimation, load modeling, recursive least squares, power distribution system, bad data.**

I. Introduction

WITH the advancement of smart grid technologies and the increasing energy cost in today's market, implementing effective and efficient energy saving and demand reduction measures, such as Conservation Voltage Reduction (CVR), is becoming more and more popular among electrical utilities. CVR is to operate an electrical distribution network in the lower band of a permissible voltage range set by National Service Voltage Standard (ANSI) residential voltage limits (120V±5%). Typically, electrical utilities use on-load tap changing transformers, voltage regulators, and capacitor banks to achieve the benefits of CVR. In past decades, CVR has been widely tested and piloted by utilities, and attractive energy-saving results have been reported, usually ranging from 0.3% to 1% load reduction per 1% voltage reduction [1]–[4]. Furthermore, a study led by Pacific Northwest National Laboratory has shown that the deployment of CVR on all distribution feeders in United States could provide 3.04% annual energy savings nationwide [5].

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There are two main technical barriers for an effective implementation of CVR [1], [6], [7]: i) the coordination between CVR and voltage-regulating devices, and ii) the effective assessment and verification of CVR effects. This paper provides a robust and viable solution to the latter one. It is imperative to have an accurate, fast, and easy-to-implement CVR assessment method to assist utilities in making decisions to apply voltage reduction, select CVR feeders, and perform cost/benefit analysis. Energy-saving from the reduction of voltage is quantified by a CVR factor, which is the ratio of percentage changes in energy to percentage changes in voltage. There are several challenges in calculating CVR factors: 1) the CVR effect is dependent on system configurations, load types, and customer behaviors, making CVR factors time variant and highly stochastic, 2) the integration of renewable distributed generators adds more complexity to load behaviors and CVR analysis, 3) the missing and bad measurements and noises in field data, and 4) the natural load variations which may bias the small CVR effect. The existing methods to quantify CVR effects can be classified into three categories: comparison-based, synthesis-based, and load modeling-based. Comparison-based methods [8] attempt to estimate what the load would be if there is no CVR using control groups or simulations. The control groups could be the same feeder on a day without voltage reduction or a different feeder with similar operation conditions. However, a good control group may not exist. Computer simulations require detailed system models which may not be available, and they cannot fully capture load behaviors. Synthesis-based methods [9] are bottom-up methods which test individual electrical appliances' CVR effects and aggregate them. But it is difficult to collect accurate load composition information and voltage responses of all existing electrical appliances. Load modeling-based methods [10] represent load consumptions as a function of voltages, and calculate CVR factors from the identified load-to-voltage (LTV) sensitivities. The advantage of this method is that real-time CVR factors can be calculated directly from measurements. Our previous work [10] has demonstrated the effectiveness of using LTV sensitivities to calculate real-time CVR factors. However, the exponential load model used in that work cannot represent different load compositions. Moreover, the conventional recursive least square (RLS) algorithm used to identify load model parameters is vulnerable to sudden parameter changes, impulsive noises and bad data which are commonly seen in practical measurements.

To deal with the above-mentioned challenges and disadvantages of existing methods, we propose a robust time-varying load modeling technique to assess CVR effects. The

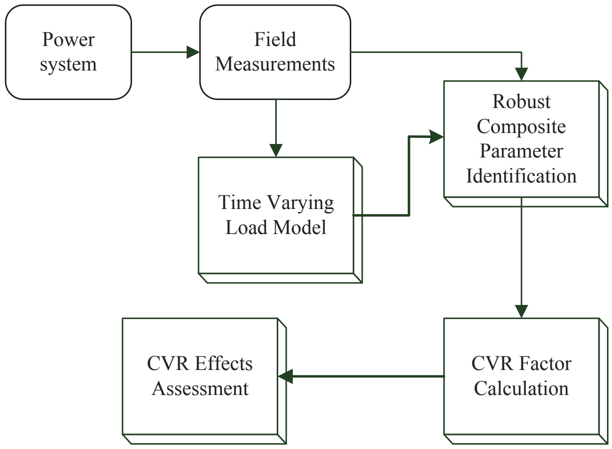

Fig. 1: Framework of the proposed robust CVR assessment method.

load behaviors at substations with CVR are captured by a time-varying ZIP load model, which is composed of constant impedance (Z), constant current (I), and constant power (P) elements, but with time-varying parameters driven by customer behaviors, weather and different system operation conditions. The intermittency of renewable generators may increase the frequency of sudden load changes, which lead to fast changes of load model parameters. This poses challenges for CVR assessment since power consumption changes resulted from the voltage reduction depend heavily on the accuracy of ZIP parameters. To effectively tracking the continuous changes of load model parameters, this paper proposes a robust recursive least squares based on the Huber M-estimator with a convex cost function and a strategically variable forgetting factor adjustment scheme. Here, the Huber M-estimator is to bound the influence of impulsive measurement noise or outliers while the variable forgetting factor is to help identify parameters of the ZIP model from measurement data in a real-time manner. Once a credible load model is identified, CVR factors can then be calculated.

II. Proposed Robust CVR Assessment Framework

Fig. 1 shows the proposed robust CVR assessment framework using filed measurements. It contains four major steps: time-varying load modeling, robust model parameter identification, CVR factor calculation and statistical assessment of CVR effects. The SCADA system continuously collects real and reactive power and voltage. After load models are selected, filed measurements are used as inputs for the proposed method to identify time-varying load parameters. CVR factors are then calculated, followed by the statistical analysis of CVR effects.

A. Time-Varying Load Modeling

The performance of CVR depends on LTV sensitivities. Therefore, the accurate assessment of LTV sensitivities is a key step of load modeling. In the literature, two types of load modeling are widely used, i.e., component-based and measurement-based modeling. The component-based approach develops load models from prior knowledge and its constituent parts. This is difficult to implement in large systems since load model parameters change with time. The measurement-based approach identifies and updates load model parameters using real-time field measurements. In this paper, the measurement-based approach is used to capture the continuous system variations.

The data required for measurement-based approach is the sampled voltage magnitude, and active and reactive power consumption at substations. Once field measurements are available, the next step is to develop a mathematical relationship between the voltage magnitude and power at the point of aggregation. There are three different approaches to relate the measured data for proper representation of a load, i.e., static, dynamic and composite models [11]. Since the purpose of CVR assessment is to analyze energy-savings under the steady-state system operation, the static load model is used.

A static load model describes the relationship between the bus voltage and the load power at a given instant of time. There are two forms of static load models, ZIP load model and exponential load model. Since CVR effects may change once LTV sensitivities change from one static load type (constant impedance) to another load type (constant power), the exponential load model may not be able to capture this transition accurately, while the ZIP load model can handle that by adaptively changing the percentage of the Z, I and P through online parameter estimators. In addition, because of human behaviors, weather conditions and continuous on/off switching of different loads, the load composition is unknown in most feeders and changes with time. Thus, a time-varying ZIP load model that captures the input-output relationship between power and voltage of the substation of interest is

$$P(t) = P_0 \left(a\left(t\right) \left(\frac{V\left(t\right)}{V_0} \right)^2 + b\left(t\right) \left(\frac{V\left(t\right)}{V_0} \right) + \gamma\left(t\right) \right), \quad (1)$$

where $a(t)$, $b(t)$ and $\gamma(t)$ are time-varying parameters that need to be estimated and $a(t)+b(t)+\gamma(t)=1$; P_0 is the base active power consumed at the nominal voltage V_0; $P(t)$ is the aggregate active power consumption at the point of interest. The expression for the reactive power has a similar form. By discretizing (1) and defining $z_k = {}^{P_k}/_{P_0}$, $Vl_k = {}^{V_k}/_{V_0}$, we obtain

$$y_k = 1 - z_k = \begin{bmatrix} 1 - Vl_k^2 & 1 - Vl_k \end{bmatrix} \begin{bmatrix} a_k \\ b_k \end{bmatrix} = \boldsymbol{H}_k^T \boldsymbol{x}_k, \quad (2)$$

where $a_k + b_k + \gamma_k = 1$.

Problem statement: Since y_k is subject to measurement errors (small or even large one that induces outliers) or measurement loss, the objective is therefore to find the parameter $\hat{\boldsymbol{x}}_k$ such that the error between measured system outputs and estimated model outputs at time instant k is minimized, i.e.,

$$\widehat{\boldsymbol{x}}_k = \arg\min_{\boldsymbol{x}_k} f\left(y_k - \boldsymbol{H}_k^T \boldsymbol{x}_k\right), \quad (3)$$

where $f(\cdot)$ is the objective function.

B. Robust Parameter Estimation

In the literature, the weighted least squares is widely used to estimate ZIP model parameters with the assumption that these parameters remain constant during the period to collect multiple measurements [12]–[14]. However, this assumption might not be valid given that load consumption is constantly changing with time due to weather and continuous voltage/VAR control device switchings. In other words, load model parameters are time variant [1], [10]. On the other hand, as investigated in [15]–[17] that owing to load variations caused by the weather, economic, social behavior, temporal correlations exist among loads in the same geographic area. Driven by the change of loads through power flow equations, the voltage should change at the same time, and as a consequence exhibiting temporal correlations. In addition, during the implementation of CVR, the time series reduction of voltage would result in time series changes of load power. Therefore, it is important to incorporate temporal correlations of voltage or power into the CVR assessment.

To capture temporal correlations and update load model parameters continuously, RLS can be used [18], [19]. The idea is to solve the following optimization problem recursively,

$$\widehat{\boldsymbol{x}}_k = \arg\min_{\boldsymbol{x}_k} \sum_{k=1}^{t} \lambda^{t-k} \left(y_k - \boldsymbol{H}_k^T \widehat{\boldsymbol{x}}_k\right)^2. \tag{4}$$

Although RLS has been successfully applied to many engineering problems and shown to have a certain degree of tracking capability, it suffers from several issues, such as slow convergence in sudden system changes because of a constant forgetting factor, vulnerability to impulsive measurement noise due to the use of least squares technique, etc [19]. Indeed, intermittent renewable-based power sources increase the probability of sudden changes in complex bus voltages along the feeder within a small timeframe, which may cause sudden changes of load model parameters [20]–[22]. There might be other reasons that could cause large parameter changes, such as switchings of voltage control devices, sudden load changes, etc. On the other hand, the field power and voltage measurements might be subject to gross errors caused by impulsive measurement noise, failures of metering devices, etc. Thus, a robust time-varying load parameter estimation method should be developed.

In this paper, the robust Huber M-estimator [23] formulation is adopted to enhance the statistical robustness of RLS, while a variable forgetting factor scheme is proposed to improve the adaptiveness of RLS for addressing sudden parameter changes. Formally, we have the following objective function,

$$J(\boldsymbol{x}_k) = \xi_k J_\varphi(\boldsymbol{x}_k) + (1-\xi_k) J_\lambda(\boldsymbol{x}_k), \tag{5}$$

where $J_\varphi(\boldsymbol{x}_k)$ and $J_\lambda(\boldsymbol{x}_k)$ represent the robust estimation criterion and RLS criterion with variable forgetting factors, respectively. They are expressed as follows

$$J_\varphi(\boldsymbol{x}_k) = \sum_{k=1}^{t} \lambda_1(k)^{t-k} \rho(r_{S_k}), \tag{6}$$

$$J_\lambda(\boldsymbol{x}_k) = \sum_{k=1}^{t} \lambda_2(k)^{t-k} \left(y_k - \boldsymbol{H}_k^T \boldsymbol{x}_k\right)^2, \tag{7}$$

where $\lambda_1(k)$ and $\lambda_2(k)$ are time-varying forgetting factors and will be shown in detail later; $r_{S_k} = {}^{r_k}/_{s}$ is the standardized residual; $r_k = y_k - \boldsymbol{H}_k^T \widehat{\boldsymbol{x}}_k$ is the residual; $s = 1.4826 \cdot b_m \cdot \text{median}_i |r_k(i)|$ ($i = 1, ..., L$ and L is the order of the median filter) is the robust scale estimate; b_m is a correction factor for unbiasedness at the Gaussian distribution; $\rho(\cdot)$ is the Huber convex function [23] of r_{S_k},

$$\rho(r_{S_k}) = \begin{cases} \frac{1}{2} r_{S_k}^2, & \text{for } |r_{S_k}| < c \\ c\,|r_{S_k}| - c^2/2, & elsewhere \end{cases}, \tag{8}$$

where c is a tuning parameter. The Huber score function is a blend of the minimum ℓ_1 and ℓ_2 norm function. As $c \to 0$, it approaches the ℓ_1 norm, which is equivalent to the median in the scalar case; and as $c \to \infty$, it tends to the ℓ_2 norm, which is equivalent to the mean in the scalar case. It is shown by Huber [23] that the ρ function is asymptotically optimally robust in the ϵ neighborhood of the Gaussian distribution. To achieve high statistical efficiency at Gaussian distribution or other non-Gaussian thick-tailed distributions, it is recommended to set c between 1 and 2.

Remark. *It is important to notice that a variable forgetting factor is useful for tracking both the slow and sudden variations of parameter values, whereas the robust Huber M-estimator plays a role in bounding the influences of outliers caused by impulsive noise, meter failures, etc. In other words, we need to distinguish the sudden system parameter changes and outliers so that the proper criterion can be chosen. This is solved by defining a binary decision parameter* $\xi_k \subseteq \{0,1\}$. *If* $\xi_k = 0$, *criterion (7) is chosen to track the sudden parameter changes, otherwise criterion (6) is adopted to suppress outliers and track slow variations of parameter values with modified variable forgetting factors. Thus, the key is to develop appropriate decision rule of* ξ_k, *which will be shown below.*

Define a ϵ-contamination model, i.e., $G = (1-\epsilon) F + \epsilon \Delta_r$, where F is the target distribution, such as Gaussian distribution; Δ_r is the unknown probability mass at r with a large variance compared to F, which is commonly used to simulate outliers or impulsive noise. ϵ is a binary independent identically distributed process so that the probability is close to zero when ϵ=1, and has a value close to one when ϵ=0. To this end, a decision rule can be developed using the statistics of mean and median [19] in each step:

$$\xi_k = \begin{cases} 1, s \cdot median\{|r_k(i)|\} < mean\{|r_k(i)|\} \\ 0, s \cdot median\{|r_k(i)|\} \geq mean\{|r_k(i)|\} \end{cases}, \tag{9}$$

where the median and mean filters of the absolute value of the residual are calculated on a sliding window with a length of L previous samples of residual.

If an abrupt parameter change is declared, the criterion (7) is used and solved recursively through (10)-(13)

$$\widehat{\boldsymbol{x}}_k = \widehat{\boldsymbol{x}}_{k-1} + \boldsymbol{K}_k \left(y_k - \boldsymbol{H}_k \widehat{\boldsymbol{x}}_{k-1}\right), \tag{10}$$

$$\boldsymbol{K}_k = \boldsymbol{\Sigma}_{k-1} \boldsymbol{H}_k^T \left(\lambda_2(k) \boldsymbol{I} + \boldsymbol{H}_k \boldsymbol{\Sigma}_{k-1} \boldsymbol{H}_k^T\right)^{-1}, \tag{11}$$

$$\boldsymbol{\Sigma}_k = \frac{1}{\lambda_2(k)} \left(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k\right) \boldsymbol{\Sigma}_{k-1}, \tag{12}$$

$$\lambda_2(k) = \begin{cases} \tau = 1 - \frac{{r_k}^2}{\eta\left[1 + H_k^T \Sigma_{k-1} H_k\right]} & if\ \tau > \lambda_{2_{\min}} \\ \lambda_{2_{\min}} & if\ \tau \leq \lambda_{2_{\min}} \end{cases}, \tag{13}$$

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where η is a constant chosen to satisfy the desired estimation quality in the stationary operation condition [19]; $\lambda_{2_{\min}}$ is the lower bound of the variable forgetting factor and is typically set around 0.8. This small forgetting factor enables RLS to emphasize on recent measurements while forgetting older information fast so as to track sudden changes.

On the other hand, if an outlier is detected, the criterion (6) is chosen. To minimize (6), one takes its partial derivative with respect to $\boldsymbol{x}_k$ and sets it to zero, yielding

$$\boldsymbol{R}_k^{H\rho}\boldsymbol{x}_k = \boldsymbol{P}_k^{x\rho}, \tag{14}$$

where

$$\begin{aligned}\boldsymbol{R}_k^{H\rho} &= \sum_{i=1}^{k} \lambda_1(k)^{k-i} q\left(r_k(i)\right) \boldsymbol{H}_i \boldsymbol{H}_i^T \\ &= \lambda_1(k)\,\boldsymbol{R}_{k-1}^{\boldsymbol{H}\rho} + q(r_k)\,\boldsymbol{H}_k\boldsymbol{H}_k^T\end{aligned}, \tag{15}$$

$$\begin{aligned}\boldsymbol{P}_k^{x\rho} &= \sum_{i=1}^{k} \lambda_1(k)^{k-i} q\left(r_k(i)\right) y_i \boldsymbol{H}_i \\ &= \lambda_1(k)\,\boldsymbol{P}_{k-1}^{x\rho} + q(r_k)\, y_k\boldsymbol{H}_k\end{aligned}, \tag{16}$$

and $q(r_k) = {\psi(r_k)}/{r_k}$; $\psi(r_k) = {\partial\rho(r_k)}/{\partial r_k}$. Equation (14) is called the M-estimator normal equation, while $\boldsymbol{R}_k^{H\rho}$ and $\boldsymbol{P}_k^{x\rho}$ represent the M-estimator correlation matrix of $\boldsymbol{H}_k$ and M-estimator cross correlation vector of $\boldsymbol{H}_k$ and y_k, respectively. Using the matrix inversion lemma and defining $\boldsymbol{\Sigma_k} = (\boldsymbol{P}_k^{x\rho})^{-1}$, (14) could be solved by

$$\widehat{\boldsymbol{x}}_k = \widehat{\boldsymbol{x}}_{k-1} + \boldsymbol{K}_k w_k \left(y_k - \boldsymbol{H}_k\widehat{\boldsymbol{x}}_{k-1}\right), \tag{17}$$

$$\boldsymbol{K}_k = \boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\left(\lambda_1(k)\boldsymbol{I} + w_k\boldsymbol{H}_k\boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\right)^{-1}, \tag{18}$$

where w_k is the weight of the kth measurement given by

$$w_k = \min\left(1, \frac{c \cdot sign\left(r_{S_k}\right)}{r_{S_k}}\right). \tag{19}$$

$\lambda_1(k)$ is a variable forgetting factor that is determined as follows: in cases that outliers are detected, $\lambda_1(k)$ retains its previous value, and its value is updated as follows when the next sample arrives,

$$\lambda_1(k) = \begin{cases} \tau = 1 - \frac{{r_k}^2}{\eta\left[1 + H_k^T\Sigma_{k-1}H_k\right]} & if\ \tau > \lambda_{1_{\min}} \\ \lambda_{1_{\min}} & if\ \tau \le \lambda_{1_{\min}} \end{cases}, \tag{20}$$

so as to track the slow variations of parameters. $\lambda_{1_{\min}}$ is the lower bound of the variable forgetting factor.

After the load parameter identification, the estimation error covariance matrix $\boldsymbol{\Sigma}_k$ needs to be updated such that the next recursive parameter estimation can be performed. It should be noted that when an outlier occurs, the error distribution no longer follows Gaussian distribution. Therefore, the updating of the parameter estimation error covariance matrix using the Gaussianity assumption, e.g., the updating of $\boldsymbol{\Sigma}_k$ through

$$\boldsymbol{\Sigma}_k = \frac{1}{\lambda_1(k)}\left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{\Sigma}_{k-1}, \tag{21}$$

$$\boldsymbol{K}_k = \boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\left(\lambda_1(k)\boldsymbol{I} + \boldsymbol{H}_k\boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\right)^{-1}, \tag{22}$$

will cause biased estimation and lead to a degraded performance of the estimator, resulting in poor tracking capabilities and lower statistical efficiency. The correct asymptotic covariance matrix of that estimator should be updated using the influence function based on the following theorem [24]:

Theorem 1. *As the number of the observation tends to be very large, the probability distribution of the estimated state tends to be the Gaussian distribution asymptotically with zero mean and an asymptotic covariance matrix* $\boldsymbol{\Sigma}_k$ *expressed as* $\boldsymbol{\Sigma}_k = \mathbb{E}[\boldsymbol{IF}\cdot\boldsymbol{IF}^T]$*, where* $\boldsymbol{IF}$ *is the influence function (IF) of an M-estimator.*

Corollary 1.1. *According to Hampel's proposal, the asymptotic variance matrix* $\boldsymbol{V}$ *of a linear regression model (*$\boldsymbol{Y} = \boldsymbol{X\theta} + \boldsymbol{\varepsilon}$ *for example) using M-estimator can be derived from the Influence Function (IF) given by*

$$\begin{aligned}\boldsymbol{V} &= \lim_{m\to\infty} Var\left(\sqrt{m}\widehat{\boldsymbol{\theta}}_m\right) = \mathbb{E}\left[\boldsymbol{IF}\cdot\boldsymbol{IF}^T\right] \\ &= \frac{\mathbb{E}\left[\psi^2(r_S)\right]}{\left(\mathbb{E}\left[\psi'(r_S)\right]\right)^2}\left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\end{aligned}, \tag{23}$$

where r_S *is the standardized residual and* m *is the number of measurements.*

Thus, following Theorem 1 and Corollary 1.1, we could also derive the asymptotic covariance matrix of our estimator as

$$\boldsymbol{\Sigma}_k = \frac{\frac{\mathbb{E}\left[\psi^2(r_S)\right]}{\left(\mathbb{E}\left[\psi'(r_S)\right]\right)^2}}{\lambda_1(k)}\left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{\Sigma}_{k-1}, \tag{24}$$

$$\boldsymbol{K}_k = \boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\left(\lambda_1(k)\boldsymbol{I} + w_k\boldsymbol{H}_k\boldsymbol{\Sigma}_{k-1}\boldsymbol{H}_k^T\right)^{-1}. \tag{25}$$

Remark. *When there is no bad measurement, all the weights of measurements are equal to 1, i.e.,* $w_k = 1$*, therefore,* $\boldsymbol{\Sigma}_k$ *reduces to* $\frac{\frac{\mathbb{E}\left[\psi^2(r_S)\right]}{\left(\mathbb{E}\left[\psi'(r_S)\right]\right)^2}}{\lambda_2(k)}\left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{\Sigma}_{k-1}$ *(in this case,* $\lambda_2(k)$ *and* $\lambda_1(k)$ *have the same updating formula).* $\frac{\mathbb{E}_F\left[\psi^2(r_S)\right]}{\left\{\mathbb{E}_F\left[\psi'(r_S)\right]\right\}^2}$ *is close to 1 [23]. For example, this value is 1.0369 for c=1.5,* $\boldsymbol{\Sigma}_k \simeq \frac{1}{\lambda_2(k)}\left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{\Sigma}_{k-1}$*, which is the error covariance matrix of RLS estimator with variable forgetting factors. Otherwise, outliers are downweighted and suppressed. This further explains the good property of the proposed robust estimator, that is, it has a good robustness to outliers while maintaining a high statistical efficiency under Gaussian distribution.*

C. CVR Effects Assessment

Definition: *CVR factor is defined as the percentage reduction of "Quantities"* ΔW *with respect to the percentage reduction of voltage* V*, where* ΔW *may refer to real or reactive power for a particular part of a distribution system (load, feeder, substation, or utility) [1], [5]*, i.e.,

$$CVR_f = \frac{\Delta W\%}{\Delta V\%}. \tag{26}$$

In this paper we use the estimated load model parameters to calculate the CVR factor as the ratio of changes in either real or reactive power to changes in voltage. This approach is able to estimate the power consumption at an arbitrary voltage level given the initial voltage and power injection values. At time instant k, $\widehat{P}_k$ and $\widehat{Q}_k$ are the estimated real and reactive

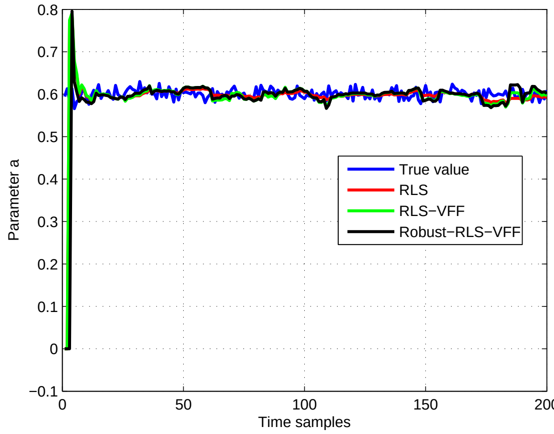

Fig. 2: Tracking parameter a without sudden changes and outliers.

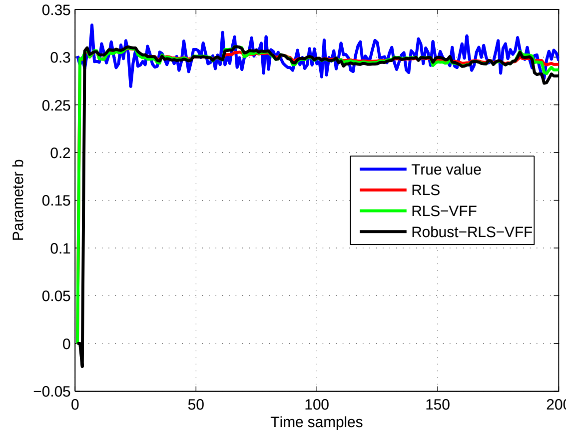

Fig. 3: Tracking parameter b without sudden changes and outliers.

power, respectively. Therefore, the ratio of changes of real power $\delta\widehat{P}_k$ (or reactive power $\delta\widehat{Q}_k$) to changes in voltage $\delta\widehat{V}_k$ can be calculated through

$$\frac{\delta P_k}{\delta V_k} = \frac{P_k - \widehat{P}_k}{V_k - \widehat{V}_k}, \tag{27}$$

$$\frac{\delta Q_k}{\delta V_k} = \frac{Q_k - \widehat{Q}_k}{V_k - \widehat{V}_k}, \tag{28}$$

where P_k, Q_k and V_k are the values without CVR.

Thus, the CVR factors associated with real and reactive power could be further calculated as

$$CVR_f^P = \frac{\delta P_k}{\delta V_k} . \frac{V_k}{P_k}, \tag{29}$$

$$CVR_f^Q = \frac{\delta Q_k}{\delta V_k} . \frac{V_k}{Q_k}. \tag{30}$$

Remark. *"CVR factor" is usually related with active power as the electricity consumption is normally billed for active power. In other words, lower active power consumption results in a lower bill. Since reactive power reduction does not lead to economic benefits in current industry settings (it may certainly be beneficial for system operations), the reactive CVR effects are not discussed in detail in this paper.*

III. Simulation Results

Utilities are interested in aggregate CVR effects at the substation level. Therefore, we focus on identifying load models for substations with CVR. The IEEE 118-bus test system is used in this paper. We randomly pick 10 load buses and assign time-varying ZIP models to these buses. All other buses are represented by the constant PQ model. For illustration, we only show the results of one of the 10 substations. To simulate the measurements at substations for load parameter identification, the power flow is executed using specified load parameters at each time instant. The calculated real and reactive power and the voltages are taken as measurements, and corrupted by Gaussian noise with zero mean and standard deviation of 10^{-2}. 200 measurement samples with different

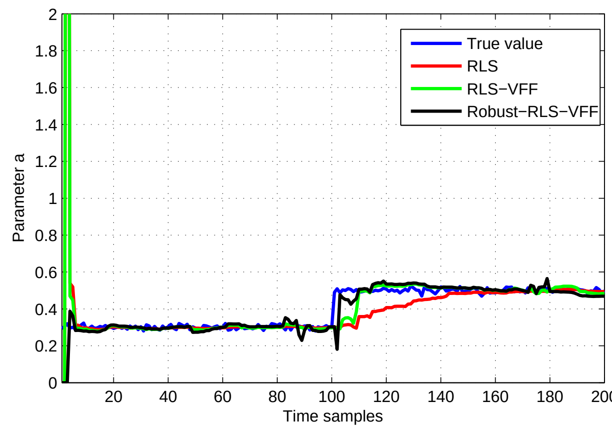

Fig. 4: Tracking parameter a with sudden parameter changes at time sample 100 but without outliers.

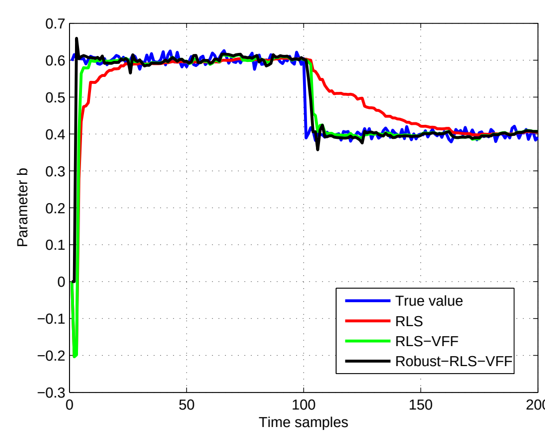

Fig. 5: Tracking parameter b with sudden parameter changes at time sample 100 but without outliers.

load parameters are obtained. Three identification methods, traditional RLS with a constant forgetting factor 0.95, RLS with variable forgetting factors (RLS-VFF) and the proposed robust RLS with variable forgetting factors (Robust-RLS-

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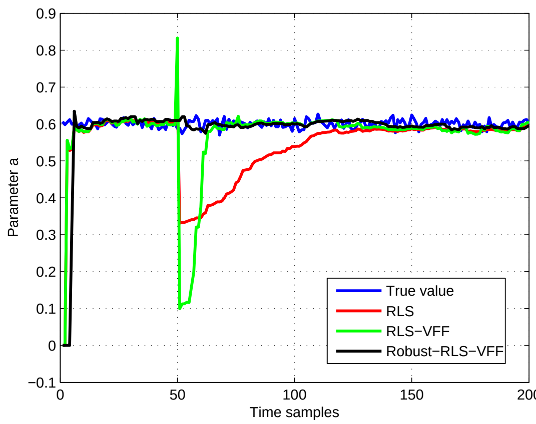

Fig. 6: Tracking parameter a without sudden parameter changes but with outliers at time sample 50.

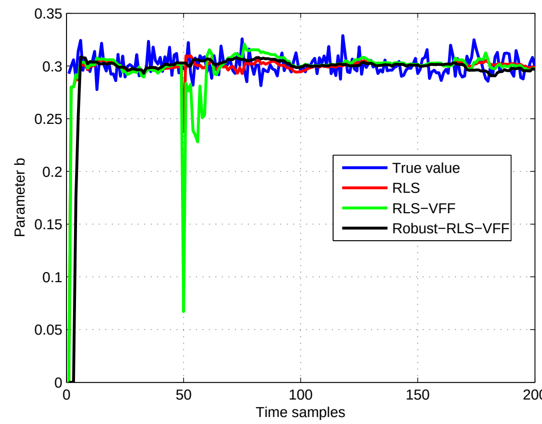

Fig. 7: Tracking parameter b without sudden parameter changes but with outliers at time sample 50.

VFF), are used for comparisons. The threshold for the Huber cost function is 1.5; $\lambda_{1_{\min}} = \lambda_{2_{\min}} = 0.8$. To make the results statistically sound, 100 Monte Carlo simulations for each case is performed and the average value is considered as the final parameter estimates.

A. Case 1: Continuous Load Parameter Changes Without Sudden Changes and Outliers

In this scenario, we assume the initial ZIP parameters are a =0.3, b =0.6 and γ =0.1. A Gaussian random variable with zero mean and standard deviation 10^{-2} is added to a and b to simulate continuous parameter changes while parameter $\gamma = 1 - a - b$. No sudden parameter change or outlier occurs. Due to the space limitation, only the results of parameters a and b are presented (parameter γ could be easily calculated by $\gamma = 1 - a - b$). Figs. 2 and 3 show the parameter tracking results. It could be observed that three methods have a similar performance under this operation condition.

B. Case 2: Continuous Load Parameter Changes with Sudden Changes but Without Outliers

In this case, we have the same assumptions as Case 1 except that at the time instant 100, parameters a and b change to 0.5 and 0.4 suddenly because of different types of load switchings or other control actions. The parameter tracking results are presented in Figs. 4 and 5. From these two figures, we can see that the proposed Robust-RLS-VFF is the fastest method to track sudden parameter changes, followed by the RLS-VFF with strategically tuned forgetting factors to handle sudden system changes. The RLS has the slowest tracking speed because of a constant forgetting factor that needs longer time to converge to the new parameter.

C. Case 3: Continuous Load Parameter Changes without Sudden Changes but with Outliers

In this case, we have the same assumptions as Case 1 except that the power measurements at the time instant 50 become outliers with 30% measurement errors due to the cyber attacks, impulsive measurement noise or meter failures, etc.

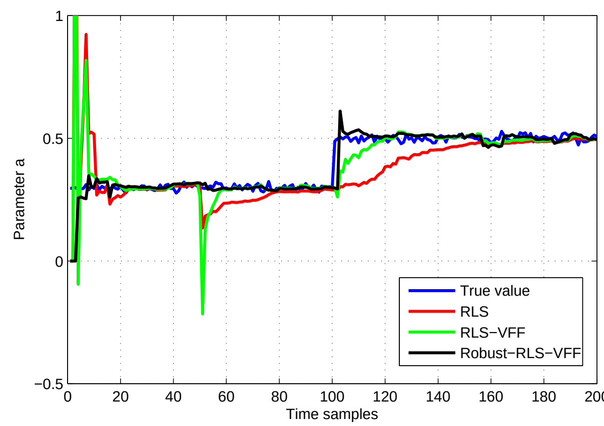

Fig. 8: Tracking parameter a with both sudden parameter changes and outliers at time samples 50 and 100, respectively.

Figs. 6 and 7 show the results of three methods. It can be seen that RLS-VFF and RLS could not suppress the impact of outliers because the LSE formulation is highly sensitive to gross errors. In addition, it is interesting to notice that RLS-VFF is more vulnerable to outliers than RLS. The reason is detailed as follows: the occurrence of outliers can cause large residuals, which makes RLS-VFF erroneously think there is a sudden system change; then the forgetting factor is changed to a smaller value, indicating large weights to current measurements while forgetting the previous information; however, the current measurements are outliers, which makes the estimation even worse. By contrast, the proposed Robust-RLS-VFF is able to effectively suppress the outliers through the strategically updating mechanism of forgetting factors and the robust Huber M-estimator.

D. Case 4: Continuous Load Parameter Changes with both Sudden Changes and Outliers

In this case, we create a more challenging situation, where the system is subject to both sudden load parameter changes and outliers, to test the performance of the three methods. This case has the same assumption as Case 3 except that at

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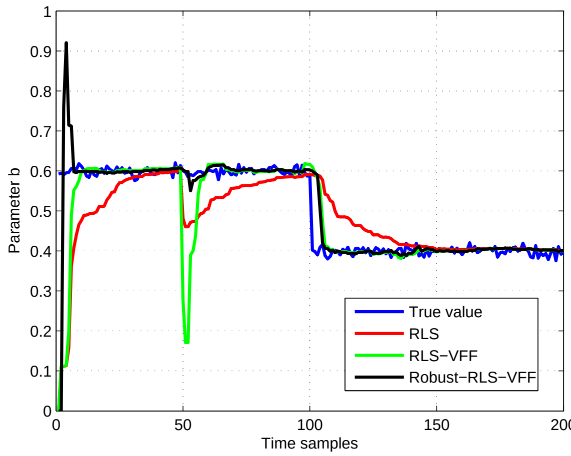

Fig. 9: Tracking parameter b with both sudden parameter changes and outliers at time samples 50 and 100, respectively.

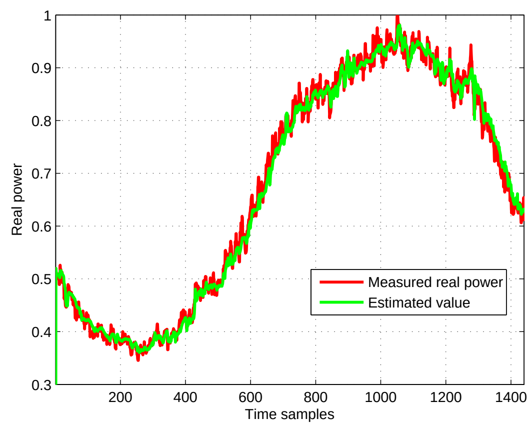

Fig. 10: Measured active power and the estimated model outputs on June 21, 2012

the time instant 100, parameters a and b suddenly change to 0.5 and 0.4, respectively. The test results are shown in Figs. 8 and 9. It can be concluded that the proposed Robust-RLS-VFF could effectively handle both sudden parameter changes and outliers, while the RLS-VFF has a better performance in tracking sudden parameter changes than RLS, but is more sensitive to outliers than RLS. The above case studies show that the proposed Robust RLS-VFF is more suitable for CVR assessment since it can capture continuingly-changing load characteristics in practical power systems.

IV. CVR Assessment with Filed Measurements

In this section, Robust-RLS-VFF is applied for CVR assessment using field measurements from a utility's substation. The CVR factors of this substation are calculated using the proposed method. The CVR tests were conducted from June 2012 to August 2012. During this period, 44 days are voltage-reduction days, while the remaining days are normal-voltage days. The measurement devices installed at the substation measure kW, kVAR, voltage and current at a one-minute interval. The performance of load modeling can be validated

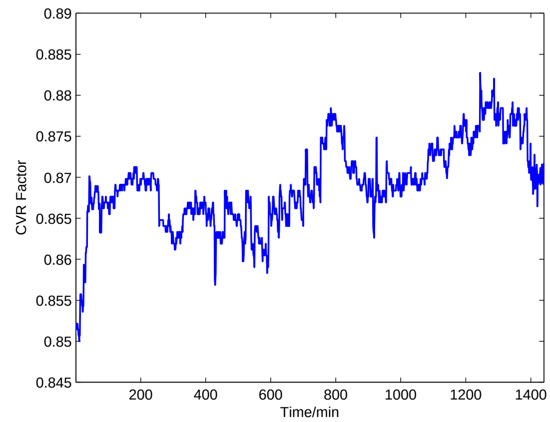

Fig. 11: CVR factors calculated by the proposed method on June 21, 2012

TABLE I: Performance Comparisons of Three Methods With Field Measurements

Index	RLS	RLS-VFF	Robust-RLS-VFF
REP	1.52%	1.35%	0.93%
MAPE	1.21%	1.06%	0.88%

by comparing the outputs of the identified model with the measured system data. To do this, we randomly pick up one of CVR test days, i.e., June 21, 2012 as an example. Fig. 10 shows the comparison results of the estimated and measured active power. We can see from this figure that the load model identified by the proposed method are very close to the system measurements, demonstrating its effectiveness. To further justify the effectiveness and robustness of the proposed method, we show the performance comparison results of our proposed method and the other two non-robust methods, in terms of relative error percentage (REP) and mean absolute percentage error (MAPE). The definitions of these two indices can be found in [25]. The results are presented in Table I. It can be seen that the proposed method outperforms the other two methods. Fig. 11 shows CVR factors calculated by the proposed method during the voltage-reduction period on a test day. It can be observed that CVR factors vary with time due to different load consumption patterns.

To further analyze the statistics of CVR factors for June, July and August, we have calculated the CVR values for the test days in 1-min time interval. Figs. 12-14 show histograms of CVR factors for each month in summer 2012. CVR factors in these three months roughly follow Gaussian distributions with mean 0.83, 0.85 and 0.87, respectively. The lower and upper quartiles are 0.77 and 0.89 in June, 0.80 and 0.91 in July, 0.84 and 0.94 in August, respectively. It shows that CVR factors are changing due to different load consumption patterns at different time, but they do not vary a lot. It is also interesting to notice that CVR results obtained here are fairly consistent with the seasonal CVR factors reported by several utilities, where CVR factors of AEP, NEEA and HQ vary within 0.78-1.01 during summer [26], [27].

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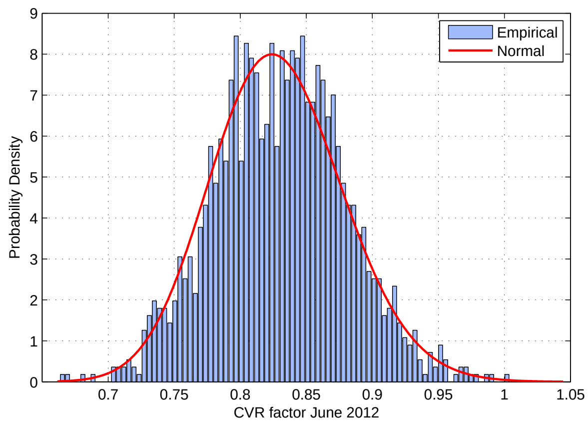

Fig. 12: Histogram of CVR factors in June 2012

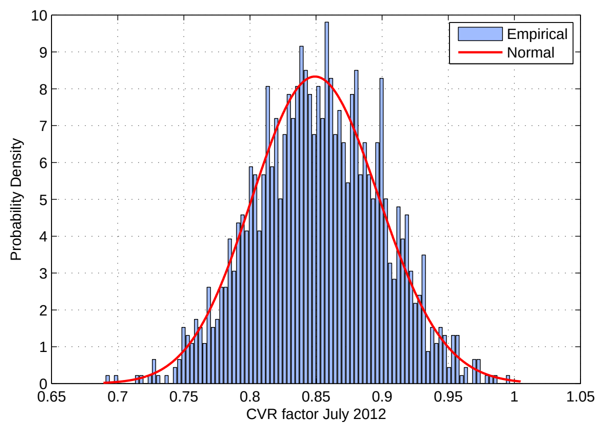

Fig. 13: Histogram of CVR factors in July 2012

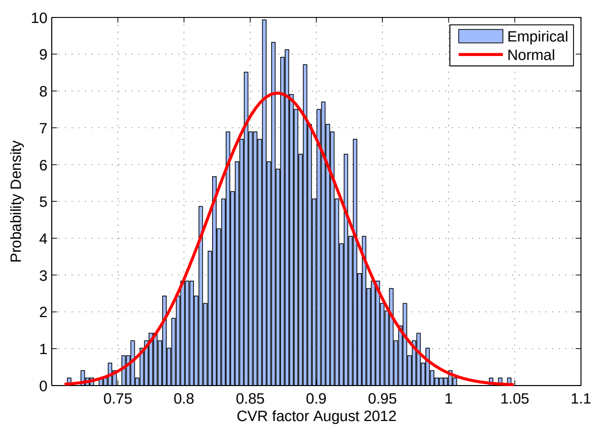

Fig. 14: Histogram of CVR factors in August 2012

V. Conclusion

This paper develops a robust time-varying load modeling technique to consider the impacts of load stochasticity on the assessment of CVR effects. The proposed robust parameter identification method can track steady and sudden step changes of load parameters, and deal with the occurrence of bad measurements thanks to variable forgetting factors and the robust Huber M-estimator. Accurate time-varying load models can be identified and CVR factors could be calculated appropriately. The proposed method is different from previous methods on CVR effect assessment as it does not require control groups or any assumptions of linear relationships between the load and its impact factors. Therefore, our method could be regarded as a measurement-based technique for practical CVR applications.

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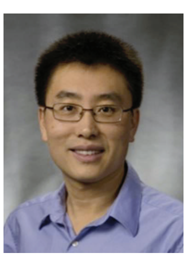

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