Nonlinear Multiple Models Adaptive Secondary Voltage Control of Microgrids

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Abstract—This article proposes a model-free secondary voltage control (SVC) for microgrids (MG) using nonlinear multiple models adaptive control. Firstly, a linear robust adaptive controller is designed to guarantee the voltage stability in the bounded-input-bounded-output (BIBO) manner so as to meet the operation requirements of MGs. Secondly, a nonlinear adaptive controller is developed to improve the voltage tracking performance with the help of artificial neural networks (ANNs). A switching mechanism for coordinating such two controllers is designed to guarantee the closed-loop stability while achieving accurate voltage tracking. By an online identification based on the input and output data of MGs, the proposed method does not resort to any a priori information of system model and primary control, thus exhibiting good robustness, ease of deployment and disturbance rejection.

Index Terms—Artificial neural network (ANN), microgrid (MG), multiple models, adaptive control, secondary voltage control (SVC).

NOMENCLATURE

Abbreviations

ANN Artificial neural network
BIBO Bounded-input bounded-output
DER Distributed energy resource
LAC Linear adaptive controller
ARMAX Auto-regressive moving average with exogenous input model
MG Microgrid
MGCC Microgrid central controller
NAC Nonlinear adaptive controller
PI Proportion-Integral
PV Photovoltaic
SVC Secondary voltage control
SNR Signal-to-noise ratio.

Variables

\( J \) Objective function
\( E^s \) Vector of voltage reference from SVC
\( e_L, e_N \) Error vectors of linear and nonlinear models
\( \hat{e} \) Voltage tracking error vector
\( h \) Linear-transformed unmodeled dynamics
\( \delta h, \hat{\delta h} \) Residual between real voltage and estimated voltage and its estimation using ANN
\( k \) Time step index
\( v_{p, \text{def}} \) Predefined voltage reference
\( v_o \) Vector of voltage magnitude
\( v_{oi} \) Terminal voltage magnitude of the \( i \)th DER
\( d_q \) components of \( v_{oi} \)
\( W \) Estimation of the ideal weight matrix
\( \Psi, \hat{\Psi} \) Vector of output and input voltage and its rearrangement
\( y \) Linear transformed output voltage vector
\( \hat{y}_L, \hat{y}_N \) Estimated transformed output voltage vectors using linear and nonlinear model identifier
\( \Delta \) Positive constant
\( \epsilon \) Small positive constant
\( \mu \) Non-negative constant
\( \Phi \) Unmodeled dynamics
\( x \) Compact state variable vector of a MG
\( \psi_i \) State vector of the \( i \)th DER
\( \xi \) Performance index of switching mechanism.

Parameters

\( A(\cdot) \) Matrix polynomial of \( n \)th-order backward shift operator
\( B(\cdot) \) Matrix polynomial of \( (n-1) \)th-order backward shift operator
\( d \) Relative degree
\( F(\cdot) \) Diagonal and stable weight matrix polynomial
\( K(\cdot), L(\cdot) \) Matrix polynomials of \( (n-1) \)th-order
\( m \) Number of DERs
\( n \) MG system order
\( R \) Diagonal real matrix
\( \rho \) Bound of magnitude of unmodeled dynamics
\( \theta \) Input-output parameter matrix
\( \hat{\theta}_L, \hat{\theta}_N \) Estimated parameter matrices using linear and nonlinear models
\( \Delta T \) Sampling time of secondary control.

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Sets

- \( \mathbb{R} \): Set of real numbers
- \( \Omega \): Set of linear parameter matrix polynomials

I. INTRODUCTION

Microgrids (MGs) are localized small-scale power systems consisting of interconnected loads and distributed energy resources (DERs), which can operate in both grid-connected and islanded modes. Compared with traditional fossil-fuel-based power grids, they have the advantages of fast demand response, low-carbon consumption, flexible utilization of DERs and high self-healing capability, etc [1], [2].

Despite of many benefits, MGs also bring some new control challenges. One of the key issues is the voltage tracking in the islanded mode. As known, hierarchical control is a popular choice for MGs, in which the primary voltage control with fast response maintains the stability while the secondary voltage control (SVC) corrects the voltage deviations [1], [2].

As per the control architecture and communication requirements, MG control methods can be classified into three main categories: centralized, decentralized and distributed [3]. Centralized approaches are usually implemented with a microgrid central controller (MGCC) and point-to-point communication network. It has well served the industry for decades and performs many practical merits. For instance, they are easy to implement and house and often less costly for small-scale systems [4]. Moreover, centralized architecture provides the best foundation for advanced control applications since all relevant data can be collected and processed in a single controller. However, it may suffer from single point of failure [5]. Redundant communication systems can be installed to enhance the reliability; nonetheless, it will lead to additional cost [6]. Another solution is using decentralized or distributed control approaches. Decentralized control is implemented with local SVC controllers without communication network, assuming that the interactions between subsystems are negligible. However, this assumption does not always hold and might result in poor system-wide performance. Distributed control consists of local controllers and a sparse communication network. Averaging-based and consensus-based distributed SVC have been well investigated [7]. In the averaging-based SVC, each DER measures its required data and transmits them to all the other units [8]. The SVC signal is then calculated by averaging the received data from other DERs [9].

By employing the broadcast gossip algorithm, the required communication links can be reduced and the algorithm can converge to an equilibrium [10]. In the consensus-based SVC, the communication network is reduced more by transferring the required data just among the neighbor DERs [11]–[13].

Conventional SVC methods are based on priori accurate models [14]. The input-output feedback linearization control [15] that builds on the full knowledge of MG models and primary control might contradict the concept of hierarchical control. Any changes of system structure or parameters could affect the control performance and could even result in instability. Some nonlinear control methods, e.g., model predictive control [16], sliding mode control [17], internal model control [18], also have similar drawbacks. Several SVC strategies are designed based on specified models of primary controllers and inner controllers [3], [19]–[21], which restricts their generalization. A finite-time control-based method [22] was proposed to overcome such drawback. To alleviate the dependence on accurate models, robust control [23], predictive control [24], and variable-structure control [25] methods have been investigated. To overcome time-varying communication delays and communication noise disturbances, robust secondary control approaches have been studied in [26], [27]. However, partial model and uncertainty dynamics are still required for robust control and variable-structure control, though they do improve robustness.

Recently, model-free control has attracted a lot of attention due to its advantages of robustness and flexibility [28]. Reference [29] proposed a data-driven adaptive voltage control scheme for interlinking converters in interlinked hybrid ac/dc MGs, where the inner loop adopts a data-driven adaptive voltage control and the SVC is essentially a Proportion-Integral (PI) controller. In [30], a bi-level distributed voltage control scheme was proposed, where the high-level controller is designed for loss minimization; the low-level controller regulates the power output and terminal voltage. In [31], a model-free sliding mode control was adopted where the parameters are tuned with heuristic techniques, nevertheless, it suffers from chattering problem due to the nature of sliding mode control [32]. Distributed averaging-based PI controllers for secondary frequency and voltage control were developed in [8], [9], [33]; however, they still require MG network information for controller parameter design.

Though most of SVC methods establish on linearized, [5], [7], [11], [19] or nonlinear system models [3], [12], [20], [21], unfortunately, the detailed MG information including network topology, line impedances and loads, may be fully or partially unavailable to establish accurate models in some cases. Moreover, since there are uncertainty dynamics and disturbances in DER-rich MGs, it is very hard to precisely capture such dynamics [34], [35]. Clearly, models with poor accuracy can significantly deteriorate the control performance. For the existing model-free control methods, they mostly resort to PI control, which often suffers from high starting overshoot, high sensitivity to controller gains and sluggish response to disturbances [36].

Our Contribution: To address these challenges, we propose a multi-variable robust adaptive SVC method for MGs, which builds on the multiple models and artificial neural networks (ANNs) that are exploited to estimate the unmodeled dynamics of MGs. The controller consists of two separate linear and nonlinear modes that are coordinated by a tailored switching strategy. In normal operation, the SVC operates under the nonlinear control mode which achieves the accurate voltage tracking. It will switch to the linear control mode so as to guarantee the stability once there are large disturbances. The proposed method is inherently model-free, in the sense that it does not rely on a priori knowledge of MG topology, line impedances and load demands, which enables independent designs between different control layers while enhancing...
the robustness against uncertainties. We rigorously prove the
global bounded-input-bounded-output (BIBO) stability of the
controller and the equivalence between the tracking error and
identification error of unmodeled dynamics. This implies that
the accurate tracking can be achieved by properly designing
the hyper-parameters of ANNs. Besides, we also analyze and
test the robustness of the controller against time delays and
communication noise disturbances.

The remainder of this article is organized as follows.
Section II briefly introduces MGs with a hierarchical control
structure. Section III presents the model-free SVC along
with the closed-loop stability analysis. Simulation results are
presented in Section IV. Section V offers conclusions and
future directions. All of the technical proofs are collected in
the Appendix.

II. PROBLEM STATEMENT

A. Hierarchical Control of MGs

The hierarchical control structure is illustrated in Fig. 1.
Primary control generally results in voltage deviations since
it follows the droop control law. SVC is therefore used to
compensate the deviations of voltage. In the islanded mode,
the reference voltages, compactly denoted by \( V_o^\text{ref} \), are
generally set as the nominal voltage of the MG, while in the
grid-tied mode, they are determined by the tertiary control [3].
SVC generates control inputs \( E_i^* \), \( i = 1, \ldots, m \), according to
the references and they are dispatched to each local primary
controller of DERs. Then, the primary control calculates the
voltage reference \( v_{oi}^* \) for the local inner control loops. Finally,
the measured output voltages of DERs \( v_{oi} \) are measured and
fed back to the SVC.

The secondary control has much slower dynamic response
compared to primary control, which decouples the primary
and secondary control [2]. This enables independent controller
design at different layers. However, the flexibility of primary
control is always limited to guarantee control performance
when model-based control algorithms (e.g., feedback lineariza-
tion and sliding mode control) are applied in the secondary
layer. The a priori structures and parameters of primary control
should be considered in SVC design and uncertainties
and disturbances of primary layer could lead to instability and
large tracking errors of MGs. This motivates us to develop a
robust model-free SVC without knowing any specifications of
primary layer.

B. Islanded MG System Description

In an islanded MG, the primary and inner control structures of
inverter-based DERs are shown in Fig. 2. In the islanded
mode, the control input of primary droop voltage controller is
\( E_i^* \), \( \forall i \), which is obtained from SVC and the system output is
the terminal voltage \( v_{oi} \). Such MG system can be compactly expressed by a nonlinear state-space model as,

\[
\dot{x}(t) = f(x(t), E^*(t)) \tag{1a}
\]
\[
v_o(t) = g(x(t)) \tag{1b}
\]

where \( v_o := [v_{o1}^*, \ldots, v_{om}^*]^T \), \( x := [x_1^T, \ldots, x_m^T]^T \), \( E^* := [E_1^*, \ldots, E_m^*]^T \); \( x_i \) denotes the internal state variables of
ith DER; \( f \) and \( g \) are the functions representing the nonlinear
dynamic system.

Remark 1: Note that, Fig. 2 is only used to illustrate how
the control signal \( E_i^* \) acts on the primary control layer, which
is actually not needed for our SVC design benefiting from the
model-free nature. In addition, this article focuses on SVC, so
the design of frequency control is not limited, which also gives
freedom to primary control, e.g., the PLL may not be needed
when droop characteristics control the frequency [37]. Besides,
functions \( f \) and \( g \) and state variables \( x \) are not necessarily
required.

III. MODEL-FREE SVC BASED ON NONLINEAR MULTIPLE
MODELS ADAPTIVE CONTROL

In this section, a novel SVC method based on nonlinear
multiple models adaptive control with unmodeled dynamics is
proposed. We first present the design of linear and nonlinear
controllers, respectively. Then, the controller parameter iden-
tification method is given. Finally, a switching mechanism is
designed to coordinate the linear and nonlinear parts.

A. Optimal Controller Design for Voltage Regulation

Given that the measurements are sampled, system (1) is
discretized as,

\[
x(k+1) = f(x(k), E^*(k)) \tag{2a}
\]
\[
v_o(k) = g(x(k)) \tag{2b}
\]

where \( E^* \in \mathbb{R}^m \), \( v_o \in \mathbb{R}^m \), \( x \in \mathbb{R}^n \). The origin is an equilibrium
of function \( f \) and \( g \).

If system (2) is observable for \( n \)th order, the state variables
of MGs \( x(k) \) can be expressed as a function of input and output
variables, \( v_o(k), \ldots, v_o(k-n+1), E^*(k), \ldots, E^*(k-n+1) \). Thus, (2) can be represented with only voltage control inputs
and voltage outputs by the auto-regressive moving-average
with exogenous input (ARMAX) model as,

\[
A(z^{-1})v_o(k+d) = B(z^{-1})E^*(k) + \varphi [v_o(k+d-1), \ldots, v_o(k+d-n), E^*(k), \ldots, E^*(k-n+1)] \tag{3}
\]
where \( A(z^{-1}) \) is a \( m \times m \) matrix polynomial of \( n \)th-order backward shift operator; \( B(z^{-1}) \) is a \( m \times m \) matrix polynomial of \( (n - 1) \)th-order backward shift operator; \( d \) (1 \( \leq d \leq n \)) is the relative degree; \( \varphi[\cdot : \cdot] \) is the unmodeled dynamics, which is a higher-order nonlinear function of \( v_o(k), \ldots, v_o(k - n + 1), E^*(k), \ldots, E^*(k - n + 1) \) [38]. \( n \) and \( d \) are unknown if the detailed model of primary controllers and MGs is not available. However, they can be determined by the method in [39]. Moreover, the following assumptions are widely believed to hold for MGs in practice.

**Assumption 1:** (i) The internal dynamics of MGs are globally uniformly asymptotically stable; (ii) matrix polynomials \( A(z^{-1}) \) and \( B(z^{-1}) \) lie in a closed and bounded set \( \Omega \).

Assumption 1(i) ensures that the voltage control input \( E^* \) will not grow faster than the output voltage \( v_o \), indicating the MG is a minimum-phase system. Note that, this assumption is not necessary if the linear part of system (2) is asymptotically stable and thus, the proposed method can be applied to this kind of non-minimum-phase nonlinear system [40].

To ensure the stability while improving the voltage tracking performance, two separate optimal controllers are designed.

We first define a cost function on voltage tracking errors, 

\[
J := \left\| F(z^{-1})v_o(k + d) - Rv_o^\text{ref}(k) \right\|^2,
\]

(4)

where \( v_o^\text{ref} \in \mathbb{R}^m \) is voltage reference vector; \( F(\cdot) \) denotes a \( m \times m \) weight matrix polynomial, which is stable and diagonal; \( R \) is a \( m \times m \) diagonal real matrix.

To minimize (4), an optimal control law is designed as,

\[
L(z^{-1})B(z^{-1})E^*(k) + K(z^{-1})v_o(k) + h[\cdot : \cdot] = Rv_o^\text{ref}(k) \quad (5)
\]

where \( L(z^{-1}) \) denotes a \( m \times m \) \((n - 1)\)th order polynomial, \( K(z^{-1}) := K_0 + K_1z^{-1} + \cdots + K_{n-1}z^{-(n+1)} \) is a \( m \times m \) matrix polynomial and \( h[\cdot : \cdot] := L(z^{-1})\varphi[\cdot : \cdot] \). \( L(z^{-1}) \) and \( K(z^{-1}) \) can be calculated by,

\[
F(z^{-1}) = L(z^{-1})A(z^{-1}) + z^{-d}K(z^{-1}).
\]

(6)

\( h[\cdot : \cdot] \) in (5) is a linear transformation of unmodeled dynamics \( \varphi[\cdot : \cdot] \), which can be estimated using ANNs. Let \( \hat{h}[\cdot : \cdot] \) be its estimation, and then substitute (5) into (3), one can obtain,

\[
F(z^{-1})v_o(k + d) = Rv_o^\text{ref}(k) + h[\cdot : \cdot] - \hat{h}[\cdot : \cdot] \quad (7)
\]

where \( F(z^{-1}) \) can be selected as a diagonal matrix such that its characteristic polynomial describes the poles of (7) and \( R \) can be chosen as \( F(1) \). If we obtain the linear parts of the system, the tracking error \( e = F(z^{-1})v_o(k + d) - Rv_o^\text{ref}(k) \) of the closed-loop system equals \( h[\cdot : \cdot] - \hat{h}[\cdot : \cdot] \). With proper configuration of the ANNs, \( e \) can be controlled to be arbitrarily small [38].

If the high-order nonlinear term \( h[\cdot : \cdot] \) is small enough, (5) can be simplified as a linear control law as,

\[
L(z^{-1})B(z^{-1})E^*(k) + K(z^{-1})v_o(k) = Rv_o^\text{ref}(k). \quad (8)
\]

B. Multiple Models Adaptive Control Based on ANNs

1) Identification of Controller Parameters: To achieve model-free control with unknown MG parameters, we propose to exploit the adaptive control method. From (3) and (6), we can obtain

\[
y(k + d) = \theta^T\Psi(k) + h[\hat{\Psi}(k)]. \quad (9)
\]

where \( y(k + d) := F(z^{-1})v_o(k + d) \) denotes the transformed output voltage; \( \theta := [K_0, \ldots, K_{n-1}, LB_0, \ldots, LB_{d-2}]^T \) denotes the input-output parameter matrix; \( \Psi(k) := [v_o(k)^T, \ldots, v_o(k - n + 1)^T, E^*(k)^T, \ldots, E^*(k - n - d + 2)^T]^T \) is the vector collecting all the output and input voltages, and \( \hat{\Psi}(k) := [v_o(k), \ldots, v_o(k - n + 1), E^*(k), \ldots, E^*(k - n - d + 2)] \). From Assumptions 1(ii), one can know that the parameter matrix \( \theta \) lies in a certain closed and bounded set. Assuming the unmodeled dynamics \( h[\cdot : \cdot] \) are globally bounded by a known positive constant \( \rho \), i.e., \( \| h[\cdot : \cdot] \| \leq \rho \), we propose the linear and nonlinear model estimators for parameter identification.

The linear estimator is designed as,

\[
\hat{y}_L(k + d) = \hat{\theta}_L(k)^T\Psi(k) \quad (10)
\]

where \( \hat{y}_L \) and \( \hat{\theta}_L(k) \) are linear estimated transformed output voltage and linear estimated parameter vectors, respectively.
The update law is designed as,
\[
\hat{\theta}_L(k) = \text{proj}\left\{\hat{\theta}_L(k)\right\},
\]  
(11)
\[
\hat{\theta}_L'(k) = \hat{\theta}_L(k) - \frac{\eta_L(k)\Psi(k-d)e_l(k)^T}{1 + \|\Psi(k-d)\|^2},
\]  
(12)
\[
\eta_L(k) = \begin{cases} 
1 & \text{if } \|e_L(k)\| > 2\rho, \\
0 & \text{otherwise},
\end{cases}
\]  
(13)

where \(e_l(k)\) is the identification error of linear model, i.e.,
\[
e_L(k) = y(k) - \hat{\theta}_L(k)^T\Psi(k-d),
\]  
(14)
\[
\hat{\theta}_L'(k) = \left[\hat{L}_{1,0}(k), \ldots, \hat{L}_{1,n-1}(k), \hat{B}_{1,0}(k), \ldots, \hat{B}_{1,n+d-2}(k)\right]^T; \text{proj}\{} \cdot \text{\} is a projection operator as
\[
\text{proj}\{\hat{\theta}_L'(k)\} = \left[\hat{\theta}_L'(k), \ldots, h_{\min}, \ldots\right]^T \text{ otherwise},
\]  
(15)

where \(h_{\min} > 0\) is defined based on prior knowledge. This aims to prevent the control signal from being too big due to the too small identification parameter \(\hat{L}_{1,0}(k)\hat{B}_{1,0}(k)\).

The nonlinear estimator is designed as,
\[
\hat{y}_N(k + d) = \hat{\theta}_N(k)^T\Psi(k) + \hat{h}\left[\hat{\Psi}(k)\right],
\]  
(16)
where \(\hat{y}_N\) and \(\hat{\theta}_N\) are nonlinear estimated transformed output voltage and nonlinear estimated parameter vectors, respectively.

\(\hat{h}\left[\hat{\Psi}(k)\right]\) is the estimation of \(\hat{h}[\Psi(k)]\) by ANNs at time instant \(k\) with \(\hat{h}[\Psi(k)] = y(k + d) - \hat{\theta}_N(k)^T\Psi(k)\). According to [41], the only requirement on the update laws of \(\hat{\theta}_N(k)\) and \(\hat{W}(k)\) is that they always lie in certain compact set. Hence, the update law of \(\hat{\theta}_N(k)\) is designed similar to that of \(\hat{\theta}_L(k)\) where the difference is the definition of identification error, i.e.,
\[
e_N(k) = y(k) - \hat{\theta}_N(k)^T\Psi(k-d) - \hat{h}\left[\hat{\Psi}(k)\right].
\]  
(17)

2) Nonlinear Identifier and Controller Based on ANNs:
The voltage tracking performance of MGs heavily depends on the accuracy of estimation of the unmodeled dynamics, i.e., \(\hat{h}[\hat{\Psi}(k)]\). As reported in [41], [42], ANNs are the universal approximators. Hence, by a proper choice of the structure and parameters of ANNs, the identification error of unmodeled dynamics \(\|\hat{h} - \hat{h}[\hat{\Psi}(k)]\|\) can be made arbitrarily small over a compact set. We choose the back propagation (BP) ANN to estimate the unmodeled dynamics \(\hat{h}[\hat{\Psi}(k)]\).

To guarantee that the hyper-parameters are well-tuned, we use the random search algorithm in [43] to calibrate the hyper-parameters based on the performance on a validation set. According to [38], with well-tuned hyper-parameters and appropriate training algorithm, one can obtain the estimation of ideal parameter matrix, \(\hat{W}(k)\) (containing weights and biases). Then, by taking \(\hat{W}(k)\) and \(\Psi(k)\) as the input vectors of the ANN function, it can achieve accurate and fast estimation of unmodeled dynamics. From a system theoretical point of view, ANNs are convenient families of nonlinear mappings as,
\[
\delta h[\hat{\Psi}(k)] = \phi\left[\hat{W}(k), \Psi(k)\right]
\]  
(18)
where \(\phi[\cdot]\) represents the function of ANNs; \(\hat{W}_i\) and \(\hat{b}_j\) denote the ideal weight and bias vectors, respectively, \(i = 1, 2, 3; \Gamma\) represents a vector of activation functions.

3) Linear Adaptive Controller and ANN-Based Nonlinear Adaptive Controller: Finally, the linear adaptive controller (LAC) is designed as
\[
\hat{\theta}_L(k)^T\Psi(k) = R_v^\text{ref}(k).
\]  
(19)
Moreover, the nonlinear adaptive controller (NAC) based on ANN is designed as
\[
\hat{\theta}_N(k)^T\Psi(k) + \delta h[\hat{\Psi}(k)] = R_v^\text{ref}(k).
\]  
(20)

4) Controller Design for Time Delays: In hierarchical control, the sampling time of SVC is larger than the primary control. The communication delays between the two levels can affect the stability and tracking performance of MGs. So, we consider a discrete-time system whose sampling time is equal to that of voltage measurement. We round the time delays to an integer multiple of the sampling period.

When there is no communication delay, i.e., \(d = 1\), the predicted output \(\hat{v}_i(k)\) is computed with \(\hat{\theta}(k-1)\). However, when time delay exists, i.e., \(d > 1\), the measured output voltage \(v_o(k)\) depends on the control input \(E^\text{ref}(k-d)\) which is calculated with estimated parameters \(\hat{\theta}(k-d)\) using the then available measurement. Therefore, the identification error \(e(k)\) using \(\hat{\theta}(k-1)\) is equal to the tracking error \(\hat{e}(k)\) with delayed measurements. To solve this problem, the update law is designed in the form of (11)–(13). It can be elaborated that the sequence \(\hat{\theta}(k)\) are divided into \(d\) subsequences and each one updates itself when data is available. Note that, \(\hat{W}\) in (18) also needs to be split into \(d\) sequences and the corresponding ANNs update their parameters in their own time-scale.

C. Switching Mechanism
LAC aims to guarantee the stability while NAC is designed to achieve accurate voltage tracking. As shown in Fig. 3, the error between the weighted output voltage \(v_o^\text{ref}\) of MGs and weighted desired voltage reference \(v_o^\text{ref}\) are given to the linear and nonlinear loops simultaneously at each time step \(k\). In the nonlinear loop, NAC generates the voltage control signals \(E_n^\text{ref}\) and transfers it to the nonlinear identified model based on ANNs. It is the same for the linear loop except that the controller and identifier are replaced by LAC and ARMAX model without ANN. Then, both linear and nonlinear identification errors, \(e_l\) and \(e_n\), are sent to the switching logic block. This block decides which controller is selected in the current time step. Finally, the selected voltage control signal is adopted in the primary control of DERs.

The performance index of switching mechanism is proposed based on a similar logic in [41]:
\[
\xi_i(k) = \sum_{s=d}^k \frac{\eta_j(s)\left(\|e_j(s)\|^2 - 4\rho^2\right)}{2(1 + \|\Psi(s-d)\|)} + \mu \sum_{s=k-M+1}^k \left(1 - \eta(j)\right)\|e_j(s)\|^2
\]  
(21)
control algorithm (10)–(22), suppose Assumption 1 holds and methods, which are detailed by the following propositions.

**D. Analysis of Stability and Tracking Error Convergence**

In this section, we analyze the stability and voltage tracking errors of the closed-loop MG system with the proposed SVC method, which are detailed by the following propositions.

**Proposition 1 (BIBO-Stability):** For the system (3) with the control algorithm (10)–(22), suppose Assumption 1 holds and

\[ h[\cdot]\| \leq \rho, \]

where \( \mu \geq 0 \) is a constant and \( M \) is a positive integer.

We select the linear or nonlinear controller according to the smaller performance index:

\[ \xi_{s} = \min [\xi_{L}, \xi_{N}]. \]

Note that, the performance index (21) is comprised of two terms. The first term is designed to differentiate signals with different rates to guarantee the boundedness of all signals, thus realizing stable switching. The second term is a measure of estimation errors over a period and is used to improve control performance [41]. When the linear or nonlinear identifier predicts the voltage with smaller errors, the second term decreases, thus the corresponding controller will be chosen. Properly selecting \( \mu \) and \( M \) can enhance the stability. An outstanding advantage of such switching mechanism is that the stability and tracking performance can be decoupled. This means the hyper-parameters and training method of ANNs do not affect the stability.

When the ANN is degraded or disturbed, \( e_{N} \) increases. Consequently, \( \xi_{L} < \xi_{N} \) and LAC is chosen. LAC keeps working to guarantee the stability until the ANN-based controller recovers. As \( e_{N} \) decreases, \( \xi_{L} \) is greater than \( \xi_{N} \) and the controller NAC is chosen to improve the performance. A proper selection of \( \mu \) and \( \rho \) can enhance the voltage tracking performance while guaranteeing closed-loop stability.

**Remark 3:** According to the switched systems theory [44], it is possible to guarantee the stability with better performance by frequently switching controllers for unstable subsystems. However, such frequent switching may deteriorate the control performance or even cause instability in subsystems. Therefore, designing an appropriate switching mechanism is essential [45]. Our switching mechanism considers both the stability and voltage tracking performance.

**Algorithm 1 Model-Free SVC**

1. Measure the MG output voltage \( v_{o}(k) \) and establish data vector \( \Psi(k-d) \) together with SVC input \( E^{*}(k) \) at current time step.

2. **procedure CONTROLLER SELECTION**

3. Calculate the identification errors \( e_{j}(k) \) and \( e_{N}(k) \) using (14) and (17), respectively.

4. Calculate \( \xi_{L}(k) \) and \( \xi_{N}(k) \) with (21) and (22).

5. if \( \xi_{L}(k) \leq \xi_{N}(k) \) then

6. \( j \) switches to position \( L \) and select linear controller

7. else

8. Let \( j = N \) and select nonlinear controller.

9. **end if**

10. **end procedure**

11. **procedure CONTROLLER CALCULATION**

12. if \( j = L \) then

13. Estimate LAC parameters \( \hat{\theta}_{L}(k) \) with (11)–(15), and calculate the SVC input \( E^{*}(k) \) using (19).

14. else

15. Estimate NAC parameters \( \hat{\theta}_{L}(k) \) with (17)–(18) and calculate \( E^{*}(k) \) using (20).

16. **end if**

17. **end procedure**

18. Let \( k = k + 1 \), and return to Step 1.

\[ \| h[\cdot] \| \leq \rho, \] the inputs \( E^{*} \) and output voltages \( v_{o} \) of MGs are uniformly bounded, i.e.,

\[ \max_{0 \leq \tau \leq k} \| v_{o}(\tau) \|, \| E^{*}(\tau) \| \leq \Delta \]

which holds for some positive constant \( \Delta \).

There are many kinds of stability definitions, such as Lyapunov stability, asymptotic stability, etc. For MG system which is nonlinear, these stability definitions only require the voltages converge to the stable operation point without boundedness. However, in practical operation, it is more important to ensure the voltage not to exceed the stability bound rather than to converge in infinite time. Therefore, in this article, we define the stability in a BIBO manner, which guarantees that all the output voltages of MGs are bounded. It is worth noting that our proposed control strategy naturally guarantees the inputs are bounded, which implies our method is much feasible in practice.

**Proposition 2 (Tracking Error Convergence):** With proper hyper-parameter calibration of ANNs and the proposed adaptive control method, the voltage tracking errors asymptotically converge to an arbitrarily small positive constant \( \epsilon \), i.e.,

\[ \lim_{k \to \infty} \| \tilde{e}(k) \| = \lim_{k \to \infty} \| F(z^{-1})v_{o}(k) - Rv_{o}^{\text{ref}}(k-d) \| < \epsilon. \]

The proofs can be found in the Appendix.

**E. Algorithm Implementation**

The overall SVC algorithm design is presented in Algorithm 1. Firstly, \( v_{o}(k) \) is measured and sent to the SVC controller. The sampling rate of secondary control can be chosen from 100 Hz to 1 kHz [46]. Combining \( v_{o}(k) \) and \( E^{*}(k) \) with the historical data, we construct the data vector \( \Psi(k-d) \). The number of historical data depends on the \( n \) and \( d \), which can be identified using the method in [39]. Then, the linear and nonlinear identifiers and controllers are established in the control center. The parameters \( \rho \) and \( \mu \) in (21)–(23) can affect the
tracking performance. As \( \rho \) decreases, the accuracy of linear parts increases. But if \( \rho \) is too small, the parameter updating process converges slowly. \( \mu \) represents the weight of tracking performance. To balance the stability and control performance, \( \mu \) is usually selected around 1.5 [38]. The widely-used two-way communication network between MGCC and DERs is required [47].

### IV. Case Studies

#### A. Simulation Setup

The proposed SVC is tested on a widely used MG system (see Fig. 4), originally consisting of four inverter-based DERs and two loads [48]. The control system of DERs has been shown in Fig. 2. The MG parameters are given in Table II.

The sampling periods of primary and secondary control are set as \( 10^{-4} \) s and \( 0.01 \) s, respectively. The total simulation time is \( 10 \) s. All the dynamic simulations are implemented in MATLAB/Simulink environment.

We establish a feed-forward ANN consisting of two hidden layers (50 and 8 nodes, respectively). To obtain the training set, we first only adopt the LAC and make \( \psi_k \) time-varying instead of a constant. When finishing this case, we select the current and historical control inputs \( E^*(k), \ldots, E^*(k-n+1), k = 1, \ldots, 1000 \) and output voltages \( v_o(k+d-1), \ldots, v_o(k+d-n) \) as the inputs of the training of ANN. While the errors between the output voltage of the linear model with LAC and real voltages \( \hat{\psi}(k) = \psi_o(k+d) - \hat{v}_o(k+d) = y(k) - \hat{\theta}_L(k)^T \Psi(k) \) are used as the output of the training set. The ANN is trained offline using back-propagation with Levenberg-Marquardt algorithm [49]. The learning rate is set as \( lr = 0.9 \), and the momentum factor is selected as \( mc = 0.8 \).

The activation function for the first and second hidden layers are selected as “tansig” and “purelin”, respectively. The offline training of ANN takes 29.36 s and 1675 iterations. The mean squared error of training and test are \( 9.94 \times 10^{-6} \) and \( 3.85 \times 10^{-3} \), respectively. The trained ANN is integrated into the NAC as an identifier.

#### B. Tracking Performance

The voltage tracking performance is shown in Fig. 5. The reference voltages are set as 300 V. The SVC is not applied until \( t = 2.5 \) s. Before that though voltages are stable, steady state errors still exist. Once SVC is implemented, the voltage magnitudes are restored to the reference values rapidly. At \( t = 5 \) s, a constant power load is attached to the system. To show the robustness of the model-free method, a parameter perturbation that \( L_c \) is reduced by 25\% is triggered since \( t = 7.5 \) s. From Fig. 5(b), we notice that when large disturbances happen, the control mode oscillates between the LAC and NAC due to the degradation of ANNs. The switching mechanism is trying to balance the tracking performance and stability. Once ANN recovers, it switches back to NAC. The results show that the proposed SVC exhibits good voltage tracking performance and robustness to the uncertain perturbations. Fig. 6 shows that the active power outputs of DERs are allocated according to their rated power.

#### C. Stability of LAC

One may doubt that what if the ANN is not well-trained or its hyper-parameters are not well-tuned. To verify the stabilization of the proposed controller, we only use the LAC by...
fixing $j \leftarrow L$. The results are shown in Fig. 7. Due to the inaccuracy of linearized model, the parameters are kept updating automatically, which leads to the oscillations of output voltages. However, the stability is still guaranteed and the errors between real and reference voltages are maintained bounded, even when large disturbances occur.

**D. Comparison With Feedback Linearization Control**

A comparison study with input-output feedback linearization control, which is well-known as nonlinear control method requiring precise model, is carried out in this section. When load 3 is attached at $t = 5$ s, we assume the information of load change is known by the secondary controller based on feedback linearization. Similarly, an unknown parameter perturbation occurs at $t = 7.5$ s. As shown in Fig. 8, though the feedback linearization controller can deal with the known large load fluctuation, it fails to restore and stabilize the output voltages in case of uncertainties. The corresponding active power outputs are shown in Fig. 9.

**E. Comparison With PID Control**

To compare the proposed method with the existing model-free approaches, in this section, we conduct simulations by the most widely-used model-free PID control. Fig. 10 and Fig. 11 show that, under the same conditions, the PID control can realize accurate voltage tracking and is robust to unknown parameter perturbation. However, compared with the proposed method, PID control performs more sluggish transient responses and much larger overshootings after large load disturbance and parameter perturbation.

**F. Effect of Time Delays**

In this section, we test the robustness of the proposed controller against time delays. The sampling time $\Delta T = 10$ ms. We set the time delays as $(d - 1) \times \Delta T | d = 1, 2, 3, 5, 10, 20)$, respectively. Any fractional time delays are rounded up. Load 3 is attached at $t = 5$ s. Fig. 12 shows the comparison of voltage tracking performances with different time delays of DER1. The blue line shows the result without time delay, i.e., $d = 1$. The red line shows the worst case with a delay of 190 ms. The result shows that the proposed controller can stabilize the system with any different delays. However,
mechanism. The unmodeled nonlinear dynamics are online i.e., LAC and NAC, which are coordinated by a switching SVC. The proposed controller consists of two separate parts, primary control are treated as a “black-box” when designing the nonlinear multiple models adaptive control. The MGs with prior- 5s, 5s, and 7.5 s, respectively.

G. Effect of Communication Noise Disturbances
The noise disturbances in communication links between secondary and primary levels widely exist in the SVC of MGs and may degrade the dynamic performance of the controller. To study the influence of communication noise disturbances on the proposed SVC method, white noises with signal-to-noise ratio (SNR) of 30 dB, 20 dB and 10 dB are added to the communication links between SVC and primary level. Note that smaller SNR indicates larger noise disturbance, and the SNR is usually between 30 to 40 dB in MGs [50], [51]. As shown in Fig. 13, the proposed SVC method can realize voltage tracking with different communication noise disturbances. Communication noises with 30 dB, 20 dB and 10 dB SNR are added at 2.5 s, 5 s, and 7.5 s, respectively.

V. CONCLUSION
In this article, we proposed a novel model-free SVC using nonlinear multiple models adaptive control. The MGs with primary control are treated as a “black-box” when designing the SVC. The proposed controller consists of two separate parts, i.e., LAC and NAC, which are coordinated by a switching mechanism. The unmodeled nonlinear dynamics are online estimated by ANNs. We have proved that the tracking errors can be achieved arbitrarily small given a proper nonlinear identification. The simulation results show that such switching mechanism can guarantee BIBO stability of the closed-loop system while achieving accurate tracking. The proposed controller is robust to uncertainties, disturbances and time delays.

Due to the advantages of flexibility and robustness, the distributed and decentralized model-free secondary voltage control will be further investigated in our future works.

APPENDIX
Proof of Proposition 1: This proof can be separated into two parts: the BIBO stability of output voltage and the convergence of voltage tracking error. For the proof of stability, we first prove that SVC input $E^*$ and output voltage $v_o$ are bounded by $e$, then we use the contradiction argument to prove that $e$ is bounded, and that means $E^*$ and $v_o$ are also bounded.

Define the parameter identification error of linear estimator as $\psi_L(k) = \dot{\theta}_L(k) - \theta$. By (12), it follows that

$$\psi_L(k) = \psi_L(k - d) + \eta_L(k)(\psi(k - d)e_L(k))^T1 + \|\psi(k - d)\|^2.$$  \hspace{1cm} (25)

Following the proof in [41] and from the logic function (22), it can be proven that $\dot{\theta}_L(k)$ is bounded. In addition,

$$\lim_{N \to \infty} \sum_{k=d}^{N} \eta_L(k)(\|e_L(k)\|^2 - 4\rho^2) < \infty,$$  \hspace{1cm} (26)

$$\lim_{k \to \infty} \eta_L(k)(\|e_L(k)\|^2 - 4\rho^2) \rightarrow 0.$$  \hspace{1cm} (27)

From (14) and (19), we have

$$e_L(k) = F(\varepsilon^{-1})v_o(k) - Rv_o^\text{ref}(k - d).$$  \hspace{1cm} (28)

Since $F(\varepsilon^{-1})$ is stable, then from (28), there exist positive constants $\ell_1$ and $\ell_2$ such that

$$\|\psi(k - d)\| \leq \ell_1 + \ell_2 \max_{0 \leq \tau \leq k} \|e_L(\tau)\|.$$  \hspace{1cm} (29)

which indicates that the input $E^*$ and output voltage $v_o$ are bounded by the linear identification error $e_L$.

To prove the boundedness of $e_L$, we utilize the proof by contradiction argument. Suppose that $e_L(k)$ is unbounded, then

Fig. 11. Active power outputs of DERs with PID control.

Fig. 12. Voltage tracking performance with different time delays; SVC is applied after 2.5 s; load 3 is attached at 5 s.

Fig. 13. Voltage tracking performance with different communication noise disturbances. Communication noises with 30 dB, 20 dB and 10 dB SNR are added at 2.5 s, 5 s, and 7.5 s, respectively.
there must exist a positive time constant $T$, such that $\|e_L(k)\| > 2\rho$ and $a_L(k) = 1$ for $k > T$, i.e., there exists a monotonic increasing sequence $\|e_L(k_n)\|$ such that $\lim_{k_0 \to \infty} \|e_L(k_n)\| = \infty$. Then, it follows that

$$\lim_{k_0 \to \infty} \eta(k_0)(\|e_L(k_n)\|^2 - 4\rho^2) = 0.$$ (30)

However, it contradicts (27) which means $e_L(k)$ is bounded.

Thus, it proves the BIBO stability for LAC. For NAC, from (17) and (20), it follows that

$$e(N) = F(z^{-1})v_o(k) - Rv_o^\ast(k - d).$$ (31)

Since $F(z^{-1})$ is stable, then from (31), there exist positive constants $\ell_3$ and $\ell_4$ such that

$$\|\Psi(k - d)\| \leq 1 + \ell_4 \max_{0 \leq r \leq k} \|e(N)(r)\|.$$ (32)

The first term in (21) is bounded according to (26), and the second term is also bounded due to the dead-zone function (22). Hence $\xi_L(k)$ is bounded. If $\xi_N(k)$ is bounded, according to the switching mechanism function (21), we have

$$\lim_{k \to \infty} \eta_N(k)(\|e_N(k)\|^2 - 4\rho^2) = 0.$$ (33)

In this case, both of linear of nonlinear identification errors of the closed-loop MG system $e(k)$, $j = (L, N)$ satisfy that

$$\lim_{k \to \infty} \eta(k)(\|e(k)\|^2 - 4\rho^2) = 0.$$ (34)

where

$$a(k) = \begin{cases} 1, & \text{if } \|e(k)\| > 2\rho, \\ 0, & \text{otherwise}. \end{cases}$$ (35)

If $\xi_N(k)$ is unbounded, considering $\xi_L(k)$ is bounded, there must exist $k_0 > 0$ such that $\xi_L(k) \leq k_0$, $\forall k \geq k_0$. Then after time $k_0$, the switching mechanism will choose the linear controller, thus the identification error $e(k) = e_L(k)$ which also satisfies (34).

Finally, from (29), (32) and (34), it can be proved that $v_o$ and $E^\ast$ are bounded, i.e., the input and output of the closed-loop switching system are bounded while the identification error $e_j(k)$ satisfies

$$\lim_{k \to \infty} \|e_j(k)\| \leq 2\rho, \quad j = (L, N).$$ (36)

which indicates that there exist positive constants $\ell_5$ and $\ell_6$ such that

$$\|\Psi(k - d)\| \leq \ell_5 + \ell_6 \max_{0 \leq r \leq k} \|e_j(r)\| \leq \ell_5 + 2\ell_6 \rho.$$ (37)

Let $\Delta = \ell_5 + 2\ell_6 \rho$, it follows that

$$\max_{0 \leq r \leq k} \|e_N(r)\|, \|E^\ast(r)\| \leq \Delta.$$ (38)

Now we have proven the BIBO voltage stability of the closed-loop MG system with the proposed SVC.

Proof of Proposition 2: Switching mechanism always selects the controller, with respect to the smaller identification error, as the SVC input for MG system. Moreover, from (28) and (31), the output voltage tracking error $\hat{e}(k)$ is equivalent to the smaller identification error. From (17), we have the nonlinear identification error,

$$e_N(k) = y_N(k) - \hat{\theta}_N(k - d)^T \Psi(k - d) - \delta \hat{h}[\Psi(k - d)].$$

$$= y_N(k) - \hat{\theta}_N(k - d)^T \Psi(k - d) - \delta \hat{h}[\Psi(k - d)].$$

When the hyper-parameters of the ANN are well-tuned, for arbitrary small positive constant $\epsilon$, the voltage tracking error always satisfies $\|\delta \hat{h}[\Psi(k - d)] - \delta \hat{h}[\Psi(k - d)]\| < \epsilon$. It means the nonlinear identification error is always smaller than the linear one, so that the tracking error will be automatically selected as the nonlinear identification error, i.e.,

$$\lim_{k \to \infty} \|\hat{e}(k)\| = \lim_{k \to \infty} \|e_N(k)\| < \epsilon.$$ (40)

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