# A Linear Solution Method of Generalized Robust Chance Constrained Real-time Dispatch

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***Abstract*—In this letter, a novel solution method of generalized robust chance constrained real-time dispatch (GRCC-RTD) considering wind power uncertainty is proposed. GRCC models are advantageous in dealing with distributional uncertainty, however, they are difficult to solve because of the complex ambiguity set. By constructing traceable counterparts of the robust chance constraints and using the reformulation linearization technique, the model is transformed into a deterministic linear programming problem, which can be solved efficiently by off-the-shelf solvers. Numerical results verify the effectiveness of the approach.**

***Index Terms*—Chance constrained programming, distributionally robust optimization, real-time dispatch, wind power.**

## I. Introduction

The uncertainty of wind power introduces significant challenges to the real-time dispatch (RTD), which operates at a time-scale of minutes to determine the base points (BPs) and participation factors (PFs) of online units. A variety of approaches, e.g., stochastic programming (SP) and robust optimization (RO), have been applied to address this problem. However, the effectiveness of the SP based approaches relies on the precise probability distribution of wind power, which is difficult to obtain in practice. Meanwhile, the RO based approaches, which make decisions according to the bounds of disturbances, are usually criticized for their conservativeness.

The robust chance constrained dispatch approaches are proposed to fill the gap between the aforementioned two kinds of approaches. A robust chance constrained optimal power flow (RCC-OPF) model and corresponding cutting-plane algorithm are proposed in [1]. In the model, the wind power forecast error (WPFE) is assumed to follow a normal distribution, and its first- and second-order moments are allowed to change within predetermined regions. In [2], a robust chance constrained model for reserve scheduling is developed, where the type of the wind power distribution is not specified, but the moments are assumed to be known. In [3], the second-order cone programming is applied to solve the RCC-OPF model, where the expectation of WPFE must be 0 and the covariance matrix must be predetermined. In practice, both the distribution type and moments are difficult to identify. In [4], a generalized ambiguity set is used to capture uncertainties, which leads to a generalized robust chanced constrained (GRCC) OPF model. The model does not require a specific distribution type or precise moments, hence, it is more generic. However, the proposed semidefinite programming based algorithm is computationally intensive for online applications.

The main contribution of this letter is to develop a fast solution method for the GRCC model so that it can be used for real-time dispatch, i.e., GRCC-RTD. The method reduces the computational burden by constructing traceable counterparts of the robust chance constraints and applying the reformulation linearization technique (RLT).

## II. Problem Formulations

Assume the mean vector and covariance matrix of WPFE vector $\boldsymbol{w}$ are $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, respectively, and the statistical ones are $\boldsymbol{\mu}_0$ and $\boldsymbol{\Sigma}_0$. Then, the model can be formulated as

$$Z = \min_{\boldsymbol{p},\boldsymbol{\alpha}} \ \boldsymbol{p}^{\mathrm{T}}\boldsymbol{c}_1\boldsymbol{p} + {\boldsymbol{c}_2}^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{c}_3, \tag{1}$$

$$\text{s.t.} \quad \boldsymbol{e}^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{v} - \boldsymbol{e}^{\mathrm{T}}\boldsymbol{d} = 0, \tag{2}$$

$$\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\alpha} = 1, \quad 0 \le \boldsymbol{\alpha} \le 1, \tag{3}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left(p_i - \alpha_i \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w} \le \overline{p}_i\right) \ge 1 - \epsilon_{1,i}, \quad \forall i \in G, \tag{4}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left(p_i - \alpha_i \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w} \ge \underline{p}_i\right) \ge 1 - \epsilon_{1,i}, \quad \forall i \in G, \tag{5}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left(\alpha_i \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w} \le p_i^u\right) \ge 1 - \epsilon_{2,i}, \qquad \forall i \in G, \tag{6}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left(\alpha_i \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w} \ge p_i^d\right) \ge 1 - \epsilon_{2,i}, \qquad \forall i \in G, \tag{7}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left\{\boldsymbol{m}_{gl}^{\mathrm{T}}\left(\boldsymbol{p} - \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\alpha}\right) + \boldsymbol{m}_{wl}^{\mathrm{T}}\left(\boldsymbol{v} + \boldsymbol{w}\right) + \boldsymbol{m}_{dl}^{\mathrm{T}}\boldsymbol{d} \le \overline{T}_l\right\} \ge 1 - \epsilon_l, \quad \forall l \in L, \tag{8}$$

$$\inf_{\boldsymbol{w}\in D} \Pr\left\{\boldsymbol{m}_{gl}^{\mathrm{T}}\left(\boldsymbol{p} - \boldsymbol{e}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\alpha}\right) + \boldsymbol{m}_{wl}^{\mathrm{T}}\left(\boldsymbol{v} + \boldsymbol{w}\right) + \boldsymbol{m}_{dl}^{\mathrm{T}}\boldsymbol{d} \ge -\overline{T}_l\right\} \ge 1 - \epsilon_l, \quad \forall l \in L, \tag{9}$$

$$D = \begin{cases} \int f(\boldsymbol{w})\, d\boldsymbol{w} = 1, \quad f(\boldsymbol{w}) \ge 0, \\ \left[E(\boldsymbol{w}) - \boldsymbol{\mu}_0\right]^{\mathrm{T}} \boldsymbol{\Sigma}_0^{-1} \left[E(\boldsymbol{w}) - \boldsymbol{\mu}_0\right] \le \gamma_1, \ \gamma_1 \ge 0, \\ E\left[(\boldsymbol{w} - \boldsymbol{\mu}_0)(\boldsymbol{w} - \boldsymbol{\mu}_0)^{\mathrm{T}}\right] \preceq \gamma_2 \boldsymbol{\Sigma}_0, \qquad \gamma_2 \ge 1, \end{cases} \tag{10}$$

where $G$ is the set of online controllable units, e.g., units with automatic generation control; $L$ is the set of transmission lines; $D$ is the ambiguity set that determines the uncertainty level of WPFE; $\boldsymbol{p}$ is the BP vector, and $p_i$ is the $i$th element of $\boldsymbol{p}$; $\boldsymbol{\alpha}$ is the PF vector, and $\alpha_i$ is the $i$th element of $\boldsymbol{\alpha}$; $\boldsymbol{c}_1$, $\boldsymbol{c}_2$ and $\boldsymbol{c}_3$ are the cost coefficient vectors; $\boldsymbol{v}$ and $\boldsymbol{d}$ are the predicted wind power and load demand vectors; $\overline{p}_i$ and $\underline{p}_i$ are the generation limits of unit $i$; $p_i^u$ and $p_i^d$ are the adjustment limits of unit $i$; $\epsilon_{1,i}$, $\epsilon_{2,i}$ and $\epsilon_l$ are the required risk levels; $\gamma_1$ and $\gamma_2$ are the conservative coefficients; $\boldsymbol{m}_{gl}$, $\boldsymbol{m}_{wl}$, and $\boldsymbol{m}_{dl}$

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are the injection shift factor vectors; $\overline{T}_l$ is the transmission limit of line $l$; $\boldsymbol{e}$ is the vector of all ones; and $f(\boldsymbol{w})$ is the joint probability distribution function of $\boldsymbol{w}$.

The model in (1)-(10) is similar to the model in [1]. However, the ambiguity set adopted from [4] is more generic. Besides, the constraints in (6)-(7) are added to consider the adjustment reserve capacity limits of the units.

In practice, the operators may have different risk attitude for the constraint violation in different directions, so all the chance constraints in the model are one-sided, and the risk levels for different directions can be different.

## III. Solution Methodology

In practice, the BPs and PFs should be updated very quickly. However, the model in (1)-(10) is difficult to solve due to the existence of the robust chance constraints and the complexity of the ambiguity set. Thus, the model has to be transformed.

Consider a robust chance constraint:

$$\inf_{\boldsymbol{w}\in D} \Pr\left(\boldsymbol{a}^{\mathrm{T}}\boldsymbol{w} \le b\right) \ge 1-\epsilon, \tag{11}$$

where $D$ is the set in (10); $\boldsymbol{a}$ is the coefficient vector; $b$ is the limit. Ref. [5] provides a theorem to construct the deterministic counterpart of the constraint.

*Theorem 1:* If $\gamma_1/\gamma_2 \le \epsilon$, (11) is equivalent to

$$\boldsymbol{\mu}_0^{\mathrm{T}}\boldsymbol{a} + \left(\sqrt{\gamma_1} + \sqrt{\left(\frac{1-\epsilon}{\epsilon}\right)(\gamma_2-\gamma_1)}\right)\sqrt{\boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}_0\boldsymbol{a}} \le b; \tag{12}$$

Or else, (11) is equivalent to

$$\boldsymbol{\mu}_0^{\mathrm{T}}\boldsymbol{a} + \sqrt{\frac{\gamma_2}{\epsilon}}\sqrt{\boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Sigma}_0\boldsymbol{a}} \le b. \tag{13}$$

According to Theorem 1, the robust chance constraints in (4)-(7) can be directly transformed into equivalent deterministic linear constraints while the deterministic counterparts of the constraints in (8)-(9) are quadratic.

For instance, assuming all constraints in (4)-(9) satisfy the condition of (12), they can be equivalently transformed into

$$\mu_s\alpha_i + k_{1,i}\alpha_i\sqrt{\Sigma_s} \le p_i - \underline{p}_i, \quad \forall i \in G, \tag{14}$$

$$-\mu_s\alpha_i + k_{1,i}\alpha_i\sqrt{\Sigma_s} \le \overline{p}_i - p_i, \quad \forall i \in G, \tag{15}$$

$$\mu_s\alpha_i + k_{2,i}\alpha_i\sqrt{\Sigma_s} \le p_i^u, \quad \forall i \in G, \tag{16}$$

$$-\mu_s\alpha_i + k_{2,i}\alpha_i\sqrt{\Sigma_s} \le -p_i^d, \quad \forall i \in G, \tag{17}$$

$$k_l^2\left(\boldsymbol{m}_{wl} - \boldsymbol{e}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\right)^{\mathrm{T}}\boldsymbol{\Sigma}_0\left(\boldsymbol{m}_{wl} - \boldsymbol{e}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\right) \le \left(T_{1,l} - \boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{\mu}_0{}^{\mathrm{T}}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\boldsymbol{e}\right)^2, \quad \forall l \in L, \tag{18}$$

$$T_{1,l} - \boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{\mu}_0{}^{\mathrm{T}}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\boldsymbol{e} \ge 0, \quad \forall l \in L, \tag{19}$$

$$k_l^2\left(-\boldsymbol{m}_{wl} + \boldsymbol{e}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\right)^{\mathrm{T}}\boldsymbol{\Sigma}_0\left(-\boldsymbol{m}_{wl} + \boldsymbol{e}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\right) \le \left(T_{2,l} + \boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{p} - \boldsymbol{\mu}_0{}^{\mathrm{T}}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\boldsymbol{e}\right)^2, \quad \forall l \in L, \tag{20}$$

$$T_{2,l} + \boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{p} - \boldsymbol{\mu}_0{}^{\mathrm{T}}\left(\boldsymbol{m}_{gl}^{\mathrm{T}}\boldsymbol{\alpha}\right)\boldsymbol{e} \ge 0, \quad \forall l \in L, \tag{21}$$

where $\mu_s = \boldsymbol{e}^{\mathrm{T}}\boldsymbol{\mu}_0$; $T_{1,l} = \overline{T}_l - \boldsymbol{m}_{wl}^{\mathrm{T}}\boldsymbol{v} - \boldsymbol{m}_{dl}^{\mathrm{T}}\boldsymbol{d} - \boldsymbol{\mu}_0{}^{\mathrm{T}}\boldsymbol{m}_{wl}$; $T_{2,l} = \overline{T}_l + \boldsymbol{m}_{wl}^{\mathrm{T}}\boldsymbol{v} + \boldsymbol{m}_{dl}^{\mathrm{T}}\boldsymbol{d} + \boldsymbol{\mu}_0{}^{\mathrm{T}}\boldsymbol{m}_{wl}$; $\Sigma_s = \boldsymbol{e}^{\mathrm{T}}\left(\mathrm{diag}(\boldsymbol{\Sigma}_0)\right)$; $k_{1,i} = \sqrt{\gamma_1} + \sqrt{((1-\epsilon_{1,i})/\epsilon_{1,i})(\gamma_2-\gamma_1)}$; $k_{2,i} = \sqrt{\gamma_1} + \sqrt{((1-\epsilon_{2,i})/\epsilon_{2,i})(\gamma_2-\gamma_1)}$; and $k_l = \sqrt{\gamma_1} + \sqrt{((1-\epsilon_l)/\epsilon_l)(\gamma_2-\gamma_1)}$. Eqs. (18)-(19) and (20)-(21) are according to the square form of (12), reflecting the two direction transmission capability constraints.

Therefore, the model in (1)-(10) can be equivalently transformed into a quadratically constrained quadratic programming (QCQP) problem, e.g., (1)-(3) and (14)-(21).

To further simplify the model, let $\boldsymbol{x} = [\boldsymbol{p}^{\mathrm{T}}, \boldsymbol{\alpha}^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}$. Then, the QCQP model can be rewritten as

$$Z = \min\ \boldsymbol{Q}_0 \circ \boldsymbol{X} + \boldsymbol{b}_0^{\mathrm{T}}\boldsymbol{x} + c_0, \tag{22}$$

$$\text{s.t.}\quad \boldsymbol{Q}_i \circ \boldsymbol{X} + \boldsymbol{b}_i^{\mathrm{T}}\boldsymbol{x} \le c_i,\ i \in I, \tag{23}$$

$$\boldsymbol{Q}_j \circ \boldsymbol{X} + \boldsymbol{b}_j^{\mathrm{T}}\boldsymbol{x} = c_j,\ j \in M, \tag{24}$$

$$\boldsymbol{l} \le \boldsymbol{x} \le \boldsymbol{u}, \tag{25}$$

where $\boldsymbol{l}$ and $\boldsymbol{u}$ are the bounds of $\boldsymbol{x}$; $I$ and $M$ are the inequality and equality constraint sets; and $\boldsymbol{A} \circ \boldsymbol{B} = \sum_{i,j=1}^{n} A_{ij}B_{ij}$.

For instance, $\boldsymbol{Q}_i$ of (18) can be expressed as

$$\boldsymbol{Q}_i = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}, \tag{26}$$

and the elements are

$$A_{ij} = A_{ji} = \boldsymbol{m}_{gl}[i]\,\boldsymbol{m}_{gl}[j], \tag{27}$$

$$B_{ij} = C_{ji} = \boldsymbol{e}^{\mathrm{T}}\boldsymbol{\mu}_0\left(\boldsymbol{m}_{gl}[i]\,\boldsymbol{m}_{gl}[j]\right), \tag{28}$$

$$D_{ij} = D_{ji} = \boldsymbol{m}_{gl}[i]\,\boldsymbol{m}_{gl}[j]\left[k_l^2\,\mathrm{sum}\left(\boldsymbol{\Sigma}_0\right) - \left(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\mu}_0\right)^2\right], \tag{29}$$

where $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D} \in \mathbb{R}^{n\times n}$; $n$ is the number of generators; $i$ and $j$ are indices from 1 to $n$; $l \in L$; $\boldsymbol{m}_{gl}[i]$ represents the $i$th element of vector $\boldsymbol{m}_{gl}$; and $\mathrm{sum}\left(\boldsymbol{\Sigma}_0\right)$ represents the sum of all elements in $\boldsymbol{\Sigma}_0$.

The RLT, which is a linear relaxation technique of QCQP problems, can transform the model in (22)-(25) into a linear programming (LP) problem. It treats each element in $\boldsymbol{X}$ as a new independent decision variable, and uses the upper and lower bounds of the original variables $\boldsymbol{x}$ to obtain valid linear constraints on $X_{ij}$ to form the LP model. More specifically, the QCQP problem can be transformed into an LP problem with the following auxiliary constraints [6]:

$$\boldsymbol{X} - \boldsymbol{l}\boldsymbol{x}^{\mathrm{T}} - \boldsymbol{x}\boldsymbol{l}^{\mathrm{T}} \ge -\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}, \tag{30}$$

$$\boldsymbol{X} - \boldsymbol{u}\boldsymbol{x}^{\mathrm{T}} - \boldsymbol{x}\boldsymbol{u}^{\mathrm{T}} \ge -\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}, \tag{31}$$

$$\boldsymbol{X} - \boldsymbol{l}\boldsymbol{x}^{\mathrm{T}} - \boldsymbol{x}\boldsymbol{u}^{\mathrm{T}} \le -\boldsymbol{l}\boldsymbol{u}^{\mathrm{T}}. \tag{32}$$

Eqs. (22)-(25) and (30)-(32) form the final linear approximation model. Here, both $\boldsymbol{x}$ and $\boldsymbol{X}$ are decision variables. The model can provide a very precise solution (lower bound of the original problem), which will be illustrated by test results.

## IV. Numerical Results

The method is tested on IEEE benchmark systems on a PC with an Intel Core i5 CPU and 4 GB RAM. Unless otherwise specified, all risk levels, i.e., $\epsilon_{1,i}$, $\epsilon_{2,i}$ and $\epsilon_l$, are set to be 0.2, while $\gamma_1$ and $\gamma_2$ are set to be 0.1 and 1.1, respectively.

A "risk neutral" model assuming no uncertainty exists and a Gaussian distribution based model assuming the distribution is well known are adopted from [3] as benchmark models. The models are tested on the IEEE 118-bus system, where three wind farms are added at buses 17, 66 and 99, respectively. The maximum probability of constraint violations [3] according to the results of different models are summarized in Table I, where DRTD means the risk neutral model, GRTD means the Gaussian distribution based model, and GRCC means the

GRCC-RTD model (in GRCC-1, $\gamma_1 = 0$, $\gamma_2 = 1$; in GRCC-2, $\gamma_1 = 0.1$, $\gamma_2 = 1.1$; and in GRCC-3, $\gamma_1 = 0.2$, $\gamma_2 = 1.1$). WPFE samples generated from three different types of distributions, i.e., Gaussian distribution, Laplace distribution and logistic distribution, are used to perform the test.

TABLE I
MAXIMUM PROBABILITY OF CONSTRAINT VIOLATIONS

| Distribution Type | DRTD | GRTD | GRCC-1 | GRCC-2 | GRCC-3 |
|---|---|---|---|---|---|
| Gaussian | 0.5031 | 0.1987 | 0.0235 | 0.0196 | 0.0185 |
| Laplace | 0.5065 | 0.1903 | 0.0249 | 0.0237 | 0.0213 |
| Logistic | 0.5029 | 0.3205 | 0.1134 | 0.0924 | 0.0836 |
| Cost (pu) | 16.695 | 17.136 | 17.2013 | 17.2306 | 17.2561 |

From the test results, it is observed that the DRTD model has the highest constraint violation risk, which is much higher than the required level (0.2 in the test). Meanwhile, if the samples are generated from the Gaussian distribution, the GRTD model can control the risk under the required level. However, if the samples are generated from other distributions, e.g., the logistic distribution, the risk may exceed the required level significantly. In contrast, the risk levels of the GRCC models are much lower than the required level for all three distribution types, which demonstrates the effectiveness of GRCC models in dealing with different types of distributions. It is also found that the higher the considered uncertainty level is, the lower the constraint violation risk will be, indicating that GRCC-RTD can prepare appropriate reserve according to the moment uncertainty level. Moreover, all stochastic models have similar costs in the table, illustrating that GRCC models will not sacrifice the operational efficiency significently.

To illustrate the effectiveness of RLT, the costs and computation time of GRCC-2 with and without RLT are listed in Table II. Here, the QCQP model is solved by Gurobi, CPLEX and MINOS, which can grantee a suboptimal solution.

TABLE II
RESULTS WITH AND WITHOUT RLT

| Results | Gurobi | CPLEX | MINOS | LP |
|---|---|---|---|---|
| Cost (pu) | 17.2435 | 17.2478 | 17.2492 | 17.2306 |
| Computation Time (s) | 2.92 | 2.83 | 3.15 | 1.43 |

In Table II, Gurobi can obtain the least cost solution among the QCQP solvers, indicating that its solution is the nearest to the global-optimal solution. Meanwhile, it is known that the RLT approximation can provide a lower bound of the original QCQP problem [6], and the cost of the LP model is very close to that of Gurobi, indicating that the RLT can provide very precise solutions. Moreover, it should be noted that Gurobi needs 104% more computation time to solve the QCQP model compared with the RLT.

To further test the proposed linear solution method, sensitivity analyses are performed on the 118-bus system, and the results are shown in Fig. 1. Fig. 1(a) illustrates the relationship between the conservative coefficients $\gamma_1$, $\gamma_2$ and the operational cost $Z$. It is observed that a higher $\gamma_1$ or $\gamma_2$ will lead to a higher $Z$. That is to say the more ambiguous the statistic result is, the more reserve should be prepared to maintain a low risk level, thus forcing the BPs moving away from the economic operating points and increasing the operational cost. However, the cost increase is not significant.

Fig. 1(b) shows the computation time for different numbers of connected wind farms. As the number of wind farms increases from 3 to 13, the computation time slightly increases from 1.43s to 1.59s, which demonstrates the effectiveness of the method in dealing with larger numbers of wind farms.

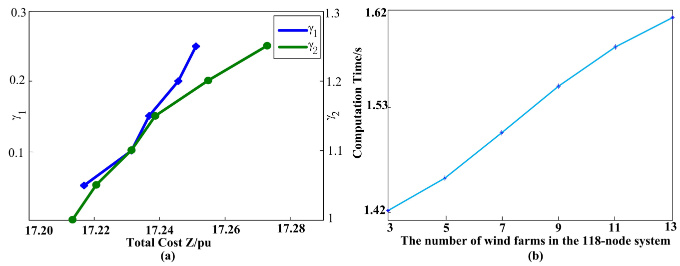


Fig. 1. Sensitivity analyses in the IEEE 118-bus system.

Fig. 2 summarizes the computation time on different IEEE benchmark systems, which further illustrates the computational efficiency of the proposed method.

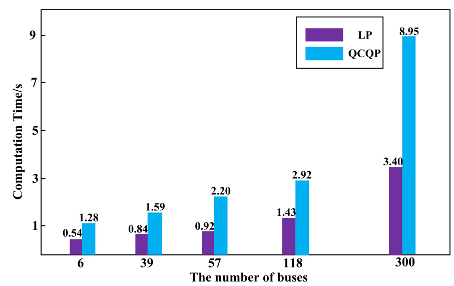


Fig. 2. Computation time with different IEEE benchmark systems.

## V. CONCLUSIONS

A novel linear solution method of GRCC-RTD is proposed in this letter. The GRCC-RTD can maintain a low constraint violation risk while achieving relatively high operational efficiency. The solution method can not only reduce the computation time significantly compared with the existing methods but also maintain high computation accuracy, revealing its potential application to large-scale power systems.

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