Considering the increased interactions between power grids and natural gas grids, this paper presents a chance-constrained maintenance scheduling model for integrated gas-electric grids with wind energy integration. Given the uncertainties of wind power, the loss of wind power probability is modeled as a chance constraint, ensuring the high utilization of wind power. To overcome the adversities caused by the nonlinear and non-convex models of natural gas systems, a piecewise linear approximation method is employed to transform the nonlinear models into a group of mixed integer linear models. A big-M formulation method is used to construct inequalities constraints for lines/pipelines to be under maintenance. In addition, unit commitment is also coordinated to achieve the best maintenance strategies. The proposed chance-constrained stochastic programming model is converted into an equivalent deterministic programming model via a strong extended formulation for the sample average approximation method by leveraging the star-inequalities. Several tests on a 4-node natural gas system with a 6-bus power system and a 20-node natural gas system with a modified IEEE 118-bus power system demonstrate the effectiveness of the proposed model.

Nomenclature

Indices and Sets

- \( w \): Index of wind farms.
- \( g \): Index of generators.
- \( g \): Index of gas wells.
- \( b, b' \): Index of gas storages, power buses.
- \( n, n' \): Index of natural gas nodes.
- \( t, t' \): Index of lines that need not to be under maintenance within the given time window.
- \( l \): Index of lines that need to be under maintenance within the given time window.
- \( p' \): Index of pipelines that need not to be under maintenance within the given time window.
- \( p \): Index of pipelines that need to be under maintenance within the given time window.
- \( b' \): Index of pipelines with compressors.
- \( s \): Index of wind power generation scenarios.
- \( t, t', k \): Index of time periods.
- \( L \): Index of piecewise segments for gas flow.
- \( P \): Index of segment sets that need to be under maintenance within the given time window.
- \( T \): Set of time periods.
- \( W \): Set of wind farms.
- \( S \): Set of wind power generation scenarios.
- \( G \): Set of generators.
- \( G_b \): Set of generators connected to power bus \( b \).

Parameters

- \( N_b \): Set of wind farms connected to power bus \( b \).
- \( N_n \): Set of gas nodes connected to gas node \( n \).
- \( N_n' \): Set of gas nodes.
- \( C_{b,t} \): Set of gas storages connected to gas node \( n \).
- \( C_{b,t} \): Set of gas wells connected to gas node \( n \).
- \( C_{b,t} \): Set of gas-tired units connected to gas node \( n \).
- \( B_b \): Set of power buses connected to power bus \( b \).
- \( B \): Set of power buses.
- \( A_{p,b,b'} \): Slopes of piecewise linear segments.
- \( B_{p,t} \): Intercept of piecewise linear segments.
- \( C_{b,t} \): Electrical susceptance of line \( b \) at \( t' \).
- \( C_{b,t} \): Maintenance costs of line \( l \) at \( t \).
- \( C_{g,t} \): Fixed cost and revenue of generator \( g \) at \( t \).
- \( C_{g,t} \): Start-up cost of generator \( g \) at \( t \).
- \( C_{g,t} \): Penalties of gas well \( g \) at \( t \).
- \( C_{n,t} \): Minimum on and off time periods of gas well \( g \) at \( t \).
- \( C_{n,t} \): Minimum on and off time periods of gas well \( g \) at \( t \).
- \( C_{n,t} \): Minimum on and off time periods of generator \( g \) at \( t \).
- \( C_{n,t} \): Duration of maintenance of line \( l \) at \( t \).
- \( C_{n,t} \): Efficiency factor of gas-fired unit \( g \) at \( t \).
- \( C_{n,t} \): Lower limits of gas flow through piecewise segments.

\( \Omega \): Set of pipelines with compressors.

\( \Omega_{\text{gas}} \): Set of gas-fired units.

\( \Omega_{\text{storage}} \): Set of gas storages.

\( \Omega_{\text{wind}} \): Set of wind farms.

\( \Omega_{\text{gen}} \): Set of generators.
Variables

\[ \alpha \]

Risk level of the chance constraint.

\[ b, b' \]

Binary variables with regard to SAA.

\[ \beta \]

Continuous variable with regard to SAA.

\[ L_{g,t} \]

Gas consumption of gas-fired unit \( g \) at \( t \).

\[ L_{n,t} \]

Power load of bus \( b \) at \( t \) and gas load of node \( n \) at \( t \).

\[ N_f^b \]

Maximum number of lines that can be under maintenance at \( t \).

\[ N_f^{b,b} \]

Maximum number of pipelines that can be under maintenance at \( t \).

\[ \bar{p}_{b,t} \]

Lower and upper limits of generator \( b \).

\[ \bar{p}_{b,b'} \]

Lower and upper capacities of line \( b-b' \).

\[ \bar{p}_{w,t} \]

Maximum capacity limit of wind generator \( w \).

\[ \bar{\tilde{p}}_{w,t} \]

Random number representing the available wind energy generated from wind generator \( w \) at \( t \).

\[ P_{g,t} \]

Power generated by wind generator \( g \).

\[ P_{m,t} \]

Power generated by wind generator \( w \) at \( t \).

\[ \bar{P}_{w,t} \]

Ramp-up and ramp-down limits of generator \( w \).

\[ \bar{P}_{g,t} \]

In-flow and out-flow limits of gas storage \( g \).

\[ \bar{p}_{b,t} \]

Lower/upper limits of phase angles at bus \( b \).

\[ \bar{p}_{b,b'} \]

Phase angle difference limit of line \( b-b' \).

\[ \bar{w}_{n,t} \]

Minimum/maximum squared pressures.

\[ \bar{\tilde{w}}_{n,t} \]

Compression factor.

\[ \epsilon \]

Risk level of the chance constraint.

\[ \alpha \]

Percent of wind energy utilized.

1 Introduction

To use more green electricity, more natural gas-fired units are deployed in power systems. According to the latest Energy Information Administration, natural gas supplied 32.1% of total U.S. electricity at utility-scale power plants in the first quarter of 2016 [1], and over 60% of new generation will be fueled by natural gas from 2025 to 2040 in the U.S. [2].

Increasing natural gas-fired units make power systems and natural gas systems have stronger coupling. An outage or an interruption in natural gas grids could cause the loss of gas supply to gas-fired units, and in consequence could jeopardize the power system security and further lead to power load shedding [3]. For example, a failure on a single pipeline in 2002 resulted in the loss of 2019 MW power generation in Chicago, and further caused a cascading power outage. More than 10000 failures in gas pipeline networks, resulting in six billion U.S. dollars of damages and losses in natural gas systems and power systems, were reported by the Pipeline and Hazardous Materials Safety Administration (2013) of U.S. Department of Transportation. In this case, it is essential to guarantee high reliability of the interdependent power grids and natural gas grids. Usually, maintenance is used as one of important means to maintain high component reliability [4]. In addition, the percentage of wind power in power systems continues to increase, led by Uruguay, Portugal and Ireland pushing over 20%, followed by Spain around 20%, Germany around 16%. The big markets of Canada, U.S. and China get 6%, 5.5% and 4% of their electricity power from wind, respectively [5]. Based on the report of U.S. Department of Energy, 20% of electricity in U.S. will be generated by wind by 2030 [6], and more than 6.5% of world electricity will be generated from wind [7]. Though wind power can help to reduce the emissions of electricity production, high inter-temporal variations of wind power and its weak predictability also cause a series of new challenges to power system operation. Considering the requirement of high reliability of gas-power grids and the increasing integration of wind power into power grids, it is imperative to coordinate maintenance on interdependent natural gas and power grids with increasing wind power.

There have been some studies on integrated gas and power grid. In [8], a two-stage expansion planning model, in which uncertainty caused by natural gas and electricity demands is included, is developed. In [9], a model for designing interdependent natural gas and electricity networks is presented, by which the types and the capacities of the components can be determined. Considering dynamic gas flows on pipeline grids, a day-ahead scheduling model is proposed in [11] for generating unit dispatch and gas transport compression operation. A decentralized optimal energy flow calculation method is developed in [12] to deal with possible multiple subsystems that are tied by means of natural gas and power grids. In [13], an approximated transient matrix-form gas flow model is developed to include the influences of dynamics of natural gas flow on power systems. [14] investigates reverse power flow for distribution systems in consideration of integrated natural gas systems under renewable power penetration. [15] analyzes the potential of potential-to-gas-to-gas systems in consideration of the characteristics and the constraints of natural gas and power grids. [16] investigates the equilibrium of the coupled electricity and natural gas markets in consideration of market clearing processes. [17] proposes a two-stage mixed-integer linear stochastic optimization model to analyze the scheduling of electricity production units under natural gas-supply uncertainty due to pipeline congestion and natural gas-price variability. [18] proposes a long-term robust co-optimization planning model for Interdependent Electricity and Natural Gas Systems to minimize the total investment and operation costs. To deal with increasing extreme events,
proposes a preventive strategy for integrated natural gas and power systems to enhance the power system resilience. [20] proposes a graph-based framework, which facilitates analysis and simulation application for coupled infrastructure networks. However, no research studies focus on coordinated maintenance on interdependent natural gas and power grids, especially when including wind power.

Due to variability and uncertainty, a large amount of reserve capacity is required to accommodate forecast errors of wind power. When reserve capacity is not enough or transmission congestion exists, power system operators need to curtail wind power to guarantee the safety of the system. Robust optimization [21] can be used to deal with variability and uncertainty in consideration of operating conditions, however, it is based on the worst case outcomes that will lead to a too conservative result. In practice, system operators desire to utilize wind power as much as possible, and be able to request a portion of wind power at a certain probability. Chance-based constraints can be used to deal with the above issue and to describe the percentage of utilized wind power [22–26].

Therefore, we use chance-constrained model to formulate the problem of scheduling maintenance for integrated natural gas grids and power grids in consideration of wind power. The main contributions of this paper are shown as follows: 1) A model for maintenance scheduling for integrated natural gas and power grids with uncertain wind power is constructed; 2) Chance constraints are used to describe the utilization of wind power, and they are converted into a group of mixed integer liner constraints by means of a strong extended formulation for the sample average approximation based on the star-inequalities; 3) A big-M formulation method is used to convert inequalities constraints for pipelines that need to be under maintenance within the given time window; 4) The influences of natural gas systems on system maintenance scheduling are analyzed.

The remainder of this paper is organized as follows. The maintenance scheduling formulation, including the optimization objective, the maintenance constraints, the operating constraints of power and natural gas grids, are shown in Section 2. Section 3 presents the strong extended formulation of sample average approximation, and Section 4 presents the case studies. The work is concluded in Section 5.

2 Maintenance Scheduling Formulation

2.1 Optimization Objective

The objective is to maximize the revenue of the natural gas and power grid under maintenance. The detailed objective can be formulated as follows.

\[
\text{max} \quad \sum_{g \in G} \sum_{b \in B_g} \left( C^b_{g, t} \cdot P_{b, t} - C^d_{g, t} \cdot \alpha_{g, t} - C^s_{g, t} \cdot \beta_{g, t} \right) + \sum_{g \in G} \sum_{b \in B_g} \left( C^b_{g, t'} - G_{b, t} \right) \cdot \left( \sum_{p \in P} \sum_{e \in T} \left( C^p_{g, t} \cdot S_{p, e} \right) - \sum_{p \in P} \sum_{e \in T} \left( C^p_{g, t'} \cdot \Delta L_{e, t} \right) - \sum_{n \in N} \sum_{e \in T} \left( C^c_{n, e} \cdot \Delta L_{e, t} \right) \right) \cdot \alpha_{g, t'} - \sum_{p \in P} \sum_{e \in T} \left( C^p_{g, t} \cdot \Delta L_{e, t} \right) - \sum_{n \in N} \sum_{e \in T} \left( C^c_{n, e} \cdot \Delta L_{e, t} \right) \right) \cdot \alpha_{g, t'}
\]

where (1a) is the revenue from the power grid, (1b) and (1c) are the revenues from the natural gas system, (1d) and (1e) are the costs of maintenance on lines and pipelines, respectively, (1f) and (1g) are the costs of penalties caused by non-served power load and gas load, respectively.

2.2 Constraints of Maintenance

When performing maintenance, some constraints of maintenance should be satisfied.

\[
m_{b, t} - m_{b, t} + m_{b, t'} \leq 1 \quad 1 \leq t' - (t - 1) \leq M \cdot \epsilon_{e, t}, \quad \forall t, e
\]

\[
m_{b, t} - m_{b, t} + m_{b, t'} \leq 1 \quad 1 \leq t' - (t - 1) \leq M \cdot \epsilon_{e, t}, \quad \forall t, e
\]

\[
\sum_{e \in T} \left( 1 - m_{e, t} \right) = D^L_{e, t} \quad \forall t
\]

\[
\sum_{e \in T} \left( 1 - m_{e, t} \right) = D^P_{e, t} \quad \forall e, t
\]

\[
\sum_{p \in P} \left( 1 - m_{p, t} \right) \leq N^L_{t, PP} \quad \forall t
\]

\[
\sum_{p \in P} \left( 1 - m_{p, t} \right) \leq N^P_{t, PP} \quad \forall t
\]

where (2) and (3) ensure the minimal durations of maintenance activities on lines and pipelines, respectively, (4) and (5) ensure that maintenance activities on lines and pipelines will be implemented during the given time window, (6) shows the maximum number of lines that can be under maintenance within one time period, and (7) shows the similar constraint for pipelines.

2.3 Operation Constraints of Power Grids

For a power grid, the operation constraints, e.g., power balance, ramping rates of generators, generation limits and voltage limits, should be satisfied. The detailed constraints are listed as follows.

\[
\Pr \left\{ \sum_{g \in G} \sum_{b \in B_g} \left( P_{b, t} \geq \alpha \cdot \sum_{g \in G} \sum_{b \in B_g} P_{b, t} \cdot \epsilon_{e, t} \right) \right\} \geq 1 - \epsilon
\]

\[
B_{b, l, t} \cdot \left( \theta_{b, l, t} - \theta_{b, t} \right) + P_{b, t} \cdot \left( \left( 1 - m_{b, t} \right) \cdot M \geq 0 \quad \forall t, l \cdot (b, l, t) \in \mathcal{B}
\]

\[
B_{b, l, t} \cdot \left( \theta_{b, l, t} - \theta_{b, t} \right) - P_{b, t} \cdot \left( 1 - m_{b, t} \right) \cdot M \geq 0 \quad \forall t, l \cdot (b, l, t) \in \mathcal{B}
\]

\[
P_{b, l, t} \cdot m_{b, t} \leq P_{b, l, t} \cdot \epsilon_{e, t} \quad \forall t, l \cdot (b, l, t) \in \epsilon
\]

\[
B_{b, l, t} \cdot \left( \theta_{b, l, t} - \theta_{b, t} \right) = P_{b, l, t} \cdot \epsilon_{e, t} \quad \forall t, l \cdot (b, l, t) \in \epsilon
\]

\[
P_{b, l, t} \leq P_{b, l, t} \cdot \epsilon_{e, t} \quad \forall t, l \cdot (b, l, t) \in \epsilon
\]

where (2) and (3) ensure the minimal durations of maintenance activities on lines and pipelines, respectively, (4) and (5) ensure that the maintenance activities on lines and pipelines will be implemented during the given time window, (6) shows the maximum number of lines that can be under maintenance within one time period, and (7) shows the similar constraint for pipelines.
where $\theta$ shows the chance constraint ensuring that the scheduled wind power is within a certain range with a probability of $1-\epsilon$, and $P_w$ denotes the probability. $\sum_{\forall n \in N} \sum_{\forall t} P_{w,n,t}$ represents that $\alpha%$ of available wind power can be dispatched. $P_{\cdot,\cdot} \leq P_{\cdot,\cdot}$ shows that the scheduled wind power is not beyond the available wind power. (9) enforces power balance at each node in each time interval. (10) and (11) represent the physical relations between voltage angles and power flows in transmission lines to be under power flows through lines to be under maintenance. (13) relates the voltage angles and power flows through the lines that need not to be under maintenance, and (14) shows the lower/upper limits of power flow through the corresponding lines. (15) and (16) denote the minimum on-time and off-time of the generator $g$. (17) represents the start-up constraint of the generator $g$. (18) is the capacity limit of the generator $g$. The constraint (19) shows the limit of the phase angle difference of the line that needs to be under maintenance, and (20) shows the limit of the phase angle difference of the line that needs not to be under maintenance. (21) enforces the limits of load shedding. (22) and (23) present the ramp-up/ramp-down constraints of generators. (24) shows the relation between real power from gas-fired units and natural gas consumption.

### Natural gas balance
Natural gas balance at each gas node should be ensured in each time period.

\[
\sum_{\forall n \in N} G_{g,t} + \sum_{\forall n \in N} (S_{g,n,t} - S_{g,n,t}) - \sum_{n \in N} F_{n,n',t} = \bar{G}_{g,t} \quad \forall n \in N, t
\]

\[
0 \leq \Delta L_{n,t} \leq L_{n,t} \quad \forall n, t
\]

### NG flow in pipelines not to be under maintenance
The pressure drops in pipelines can be expressed as the nonlinear Weymouth (8).

\[
\text{sign}(F_{n,n',t}) \cdot P_{n,n',t} = \bar{C}_{n,n',t}^2 \cdot (P_{S_{p,n,t}}^2 - P^2_{S_{p,n',t}}) \quad \forall n, n', t
\]

However, (32) leads to difficulty in solving the model. In this paper, a piecewise linear approach, as shown in Fig. 1, is employed by using the mixed integer programming. After substituting $PS^2$ with $\bar{n}$, the left side of (32) can be written as the sum of a series of linear functions approximately.

![Fig. 1. Piecewise linear approximation of pipeline flow.](image)

The model can be expressed as (77) for the pipelines that need not to be under maintenance within the given time window.

\[
\sum_{k} \left( A_{k,n,t} \cdot f^{h}_{k,n,t} + B_{k,n,t} \cdot \eta^{h}_{k,n,t} \right) = \bar{C}_{n,n',t} \cdot (\pi_{n,t} - \pi_{n',t}) \quad \forall n, n', t
\]

\[
\sum_{k} \eta^{h}_{k,n,t} = 1 \quad \forall n, t
\]

\[
\eta^{k}_{n,t} \cdot L^{k}_{p} \leq f^{k}_{p,n,t} \leq \bar{h}^{k}_{p,n,t} \cdot T^{k}_{p} \quad \forall k, t, p
\]

\[
F_{n,n',t} = \sum_{k} f^{h}_{k,n,t} \quad \forall (n, n') \in p, p'
\]

\[
\eta^{h}_{k,n,t} \in \{0, 1\} \quad \forall n, t, k
\]

where the term $F_{n,n',t}$ in (32) is approximated by the sum of piecewise linear functions on the left side of (33). For example, six piecewise linear functions, i.e., $k = \{1, 2, \ldots, 6\}$, exist in Fig. 1. (34) guarantees that only one function can be selected with the constraint $\eta^{h}_{k,n,t}$ in $\{0, 1\}$. For the $k^{th}$ linear function, the variable $f^{h}_{k,n,t}$ is limited by (35). (36) shows the gas flow through the pipeline $n \rightarrow n'$. For a big-M approach, the gas flow through the pipeline $n \rightarrow n'$.
employed to model the pipelines that need to be under maintenance.

\[
\sum_{k} \left( A_{k}^{t} \cdot p_{k}^{t} + P_{k}^{t} \cdot \eta_{k}^{t} \right) \leq C_{n,n}^{t} \cdot \left( \pi_{n,t} - \pi_{n,n'}^{t} \right) + \left( 1 - m_{n,n'}^{t} \right) \cdot M \quad \forall \left( n,n' \right) \in p,t,p
\]

(38)

\[
\sum_{k} \left( A_{k}^{t} \cdot p_{k}^{t} + P_{k}^{t} \cdot \eta_{k}^{t} \right) \geq C_{n,n}^{t} \cdot \left( \pi_{n,t} - \pi_{n,n'}^{t} \right) - \left( 1 - m_{n,n'}^{t} \right) \cdot M \quad \forall \left( n,n' \right) \in p,t,p
\]

(39)

\[
\eta_{k}^{t} \sum_{p} f_{p,k}^{t} \leq \eta_{k}^{t} \cdot \tilde{T}_{k}^{t} \quad \forall p,k,t
\]

(40)

\[
\eta_{k}^{t} = \eta_{p,k}^{t} \quad \forall p,t
\]

(42)

where gas flows through pipelines are represented as (38) and (39). When the pipelines are under maintenance, (38) and (39) are redundant by introducing a sufficiently large \( M \) and a binary variable \( \eta_{k}^{t} \). (40) and (41) present the similar meanings with (35) and (36), respectively. (42) shows the relation between the variable \( \eta_{k}^{t} \) and \( \eta_{p,k}^{t} \). When \( m_{n,n'}^{t} = 1 \), the corresponding pipeline is not under maintenance and the corresponding models are the same as (33)-(36). When \( m_{n,n'}^{t} = 0 \), the corresponding pipeline is under maintenance, and the corresponding constraints are redundant.

2.4.6 NG pressure constraints: In natural gas grids, the pressure of each node needs to be within the limit.

\[
\sum_{t} \pi_{n,t} \leq \pi_{n} \quad \forall n,t
\]

(44)

\[
\pi_{n,n'}^{t} \leq \lambda_{p} \cdot \pi_{n,t} \quad \forall p,t \left( n,n' \right) \in \bar{\rho}
\]

(45)

where (44) shows the pressure constraint. For a pipeline with a compressor, the pressures between the out-going gas node and the in-coming gas node should satisfy the constraint (45).

3 Strong Extended Formulation of Sample Average Approximation

This section first introduces the conventional sample average approximation, and then a strong extended formulation of the sample average approximation is presented.

3.1 Conventional Sample Average Approximation

Let \( \tilde{P}_{w,t,s} \), \( w \in \mathcal{W} \), \( t \in \mathcal{T} \), \( m \in \mathcal{S} \) be independent Monte Carlo samples of the random parameter \( P_{w,t} \) in (8). With \( \tilde{P}_{w,t,s} \), we can build the following formulation (46)-(49) to replace the chance constraint (8) by using the Sample Average Approximation (SAA) approach.

\[
\sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} + \nu_{s} \cdot \tilde{M} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \quad \forall s
\]

(46)

\[
P_{w,t} - \nu_{s} \cdot \tilde{M} \leq \tilde{P}_{w,t,s} \quad \forall w,t,s
\]

(47)

\[
\sum_{s \in [0,1]} \nu_{s} \leq \epsilon \cdot |\mathcal{S}|
\]

(48)

\[
\nu_{s} \in \{0,1\} \quad \forall s
\]

(49)

where \( \mathcal{S} \) denotes the number of samples, and \( \tilde{M} \) is a large positive constant. \( \nu_{s} \) is a binary variable, representing if \( s \)th scenario is satisfied or not. When \( \nu_{s} = 0 \), the constraints \( \sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) and \( P_{w,t} \leq \tilde{P}_{w,t,s} \) holds. When \( \nu_{s} = 1 \), the constraints with respect to the \( s \)th scenario become redundant. The constraint (48) shows that the number of scenarios that violate the chance constraint does not exceed \( \epsilon \cdot |\mathcal{S}| \). In this way, the original chance constraint is approximated by counting the number of scenarios that satisfy some deterministic constraints.

3.2 Strong Extended Formulation of SAA

With the conventional SAA, the size of the formulation grows rapidly as the number of samples increases. To deal with this issue, a strong extended formulation of SAA is employed. We rewrite the formulation (46)-(49).

\[
\sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} + \nu_{s} \cdot \tilde{M} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \quad \forall s
\]

(50)

\[
\beta_{w,t} + \nu_{s} \cdot \left( \tilde{P}_{w,t,s} - \tilde{P}_{w,t,s} \right) \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \quad \forall w,t,s
\]

(51)

\[
\beta_{w,t} = P_{w,t} - \tilde{P}_{w,t,s} \quad \forall w,t
\]

(52)

\[
\nu_{s} \leq \epsilon \cdot |\mathcal{S}|
\]

(53)

\[
\nu_{s} \in \{0,1\} \quad \forall s
\]

(54)

where (46) is converted to (50), and (47) is converted to (51) and (52). Take the constraint (46) as an example. There are two scenarios: when \( \nu_{s} = 0 \), \( \sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) holds; when \( \nu_{s} = 1 \), \( \sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} + M \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) holds. For the new constraint (50), when \( \nu_{s} = 0 \), \( \sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) holds; when \( \nu_{s} = 1 \), \( \sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} + \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) holds since \( P_{w,t} \geq 0 \). This shows that (50) is sufficient for (46).

For the formulation (50)-(51), the star-inequalities can be used to build the strong extended formulation [27]. To this end, we first sort \( \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s} \) and \( \tilde{P}_{w,t,s} - \tilde{P}_{w,t,s} \) in descending order and define the index of the element at the \( i \)th position of the sequence as \( s \) (where \( i \in \mathcal{I} \) and \( \mathcal{I} = \{1,2,\ldots,|\mathcal{S}|\} \)). \( |\mathcal{S}| \) denotes the largest integer but less than \( \epsilon \). By introducing new binary variables \( r_{i} \) and \( \gamma_{w,t,s} \), the strong extended formulation can be rewritten as follows.

\[
\sum_{w \in \mathcal{W}, t \in \mathcal{T}} P_{w,t} + \nu_{s(i)} \cdot \tilde{M} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s(i)} - r_{i} \cdot \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s(i)} \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s(i)}
\]

(55)

\[
r_{i} - r_{i+1} \geq 0 \quad \forall i
\]

(56)

\[
\nu_{s(i)} - r_{i} \geq 0 \quad \forall i
\]

(57)

\[
\beta_{w,t} + \sum_{s \in \mathcal{I}} \left( \tilde{P}_{w,t,s(i)} - \tilde{P}_{w,t,s(i)} \cdot \gamma_{w,t,s} \right) \geq \alpha \sum_{w \in \mathcal{W}, t \in \mathcal{T}} \tilde{P}_{w,t,s(i)} \quad \forall w,t
\]

(58)
The costs of maintenance activities are implemented with $500 per time interval, and the duration of maintenance activity is 31.

$$\gamma_{w,t,i} - \gamma_{w,t,i+1} \geq 0 \quad \forall i, w, t \quad (59)$$

$$\nu_{s(i)} - \gamma_{w,t,i} \geq 0 \quad \forall i, w, t \quad (60)$$

$$\beta_{w,t} = P_{\text{max}} - P_{w,t} \quad \forall w, t \quad (61)$$

$$\sum_{s \in S} \nu_s \leq \epsilon |S| \quad (62)$$

$$\nu_s, \gamma_{w,t,i}, r_i \in \{0, 1\} \quad \forall s, w, t, i \quad (63)$$

### 3.3 Reformulated Optimization Model

The reformulated optimization model can be rewritten as follows.

**Obj:**

$$\text{max} \quad (1)$$

**s.t.**

$$\gamma_{w,t,i} - \gamma_{w,t,i+1} \geq 0 \quad \forall i, w, t \quad (7)$$

$$\nu_{s(i)} - \gamma_{w,t,i} \geq 0 \quad \forall i, w, t \quad (8)$$

$$P_{\text{max}} - P_{w,t} \quad \forall w, t \quad (9)$$

$$\sum_{s \in S} \nu_s \leq \epsilon |S| \quad (10)$$

$$\nu_s, \gamma_{w,t,i}, r_i \in \{0, 1\} \quad \forall s, w, t, i \quad (11)$$

where the reformulated optimization model is a mixed integer linear programming model, which is solved by the solver CPLEX.

### 4 Case Studies

The proposed model is verified by two test systems. The first one is a 4-node natural gas grid with a 6-bus power grid, and the second one is a 20-node natural gas grid with a modified IEEE 118-bus power grid.

#### 4.1 4-node natural gas grid with 6-bus power grid

**Test system description:** The 4-node natural gas grid and the 6-bus power grid are detailed in [28] and [29]. The integrated systems are shown in Fig. 2. The trend of wind power generation is the Belgian wind-power forecasting from 21st to 24th April 2016 [30]. The efficiency factor of each gas-fired unit is assumed to be 0.004 MJ/m³. The parameters for generating units are listed in Table 1.

![Fig. 2: Topology of a 4-node natural gas grid with a 6-bus power grid.](image)

The time window includes 48 time intervals, and each time interval covers two hours. The lines 3-6 and 1-4 in the power grid need to be under maintenance in the given time window, and the corresponding durations of maintenance activities are 24 and 22, respectively. The costs of maintenance activities are $600 and $500 per time interval. For the natural gas grid, maintenance on the pipeline 2-3 should be implemented with $500 per time interval, and the duration of maintenance activity is 31.

<table>
<thead>
<tr>
<th>Parameters for Generators</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Limits (MW)</td>
<td>100</td>
<td>80</td>
<td>150</td>
</tr>
<tr>
<td>Upper Limits (MW)</td>
<td>300</td>
<td>200</td>
<td>350</td>
</tr>
<tr>
<td>Ramping Rates (MW/h)</td>
<td>25</td>
<td>20</td>
<td>7.5</td>
</tr>
<tr>
<td>Fixed Cost ($)</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>Linear Cost (5/MW-h)</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>Restart Cost ($)</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>Minimum Up Periods</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Minimum Down Periods</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

#### 4.1.2 Influences of $\epsilon$ and $\alpha$ on maintenance scheduling:

This section analyzes the influences of wind power on maintenance scheduling for the integrated natural gas and power grid. Different percentages of wind power utilization are investigated. Table 2 shows the maintenance strategies with different wind power utilization, i.e., different values of $\epsilon$ and $\alpha$. When we have $\epsilon = 0.3$ and $\alpha = 0.7$, maintenance on line 1–4, line 3–6, and pipeline 2–3 should be performed from the periods 25 to 46, 25 to 48, and 7 to 37, respectively. When we have $\epsilon = 0.1$ and $\alpha = 0.9$, maintenance on line 1–4, line 3–6, and pipeline 2–3 should be performed from the periods 25 to 46, 25 to 48, and 2 to 32, respectively. It is observed that the maintenance on natural gas grid has a marked difference. When we have a smaller $\epsilon$ and a larger $\alpha$, there will be more wind power utilized in the power system, and in this case power generation from the units $G_1$, $G_2$, and $G_3$ will decrease. Because $G_1$ and $G_2$ are gas-fired units, power generation reduction from them leads to the reduction of natural gas consumption of the gas-fired units, and in consequence has an impact on maintenance on the natural gas grid.

![Fig. 3: Table 2 Maintenance Scheduling with different parameters](image)
4.1.3 Influences of Scenario Number on Maintenance scheduling: Because the revised sample average approximation method is used to deal with wind power uncertainty, the number of wind power scenarios impacts the maintenance scheduling. This section shows the influences of wind power scenarios on maintenance scheduling. Fig. 4 shows the optimization objectives with different wind power scenarios when having different values of the parameter $\alpha$. For each value of the parameter $\alpha$, the objective decreases and goes to a constant value. This constant value can be obtained with enough wind power scenarios. This indicates that more wind power scenarios result in a more accurate solution. From the perspective of the mathematical model, more wind power scenarios result in more constraints due to (46) and (47), i.e., a tight feasible region, and in consequence lead to a smaller objective since the optimization model is to find a maximum objective.

Table 3 shows maintenance strategies with different wind power scenarios. It is observed that we have the same maintenance strategies with different wind power scenarios. This also shows that more scenarios of wind power leads to a more accurate solution.

4.1.4 Influences of natural gas system on power system: Because natural gas to gas-fired units are limited by the natural gas grid, maintenance on pipelines directly impacts maintenance on the power system. Fig. 5 shows the loss of load in the power system when having different pipeline maintenance durations. It is observed that the loss of load is about 50MW when the pipeline maintenance duration is from 21 to 24 time intervals. This indicates that maintenance on pipeline $P_{L1}$ does not impact gas supply to gas-fired units in the power system. However, when the pipeline maintenance duration is longer than 24 time intervals, the loss of load in the power system increases gradually. This means that maintenance over 24 time intervals or more will impact gas supply to gas-fired units in the power system and in consequence will lead to more loss of loads.

Table 3 Maintenance Scheduling with different scenarios

<table>
<thead>
<tr>
<th>Scenario Number</th>
<th>Maintenance Scheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$L_{14}:25 - 48$, $L_{16}:25 - 48$, $PL_{L1}:12 - 42$</td>
</tr>
<tr>
<td>400</td>
<td>$L_{14}:27 - 48$, $L_{16}:25 - 48$, $PL_{L1}:2 - 32$</td>
</tr>
<tr>
<td>600</td>
<td>$L_{14}:27 - 48$, $L_{16}:25 - 48$, $PL_{L1}:4 - 34$</td>
</tr>
<tr>
<td>800</td>
<td>$L_{14}:27 - 48$, $L_{16}:25 - 48$, $PL_{L1}:4 - 34$</td>
</tr>
<tr>
<td>1000</td>
<td>$L_{14}:27 - 48$, $L_{16}:25 - 48$, $PL_{L1}:4 - 34$</td>
</tr>
</tbody>
</table>

4.2 20-node grid with 118-bus power grid

4.2.1 Data description: The data of the natural gas grid and the power grid can be found in [31] and [32], respectively. Wind power trends are the same as the trends in the first case, and the buses 2, 3, 5, 13, 14, 16, 44, 50, 52, 53, 82, 83, 84, 85, and 86 are connected to wind farms. The detailed data for wind power refer to [33]. The topology of the integrated system is shown as Fig. 6. Seven transmission lines 69-47, 69-70, 69-75, 69-77, 69-68, 49-69, and 49-47 need to be under maintenance within the given time window, and the durations of maintenance are 15 time intervals. Two pipelines 10-14 and 10-11 need to be under maintenance, and the durations of maintenance are 18 time intervals. The maximum numbers of pipelines and lines that can be under maintenance in one time interval are 1 and 2, respectively.

4.2.2 Influences of wind power on system maintenance: We first analyze the influences of wind power utilization on maintenance scheduling. Table 4 shows the objectives with different values of $\epsilon$ and $\alpha$. A smaller $\epsilon$ and a larger $\alpha$ indicate more wind power utilized in power system. In this case power generation from generating units decreases, and in consequence the objective decreases accordingly.

Table 4: Objectives with different wind power scenarios.

<table>
<thead>
<tr>
<th>Scenario Number</th>
<th>Percentage of wind power utilized $\alpha$</th>
<th>Objective ($)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$0.75$</td>
<td>1.165</td>
</tr>
<tr>
<td>400</td>
<td>$0.90$</td>
<td>1.167</td>
</tr>
<tr>
<td>600</td>
<td>$0.80$</td>
<td>1.166</td>
</tr>
<tr>
<td>800</td>
<td>$0.80$</td>
<td>1.168</td>
</tr>
</tbody>
</table>

Fig. 4: Objectives with different wind power scenarios.

Fig. 5: Loss of load under different maintenance durations.
Influences of natural gas system on system maintenance: This section analyzes the influences of the natural gas system on system maintenance. Fig. 8 (a) shows the average loss of load for each time period when considering the influences of the natural gas grid, e.g., natural gas transmission capacity. Fig. 8 (b) shows the average loss of load for each time period when the influences of the natural gas grid are not included, i.e., the natural gas to the gas-fired units is not limited by the natural gas grid. It is observed that the first case in Fig. 8 (a) has larger loss of load compared to the second case in Fig. 8 (b) when having the same power forecast error. The main reason for more loss of load in the first case is the limited natural gas to the power grid due to the constraints of the natural gas grid. Therefore, the influences of natural gas systems on system maintenance should be included in practice.

We also analyze the influences of the number of wind power generators and wind power forecast errors. Fig. 7 shows the objective values with different wind power generators and wind power forecast errors. Table 5 shows the locations of different wind generators in different cases. It is observed that smaller wind power forecast errors result in a smaller wind power forecast error. From the perspective of mathematical model, more piecewise functions result in more piecewise functions on maintenance scheduling. Table 6 shows the objectives with different numbers of piecewise functions. It is observed that more piecewise functions lead to a smaller objective. From the perspective of mathematical model, more piecewise functions result in a relaxed feasible solution region according to the constraints (46) and (47).

### Table 5: Locations of wind generators in different cases

<table>
<thead>
<tr>
<th>Number</th>
<th>Locations (Buses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2, 3, 5</td>
</tr>
<tr>
<td>6</td>
<td>2, 3, 5, 13, 14, 16</td>
</tr>
<tr>
<td>9</td>
<td>2, 3, 5, 13, 14, 16, 44, 50, 52</td>
</tr>
<tr>
<td>12</td>
<td>2, 3, 5, 13, 14, 16, 44, 50, 52, 82, 83</td>
</tr>
<tr>
<td>15</td>
<td>2, 3, 5, 13, 14, 16, 44, 50, 52, 82, 83, 84, 85, 86</td>
</tr>
</tbody>
</table>

### Table 4: Objective Values ($10^9$) with Different Values of $\epsilon$ and $\alpha$

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$\alpha = 0.7$</th>
<th>$\alpha = 0.8$</th>
<th>$\alpha = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.109</td>
<td>3.096</td>
<td>3.085</td>
</tr>
<tr>
<td>0.2</td>
<td>3.125</td>
<td>3.101</td>
<td>3.091</td>
</tr>
<tr>
<td>0.3</td>
<td>3.131</td>
<td>3.104</td>
<td>3.095</td>
</tr>
</tbody>
</table>

Fig. 7: Objectives with different wind power generators and wind power forecast errors

Fig. 8: Objectives with different wind power generators and wind power forecast errors
a tighter feasible solution region according to the constraints (33)-(43). Because the optimization is to find a maximum objective value, a tighter feasible solution region leads to a smaller objective. More piecewise functions result in a more accurate solution, but also lead to a longer computation time. Take $\varepsilon = 0.3, \alpha = 0.7$ as an example, the computation times are 3.6(h) and 8.1(h) when having 3 and 9 piecewise functions, respectively. Because maintenance scheduling is made off-line, a long computation time is acceptable in practice.

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Objective Values ($$/h) with Different Piecewise Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 3$</td>
<td>$3.149 \quad 3.138 \quad 3.131 \quad 3.127$</td>
</tr>
<tr>
<td>$k = 5$</td>
<td>$3.127 \quad 3.115 \quad 3.109 \quad 3.104$</td>
</tr>
<tr>
<td>$k = 7$</td>
<td>$3.104 \quad 3.092 \quad 3.081 \quad 3.070$</td>
</tr>
<tr>
<td>$k = 9$</td>
<td>$3.063 \quad 3.047 \quad 3.032 \quad 3.020$</td>
</tr>
</tbody>
</table>

5 Conclusions

This paper presents a chance-constrained maintenance scheduling model for interdependent natural gas systems and power systems with wind energy integration. Chance constraints are used to model uncertainties of wind power, and these chance constraints are transformed into equivalent deterministic constraints by means of extended star-inequalities. The proposed model is solved by an extended star-inequalities-based deterministic-constraint MILP model, and the major findings are listed as follows. 1) Under the maintenance window, especially when including the interactions between interdependent power systems and natural gas systems, the maintenance window can reduce the potential loss of load, and accurate wind power forecasting improves the system revenue during the maintenance window. 2) Shorter maintenance durations for power systems and natural gas grids can increase the system revenue during the maintenance window, especially when including the interactions between interdependent power systems and natural gas systems. At present, a piecewise linear approximation approach is employed to transform the nonconvex and nonlinear Weymouth gas flow equations into a series of piecewise linear functions, which cause a gap to the original model. In the future work, several state-of-the-art methods on second order conic relaxation can be used to improve the accuracy. In addition, second-order cone program (SOCP) formulations for describing AC power flow can replace DC power flow to improve the accuracy.

6 Acknowledgments

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7 References