

## A Data-Driven Stackelberg Market Strategy for Demand Response-Enabled Distribution Systems

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### Abstract

A data-based Stackelberg market strategy for a distribution market operator (DMO) is proposed to coordinate power dispatch among different virtual power plants, i.e., demand response (DR) aggregators (DRAs). The proposed strategy has a two-stage framework. In the first stage, a data-driven method based on noisy inverse optimization estimates the complicated price-response characteristics of customer loads. The estimated load information of the DRAs is delivered to the second stage, where a one-leader multiple-follower stochastic Stackelberg game is formulated to represent the practical market interaction between the DMO and the DRAs that considers the uncertainty of renewables and the operational security. The proposed data-driven game model is solved by a new penalty algorithm and a customized distributed hybrid dual decomposition-gradient descent algorithm. Case studies on a practical DR project in China and a distribution test system demonstrate the effectiveness of the proposed methodology.

### Index Terms

Market strategy, demand response, noisy inverse optimization, Stackelberg game, Lagrange dual decomposition.

### Nomenclature

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### Parameters

- $c^p_{i,t}$: Historical price data of aggregator $i$ at time $t$
- $\omega_i$: Historical consumption data of aggregator $i$ at time $t$
- $M$: Penalty factor of the reformulation
- $F$: Forgetting factor
- $p^d_{i,n,t}/q^d_{i,n,t}$: Demand active/reactive power at bus $n \in \Omega_n$, $n \notin T$
- $c_p$: Generation cost of DG $g$
- $p^{d}_i$: Maximum generation of DG $g$
- $p^d_{i,s}$: Renewable generation of aggregator $i$ at time $t$ in scenario $s$
- $\bar{c}_t$: Electricity prices set by ISO at time $t$
- $\bar{g}_t$: Planned purchased electricity set by ISO at time $t$
- $\bar{p}^d_{i}/\bar{q}^d_{i}$: Maximum active/reactive exchange power at bus connected to aggregator $i$
- $\bar{P}_{nm}/\bar{Q}_{mn}$: Active/reactive power limit of line $(n,m)$
- $\bar{e}$: Maximum electricity price set by DMO
- $b^1_n/b^2_n$: Resistance/reactance between buses $n$ and $n+1$
- $\mu$: Power redispatch cost
- $\gamma_t$: Voltage deviation
- $\bar{y}_i$: Price penalty paid for mismatch between energy generation and consumption
- $T$: Total number of samples
- $N$: Proportion of different samples
- $\bar{r}_{i,d}:\bar{y}_i$ and $\bar{d}_i$: Pick-up/drop-off rate of DGs
- $Pr(s)$: Probability of realization for $s \in S$
- $\bar{E}_s$: Expectation with respect to $S$
- $K$: Number of concatenated elements

### Variables

- $Q(p)$: Second-stage stochastic problem
- $\Delta p^d_{i,t}$: Mismatch variable
- $\phi(p, s)$: Mismatch problem

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with electricity price and energy consumption as variables to
model customer reactions to electricity prices. In [5], the price-elasticity pattern was modeled as a bilinear function, which increases the effective capacity of the resulting aggregator resource to achieve dispatchability through an averaging of local conditions. A demand response aggregator (DRA) or virtual power plant (VPP) serves this purpose.

One key issue for demand-side resources is their relatively small individual capacities. A second key issue is that their degree of flexibility can depend on local environmental conditions and the local objectives of their owners or managers.

Harnessing useful service flexibility from these resources thus requires some form of aggregation of their service capabilities, which increases the effective capacity of the resulting aggregated resource to achieve dispatchability through an averaging of local conditions. A demand response aggregator (DRA) or a virtual power plant (VPP) serves this purpose.

Recently, there has been much interest in adopting the Stackelberg game to build hierarchical models for practical decision-making problems in power markets [13]. A bilevel game between power service providers and users was proposed in a retail market [14]. This model aimed to assist providers to set optimal strategies and encourage users to adjust their power usage. Reference [15] presented a real-time DR algorithm based on the Stackelberg game to control smart appliances. A virtual electricity trading process was designed to balance local objectives between followers (devices) and the
II. PROPOSED MARKET-BASED FRAMEWORK

We assume that one distribution system consists of one distribution market operator (DMO) and multiple DRAs [24]. A DRA, which includes a cluster of customer loads and DGs, can access the aggregated load data of its own cluster. However, it cannot access the information of other DRAs. DRAs compete with each other on behalf of their customers, while the DMO [25] leverages price signals to coordinate the utility company and different DRAs.

Fig. 1 depicts the proposed two-stage hierarchical framework consisting of a data-driven stage and a Stackelberg-pricing stage. The first stage is based on an inverse optimization scheme that leads to a bilevel optimization problem.

To deal with the above-mentioned limitations, in this work we formulate a data-driven Stackelberg game for the distribution market. The proposed market strategy is a two-stage framework with bilevel programming models in each stage. In the first stage, the noisy data-based inverse scheme is designed to perform a data-driven modeling of price responses. In the second stage, the market strategy is modeled using a Stackelberg game, which is reformulated as a bilevel stochastic programming.

As an effective state estimator, the inverse optimization framework has been widely used in a variety of research areas [21]–[23]. In this paper, the proposed data-driven method is based on the inverse optimization scheme with two major modifications. First, the training dataset is considered as a known state vector and the estimator is based on customer electricity consumption behavior. Second, a penalty factor $M$ is used to minimize the out-of-sample prediction error.

The key contributions of this paper are threefold:

1) An innovative data-driven model is designed to estimate demand-side flexibility via historical price-consumption data. This model avoids the complexity of traditional load modeling, guarantees the execution of user response under optimized prices, and reduces computational burden. Moreover, the inverse optimization algorithm in this paper is different from conventional ones as it is based on noisy data, which not only estimates price-response parameters but also minimizes their prediction errors.

2) The proposed Stackelberg game-based market strategy considers market information from the independent system operator (ISO), the operation security constraints, and the stochasticity of distributed renewable generators. The proposed distribution-level regulation model can directly fit into ISOs' day-ahead/real-time wholesale markets and end participants in retail markets, and bridge the gap between wholesale markets and end participants.

3) Two customized algorithms are applied: one is a tuned penalty algorithm with fast computation that precisely predicts customers’ DR responses based on a large amount of historical data; the other is a distributed hybrid dual decomposition-gradient descent (HDDGD) algorithm that caters to the distributed market structure and converges to the optimal solution of the Stackelberg game through efficient parallel computation.

The proposed methods are validated in a test case and simulation using real DR data.

The rest of this paper is organized as follows. Section II introduces the proposed market strategy architecture. The first and second stage problem formulations are presented in Sections III and IV, respectively. Solution algorithms are implemented in Section V. Section VI analyzes numerical results, followed by concluding remarks in Section VII.
problem for each DRA. The upper level utilizes historical
price-consumption data to estimate load ramp rates and con-
sumption limits subject to the price response models in the
lower level. The estimated parameters are then delivered to the
DRA (follower) of the Stackelberg game model. In the second
stage, the DRA uses stochastic programming to minimize the
costs of its aggregated energy consumption, unbalanced power,
and controllable DG generation. According to the expected
wholesale market information from the ISO, network opera-
tion constraints, and the expected consumption of DRAs, the
DMO (leader) calculates a pricing strategy for the DRAs and
the electricity that needs to be purchased from the wholesale
market. Through iterative interactions between the DMO and
DRAs, all players in the game reach an equilibrium.

III. STAGE ONE: DATA-DRIVEN DR PREDICTION

To capture DR characteristics and determine the correspond-
ing model parameters for a DRA, the training procedure in the
data-driven stage is cast as a bilevel programming problem.
The upper level is a parameter-estimation problem, where the
load parameters of the lower-level are evaluated to minimize
data prediction errors.

A. Lower-Level Problem: Price Response
of a DRA’s Consumers

The lower-level problem is formulated using the consump-
tion decisions of the DRAs’ consumers, where parameters
$\theta_{i,t}$ are determined by the upper-level optimization (this
optimization is described in Section III-B). In the noisy
inverse theory [26], the consumption decision model is given
by a parameter vector $\theta_{i,t} = (a_i, \alpha_i, r_i^d, r_i^u, \lambda_i^d, \lambda_i^u, \psi_i, \bar{\psi}_i, \bar{\bar{\psi}}_i)$, at time
t $t \in T \equiv \{ t : t = 1 \ldots T \}$. Aggregated consumers of a
DRA behave as welfare-maximizing individuals, whose utility
function represents total economic benefits and customer satis-
faction. This means we can use the following model to mimic
the electricity consumption decision-making of the customers.
Compared to existing methods, there is no need to assume the
model empirically or hypothetically since the upper level eval-
uates $\theta_{i,t}$ according to historical records and prediction errors.

For $\forall t \in T$:

$$\max_{x_{i,t}} \sum_{t \in T} \left( a_i x_{i,t} - c_{i,t} x_{i,t} \right)$$  \hspace{1cm} (1a)

Let $T_{-1} = \{ t : t = 2, \ldots, T \}$. The objective function is then
subject to:

$$x_{i,t} - x_{i,t-1} \leq r_{i,t}^d, t \in T_{-1}$$  \hspace{1cm} (1b)

$$x_{i,t-1} - x_{i,t} \leq r_{i,t}^u, t \in T_{-1}$$  \hspace{1cm} (1c)

$$x_{i,t} \leq \bar{P}_{i,t}, t \in T$$  \hspace{1cm} (1d)

$$x_{i,t} \geq \bar{P}_{i,t}, t \in T$$  \hspace{1cm} (1e)

Through load ramp rates and consumption limits, con-
straints (1b)–(1e) impose a feasible region on DR activi-
ties [27]. In addition, this feasible region and parameters in
the objective function change over time as customer behavior
is time variant. To recast the bilevel optimization problem in
the data-driven stage as a single-level problem, the following
Karush-Kuhn-Tucker (KKT) reformulations will be utilized in the
next subsection:

$$-x_{i,t}^u + \lambda_i^d - \psi_i + \bar{\psi}_i = a_i - c_i$$  \hspace{1cm} (2a)

$$a_i - c_i = x_{i,t}^u - \lambda_i^d - \lambda_i^{d+} + \lambda_i^u - \psi_i - \bar{\psi}_i$$  \hspace{1cm} (2b)

$$x_{i,t} - x_{i,t-1} \leq r_{i,t}^d, t \in T_{-1}$$  \hspace{1cm} (2c)

$$x_{i,t-1} - x_{i,t} \leq r_{i,t}^u, t \in T_{-1}$$  \hspace{1cm} (2d)

$$x_{i,t} \leq \bar{P}_{i,t}, t \in T$$  \hspace{1cm} (2e)

$$x_{i,t} \geq \bar{P}_{i,t}, t \in T$$  \hspace{1cm} (2f)


B. Upper-Level Problem: DR Characteristics Estimation

Given a time series of pairwise price-consumption data
$(c_{i,t}^h, x_{i,t}^h)$, the inverse optimization estimates the value
of parameter vector $\theta_{i,t}$, which defines the lower-level
problem (1), such that the optimal solution of $x_{i,t}$ resulting
from this problem is as close as possible to the historical data
$x_{i,t}^h$ in terms of a certain norm. The parameters in $\theta_{i,t}$, in turn,
best represent the price-response characteristics of aggrega-
tor $i$’s consumers. The mathematical formulation is described
below for $\forall i \in I$:

$$\min_{x_{i,t}, \theta_{i,t}} \sum_{t \in T} \omega_t |x_{i,t} - x_{i,t}^h|$$  \hspace{1cm} (3a)

$$\text{s.t.} (2)$$  \hspace{1cm} (3b)

where constraints (3b) correspond to the KKT conditions of
lower-level problem (1). The variables $\theta_{i,t}$ in (3), which repre-
sent the parameter vector in (1), are constrained by optimality
conditions, thus guaranteeing that $x_{i,t}$ is optimal for (1).

The weight of the estimation error at time $t$ is represented
by parameter $\omega_t$ in (3a). These weights have a two-fold mean-
ing. For the day-ahead market, the weights represent the price
of the balancing power at time $t$. In this case, consumption
at time $t$ with a higher balancing price matches the original
data better. For the real-time market, $\omega_t = (t/T)^F$, where para-
meter $F$ indicates how rapidly the model forgets previous data.
To save computational costs and achieve faster convergence, a
forgetting factor $F$ is integrated to apply exponentially decay-
ing weights to previous observations. As $F \geq 0$ increases,
the weights of more recent observations become larger than
the old ones, and when $F = 0$, all observations are weighted
equally. The proposed data-driven method can be separately
applied to both the day-ahead scenario and the real-time
scenario.

To remove the absolute value sign, problem (3) can be reformu-
lated as the following linear objective function plus two
additional constraints for $\forall i \in I$:

$$\min_{x_{i,t}, \theta_{i,t}, \alpha_{i,t}, \beta_{i,t}} \sum_{t \in T} \omega_t (\alpha_{i,t}^+ + \beta_{i,t})$$  \hspace{1cm} (4a)

$$\text{s.t.} (2)$$  \hspace{1cm} (4b)

$$x_{i,t} - x_{i,t}^h = \alpha_{i,t}^+ - \beta_{i,t}, t \in T$$  \hspace{1cm} (4c)

$$\alpha_{i,t}^+, \beta_{i,t} \geq 0, t \in T$$  \hspace{1cm} (4d)
In summary, we established (1) to represent the parametric price-response model for DRA \(i\)'s flexible consumers. Compared to traditional load modeling that has polynomial, exponential, or other complicated forms, the proposed price-response model uses linear constraints (1b)–(1e) to precisely describe the feasible region of DR activities. The accuracy of the proposed model is ensured because time-varying parameters \(\theta_{i,t}\) are included in the linear constraints to represent the feasible region at time \(t\), and estimation problem (4) is proposed to best estimate \(\theta_{i,t}\) by using the sum of the weighted absolute values of residuals, i.e., the measure of prediction errors. In addition, according to the theory of statistical learning, the predicted data set should have the same type of information as the training data set. Hence, to predict a certain type of operation, the training data must be based on the same type of price signals (e.g., the prediction of day-ahead operation is based on the training data resulting from day-ahead prices).

IV. STAGE TWO: STACKELBERG PRICING STRATEGY

A typical Stackelberg game provides a framework to model the problems wherein one player (leader) has the ability to enforce its strategy on the other player (follower). As an extension to the original single-follower Stackelberg game, a one-leader, \(n\)-follower game is presented in stage two to model the practical interaction between the DMO and non-cooperative DRAs.

A Stackelberg game is composed of players, the strategy sets of the players, and utility functions. The proposed game model is defined in its normal form as \(G = \{\Omega_D, \Omega_i\}_{i \in I}; U_D, \{U_i\}_{i \in I}\) [28], where the DMO acts as the leader and the DRAs act as followers. The leader’s strategy is constituted by a time series of prices and purchased electricity from the wholesale market. Each follower’s strategy includes its aggregated energy consumption and controllable DG outputs. The strategy set of each player is determined according to certain constraints. The utility functions are defined as the quantified benefits of the leader and its followers, respectively [15].

The proposed game is played in the following sequence.

1. The leader first announces its strategy to the followers.
2. Each follower then decides an optimal strategy as its best response to the leader’s strategy and infers the leader of its best response. The leader then updates its strategy based on this feedback and announces its updated strategy.
3. This interactive process is iterated until all players obtain their desired outcomes, i.e., a Stackelberg equilibrium (SE) is achieved, where the leader maximizes its benefit based on the identified best-response strategies of all followers.
4. Thus, the SE can be expressed as a portfolio of equilibrium over strategy sets. Each player will not deviate from this equilibrium.

Thus, the proposed game model can be reformulated as a bilevel programming with the DMO in the upper level and the DRAs in the lower level. This approach is detailed in the following subsections.

A. Lower-Level Follower (DRA \(i\)) Model

For each follower, let \(C_i = \{c_{i,1}, c_{i,2}, \ldots, c_{i,T}\}\) be the pricing strategy of DRA \(i\), then \(\Omega_C = \{C_i : i \in I\}\) is the pricing strategy of the DMO for all DRAs. We develop a two-stage stochastic formulation of DRA \(i\)'s utility function and strategy set below, that takes into consideration the uncertainty of any renewable DGs.

1) The First-Stage Stochastic Problem: When \(C_i\) is revealed to DRA \(i\), the deterministic costs resulting from competing with other DRAs and interacting with the DMO includes two parts: the operational cost of its controllable DGs and the cost of purchasing electricity to meet aggregated power consumption. To minimize these costs, the formulation is:

\[
\min_{p_{i,t}} -U_i = \sum_{t \in T} \left( c_p^i p_{i,t}^l + c_p^r p_{i,t}^r \right) + Q(p) \tag{5a}
\]

s.t. \( p_{i,t}^l - p_{i,t-1}^l \leq r_{i,t}^l, t \in T \) - (5b)

\( p_{i,t-1}^l - p_{i,t}^l \leq r_{i,t}^l, t \in T \) - (5c)

\( p_{i,t}^l \leq \bar{P}_{i,t}, t \in T \) - (5d)

\( p_{i,t}^l \geq \underline{P}_{i,t}, t \in T \) - (5e)

\( p_{i,t}^r - p_{i,t-1}^r \leq r_{i,t}^r, t \in T \) - (5f)

\( p_{i,t-1}^r - p_{i,t}^r \leq r_{i,t}^r, t \in T \) - (5g)

\( 0 \leq p_{i,t}^l \leq \bar{P}_{i,t} \) - (5h)

where

\[
Q(p) = \mathbb{E}_x \phi(p, s) = \sum_{s \in S} Pr(s)\phi(p, s) \tag{5i}
\]

The objective function (5a) includes the first-stage cost and the second-stage expected cost. In Section III, load parameters such as ramp rates \((r_{i,t}^l, r_{i,t}^r)\) and power limits \((\bar{P}_{i,t}, \underline{P}_{i,t})\) are used in constraints (5b)–(5e) and constitute the predicted feasible region of DR activities. However, these parameters cannot be directly applied because of potential data synchronization issues. The data training in (3) and the operations for the aggregators in (5) are usually in different time scales. For example, the data extraction in (3) might be performed every 15 min, while the operation in (5) might be hourly. We propose the following technique to deal with this synchronization. Let index 0 and index 1 denote the original parameters and the applied parameters, respectively. When \(T^0 > T^1\):

\[
N = \left\lfloor T^0 / T^1 \right\rfloor \tag{6a}
\]

\[
\theta_{i,t}^1 = \frac{1}{N+1} \sum_{n=Nt-1}^{Nt} \theta_{i,n,t}^0, t = 1, 2, \ldots, T^1 \tag{6b}
\]

When \(T^1 > T^0\):

\[
N = \left\lfloor T^1 / T^0 \right\rfloor \tag{6c}
\]

\[
\theta_{i,t}^0 = \theta_{i,n,t}^0, t \in (Nt^0 - N, Nt^0), t = 1, 2, \ldots, T^0 \tag{6d}
\]

2) The Second-Stage Stochastic Problem: The second-stage problem is established after the energy consumption and controllable DG outputs are determined. The objective function of this stage is to minimize the penalty cost of the mismatch

\[
\min_{p_{i,t}} -U_i = \sum_{t \in T} \left( c_p^i p_{i,t}^l + c_p^r p_{i,t}^r \right) + Q(p) \tag{5a}
\]

s.t. \( p_{i,t}^l - p_{i,t-1}^l \leq r_{i,t}^l, t \in T \) - (5b)

\( p_{i,t-1}^l - p_{i,t}^l \leq r_{i,t}^l, t \in T \) - (5c)

\( p_{i,t}^l \leq \bar{P}_{i,t}, t \in T \) - (5d)

\( p_{i,t}^l \geq \underline{P}_{i,t}, t \in T \) - (5e)

\( p_{i,t}^r - p_{i,t-1}^r \leq r_{i,t}^r, t \in T \) - (5f)

\( p_{i,t-1}^r - p_{i,t}^r \leq r_{i,t}^r, t \in T \) - (5g)

\( 0 \leq p_{i,t}^l \leq \bar{P}_{i,t} \) - (5h)

where

\[
Q(p) = \mathbb{E}_x \phi(p, s) = \sum_{s \in S} Pr(s)\phi(p, s) \tag{5i}
\]
\( \Delta p_{t,i}^{x} \) caused by the stochastic nature of renewable energy generation.

\[
\phi(p, s) = \min \sum_{i \in T} y_i \Delta p_{t,i}^{x} \tag{7a}
\]

Due to the continuity of the probability distributions, it is difficult to analytically address these uncertainties. To handle this difficulty, the sample average approximation (SAA) method is applied to generate a certain number of scenarios to represent the probability distribution of the random parameters [29]. Therefore, (5i) can be replaced by its approximated form

\[
Q(p) = \frac{1}{S} \sum_{s \in S} \sum_{i \in T} y_i \Delta p_{t,i}^{x} \tag{8}
\]

where the scenario set has \( S \) realizations of random variable \( p_{t,i}^{x} \). Studies have proved that the optimal solution of the reformulated problem (5) will converge to the original solution if a sufficient number of scenarios are performed [30]. Hence, the original stochastic problem can be reformulated as a continuous deterministic optimization problem. Additionally, the feasible strategy set of aggregator \( i \) can be defined as

\[
\Omega_i = \left\{ p_{t,i}^{x}, p_{t,i}^{y} \mid (5b) - (5h), (7b), (8) \right\} \tag{9}
\]

B. Upper-Level Leader (DMO) Model

Let \( P_i = [p_{t,1,i}^{x}, p_{t,2,i}^{x}, \ldots, p_{t,T,i}^{x}] \) be the consumption strategy of DRA \( i \), then \( \Omega_P = \{P_i : i \in I\} \) is the strategy profile containing all of the optimal strategies of its followers. When DRAs respond to the DMO with \( \Omega_P \), the utility function of the leader can be defined as

\[
U_D = \sum_{i \in I} \sum_{t \in T} c_{l,t}^{x} p_{t,i}^{x} - \sum_{i \in I} c_{g,t}^{x} - \sum_{i \in I} \mu(g_{t,i} - \gamma_{t,i})^{2} \tag{10}
\]

The leader updates its strategy based on the followers’ strategies, so the first term of (10) includes the benefit gained from the energy consumption of each DRA. The second term is the cost of purchasing electricity from the wholesale market. The third term is the cost of dispatching the re-dispatched power. To maintain operational security, the following power flow and voltage constraints should be applied:

\[
P_{t,i} = g_{t,i}, t \in T \tag{11a}
\]

\[
P_{n+1,i} = P_{n,i} - P_{n+1,i}^{d}, \forall n \in \Omega_N, t \in T \tag{11b}
\]

\[
Q_{n+1,i} = Q_{n,i} - Q_{n+1,i}^{d}, \forall n \in \Omega_N, t \in T \tag{11c}
\]

\[
V_{n+1,i} = V_{n,i} - \left( b_{1,n} P_{n,i} + b_{2,n} Q_{n,i} \right), \forall n \in \Omega_N, t \in T \tag{11d}
\]

\[
1 - \epsilon < V_{n,i} < 1 + \epsilon, \forall n \in \Omega_N, t \in T \tag{11e}
\]

\[
0 \leq p_{t,i}^{x} \leq p_{t,i}^{y}, \forall i \in I, t \in T \tag{11f}
\]

\[
0 \leq q_{t,i}^{y} \leq q_{t,i}^{l}, \forall i \in I, t \in T \tag{11g}
\]

\[
0 \leq P_{n,i} - P_{n,i}^{d} \leq \bar{P}_{mn}, \forall (n, m) \in \Omega_L, t \in T \tag{11h}
\]

\[
0 \leq Q_{n,i} - Q_{n,i}^{d} \leq \bar{Q}_{mn}, \forall (n, m) \in \Omega_L, t \in T \tag{11i}
\]

Constraints (11a)-(11i) are the linearized DistFlow equations, which have been widely applied to calculate the complex power flow and voltage profile in distribution systems [31]. In addition, prices are limited by

\[
\sum_{i \in I} c_{l,i}^{x} = \tau, \forall i \in I \tag{11j}
\]

The foregoing utility function (10) and constraints (11a)-(11i) can be used to formulate the following optimization problem

\[
\min_{c_{l,i}^{x}, p_{t,i}^{x}, g_{t,i}^{y}} -U_D \tag{12a}
\]

\[
s.t. \ (11a) \tag{12b}
\]

Moreover, the feasible strategy set of the DMO can be defined by

\[
\Omega_D = \left\{ c_{l,i}^{x}, g_{t,i}^{y} \mid (11a) \right\} \tag{13}
\]

C. Stackelberg Equilibrium

The desired outcome of the game leads to a Stackelberg Equilibrium. The formal description of the SE corresponding to the proposed one-leader, non-cooperative n-follower Stackelberg game can be described as follows [15]. Given the notation of \( \Omega_C \) and \( \Omega_P \), \( (\Omega_C^{*}, \Omega_P^{*}) \) is a SE for the proposed game if it corresponds to the solution of the following bilevel optimization problem:

\[
\begin{align*}
\min_{c_{l,i}^{x}, p_{t,i}^{x}, g_{t,i}^{y}} & -U_D \\
\text{s.t.} & \ (11a) \\
\end{align*}
\]

Subsequently, we utilize the following theorem to prove the existence of the SE between the DMO and DRAs in the proposed game.

**Theorem 1:** For the proposed game, a SE exists if the following conditions are satisfied [20]:

1) The strategy set of each player is nonempty, convex, and a compact subset of some Euclidean space \( \mathbb{R} \).

2) \( U_D \) is continuous and concave in \( \Omega_C \).

3) \( U_i \) is continuous in \( P_i \) and concave in \( P_i, \forall i \in I \).

**Proof 1)**: Because \( \Omega_D \) and \( \Omega_L \) are linear, these sets are readily defined as nonempty, convex, and a compact subset of some Euclidean space \( \mathbb{R} \).

**Proof 2)** and 3): Because \( \partial^2 U_D / \partial c_{l,i}^{x} = 0 \) and \( \partial^2 U_i / \partial p_{t,i}^{x} = 0, \forall i \in I \), the SE exists between the leader’s side and followers’ side.

The uniqueness of the SE is explained in Section V-B, where the optimal solution of the HDDGD algorithm is a SE.

V. SOLUTION ALGORITHM

A. M Penalty Algorithm for the Data-Driven Stage

The data-driven problem (4) with the KKT reformulations can be solved by several off-the-shelf approaches such as CPLEX non-linear solvers. However, since (4) is NP-hard, these methods cannot provide a good result for large-scale applications in a reasonable computational period. Therefore, this subsection develops a new algorithm to tackle (4), that
is to solve a linear relaxation of the mathematical program
with equilibrium constraints (4) by penalizing violations of
the complementarity constraints.

Instead of directly finding the optimal solution to (4), the
proposed algorithm leverages historic data to calibrate the
solution through the penalty factor \( M \) to minimize out-of-
sample prediction errors.

1) Algorithm Description: The penalty algorithm utilizes
a linear (convex) relaxation of a mathematical programming
problem with equilibrium constraints, whereby the comple-
mentarity conditions of the lower-level problem in the data-
driven stage are transferred to the objective function (4a),
penalizing the sum of dual variables in the non-linear con-
straints of \( (2) \) and slacks of \( (1) \). In this paper, the slack is
defined as the right side minus the left side in the form of
a “\( \leq \)” constraint. For example, the slack of constraint \( (1b) \)
is: \( r^u_{i,t} - x_{i,t} + x_{i,t-1} \). This ensures that the slack is always
nonnegative.

The penalty method achieves an approximate solution,
which helps to provide precisely predicted parameters for
the Stackelberg-pricing stage and save computational costs.

With the above relaxation of constraints \( (2d)–(2g) \), the original
objective function \( (4a) \) can be recast as:

\[
\min_{\Omega_R} \sum_{t \in T} \omega_t \left( \alpha^+_i - \alpha^-_{i,t} \right) \\
+ M \left( \sum_{t \in T} \omega_t \left( \bar{\psi}_{i,t} + \psi_{i,t} + \bar{P}_{i,t} - P_{i,t} \right) \\
+ \sum_{t \in T-1} \omega_t \left( \bar{\lambda}^u_{i,t} + \lambda^t_{i,t} + r^p_{i,t} + r^d_{i,t} \right) \right)
\]  

\( (15a) \)

where \( \Omega_R = \{ x_{i,t}, \theta_{i,t}, \alpha^+_i, \alpha^-_{i,t}, \bar{\psi}_{i,t}, \bar{\lambda}^+_{i,t}, \bar{\lambda}^-_{i,t} \} \) subject to:

\[
(4b) - (4c), \quad (1b) - (1d), \quad (2a) - (2c) \tag{15b}
\]

\[
\bar{\lambda}^u_{i,t}, \bar{\lambda}^t_{i,t} \geq 0, t \in T-1 \tag{15c}
\]

\[
\bar{\psi}_{i,t}, \psi_{i,t} \geq 0, t \in T \tag{15d}
\]

The relaxed objective function \( (15a) \) includes two items.
The first term is the original objective \( (4a) \). The second, which
includes the sum of dual variables in non-linear complementar-
ity constraints plus their slacks, is multiplied by a penalty
coefficient \( M \). Note that the effect of \( \omega_t \) that multiplies the
second and third items is the same as the weights in \( (3) \). In ad-
dition, the introduction of \( M \) minimizes the out-of-sample
prediction error. \( M \) penalizes the sum of dual variables in non-
linear constraints of equation set \( (2) \) and the slacks of equation
set \( (1) \). In this way, the relaxed problem \( (15) \) is parameterized
on \( M \), which provides our solution approach with a degree
of freedom over directly solving \( (4) \). Indeed, we can let the
data decide which value of \( M \) minimizes the out-of-sample
prediction error.

Objective function \( (15a) \) is subject to two groups
of constraints. The first group includes auxiliary con-
straints \( (4b)–(4c) \). The second group contains the primal and
dual feasibility constraints of \( (1b)–(1d), (2a)–(2c), \) and \( (5c)–
(5d) \). Due to the linearity of \( (15) \), we can obtain its global
optimum by using linear solvers with a reasonable compu-
tational cost.

2) Statistical Significance of the Developed Algorithm:
It is obvious that the original objective function \( (4a) \) only
minimizes in-sample prediction errors. In statistical learning
theory \([32]\), it is well known that the minimization of in-
sample prediction errors is not equivalent to minimizing
out-of-sample prediction errors. According to Dror et al., the
estimated DR parameters, i.e., the optimal solution of \( (4) \) that aims
to minimize in-sample prediction errors, may not be the ones that
perform best in future. For example, the in-sample prediction
error can be reduced to zero by enlarging the parameter space
defining the market bids and overfit the data; however, the
out-of-sample prediction error would dramatically increase as a
result. As the ultimate goal of the data-driven stage is to
minimize out-of-sample errors, the experiment-based penalty
algorithm we have developed has a twofold significance com-
pared to solving \( (4) \) to optimality. First, it saves computational
cost and thus can be applied to both real-time and day-ahead
market scenarios. Second, the value of \( M \) can be adjusted by
users to minimize the out-of-sample errors. This means that
the developed algorithm can provide more accurate predictions
and results.

B. HDDGD Algorithm for Stackelberg-Pricing Stage

Since the proposed n-follower Stackelberg game has \( n \)
parallel lower-level optimization problems and time-variant
variables, it is difficult to solve and computationally intensive.
Given a fixed pricing strategy \( C_i, (5) \) is a linear programming
problem. Similarly, given a fixed consumption strategy \( P_i, (12) \)
is convex. Thus, a Lagrange dual decomposition can be applied
to cater the parallel structure and time-series variables of the
proposed model, so that the solution can be obtained more
easily and in a shorter period of time.

1) Compact Notation of the Primary Problem: To demon-
strate the proposed HDDGD algorithm, a compact notation is
established to denote the Stackelberg-pricing stage. For the
follower:

\[ \min_{\mathbf{p}_1} c^T \mathbf{p}_1 + Q(R) \] \tag{16a}

s.t. \[ A_1 \mathbf{p}_1 \leq \mathbf{b}_1 \] \tag{16b}

\[ \mathbf{p}_1 \in \mathcal{P} \] \tag{16c}

where vector \( \mathbf{p}_1 \in \mathbb{R}^m \) includes decision variables with
respect to consumption and controllable DG generation, vec-
tor \( \mathbf{c} \in \mathbb{R}^m \) represents electricity prices, \( \mathbf{R} \in \mathbb{R}^{m \times m} \)
denotes the constants in \( (7) \), \( \mathbf{b}_1 \in \mathbb{R}^m \) and \( \mathbf{A}_1 \in \mathbb{R}^{m \times m} \)
denote load ramp constraints \( (5b)–(5c) \), and \( \mathcal{P} \) indicates consumption
limits \( (5d)–(5i) \). For the leader:

\[ \min_{\mathbf{p}_2, \mathbf{c}_2} \mathbf{p}_2^T \mathbf{c}_2 + f(g) \] \tag{17a}

s.t. \[ A_2(g, \mathbf{c}_2)^T = \mathbf{b}_2 \] \tag{17b}

where vector \( \mathbf{p}_2 \in \mathbb{R}^{p_2} \) denotes the consumptions of the
DRAs, \( \mathbf{c}_2 \in \mathbb{R}^{m \times n} \) represents the pricing strategies for all
DRAs, \( g \in \mathbb{R}^{d_2} \) is the purchased electricity in all related
expressions \( f(g) \), and \( \mathbf{A}_2 \in \mathbb{R}^{d_2 \times (d_2+n_1 \times n)} \) and \( \mathbf{b}_2 \in \mathbb{R}^{d_2} \)
denote operational security and price constraints \( (11a)–(11j) \).
2) Dual Decomposition and Gradient Descent Method: For the follower, according to the compact notation, the Lagrangian is:

\[ L(u_1, p_1) = c_1^T p_1 + Q(R) - u_1^T (A_1 p_1 - b_1) \]  \hspace{1cm} (18)

with a vector of nonnegative Lagrange multipliers \( u_1 \in \mathbb{R}^{n_1} \). The dual objective is

\[ L(u_1) = \min_{p_1 \in \mathcal{P}} L(u_1, p_1) \]  \hspace{1cm} (19)\]

and the dual problem is to find

\[ u_1 = \max L(u_1) \]  \hspace{1cm} (20)\]

Let \( p_k \) and \( c_k \) be an element of \( p_1 \) and \( c_1 \), respectively, let \( A_k \)

be the coefficient vector of \( p_k \) in \( A_1 \), let \( u_k \) be the Lagrange multiplier vector of \( p_k \), \( k = 2n + 1 \), \( n \in \mathbb{Z} \), and let \((\kappa)^K_{k=1}\)

denote the operation of concatenating all elements \( \kappa_1, \ldots, \kappa_K \)
to a single column vector. Then, (18) with respect to \( p_k \) can be rewritten as

\[ \min_{p_k \in \mathcal{P}} L(u_1, p_k) = (c_k p_k)^{n_1-1}_{k=1} + Q(R) - u_1^T (A_1 (p_k^{n_1-1}_{k=1} - b_1) \] \hspace{1cm} (21)\]

To facilitate the calculation of time-dependent \( p_k \) in the coupling constraint (16b), we apply a sub-gradient algorithm to the dual decomposition [33]:

\[ (p_k^{(q)})^{n_1-1}_{k=1} = \arg \min_{p_k \in \mathcal{P}} L(u_1^{(q-1)}, p_k) = (\arg \min_{p_k \in \mathcal{P}} c_k p_k)_1^{n_1-1} \] \hspace{1cm} (22a)\]

\[ u_1^{(q)} = u_1^{(q-1)} + \delta_1 (A_1 (p_k^{(q-1)}_{k=1} - b_1) \] \hspace{1cm} (22b)\]

where \( \delta_1 > 0 \) is a step size and \( c_k = c_k + A_1^T u_k \). Thus the dual problem decomposes into \( n_1/2 \) maximization problems that can be easily solved with zero duality gap.

The leader offers an optimal price vector \( c_2^* \) given the best response \( p_2^* \). With a vector of nonnegative Lagrange multipliers \( u_2 \in \mathbb{R}^{(d_2+n_2) \times n_2} \), the Lagrangian dual objective is:

\[ \min_{u_2} L(u_2, c_2, p_2^*) = p_2^T c_2 + f(g) - u_2^T (A_2 g, c_2)^T - b_2 \] \hspace{1cm} (23)\]

Similar to (22), we have:

\[ (c_2^{(q)}, g^{(q)}) = \arg \min_{c_2, g} L(u_2^{(q-1)}, c_2, p_2^*) \] \hspace{1cm} (24a)\]

\[ u_2^{(q)} = u_2^{(q-1)} + \delta_2 (A_2 (g^{(q)}, c_2^{(q)})^T - b_2) \] \hspace{1cm} (24b)\]

If there exists a solution (no matter whether it is locally optimal), the global optimal solution can be obtained by applying the above gradient descent method [34]. From Sections IV-C and V-B2, we know that each player obtains the unique SE after the proposed distributed algorithm is applied. This unique SE also represents the global optimality of the problem.

![Fig. 2. Detailed operation and implementation of Algorithm 1, where the red arrows represent steps 2–3 and 7–10, and the black arrows correspond to steps 12–13.](image)

**Algorithm 1: Distributed HDDGD Algorithm Combined With Penalty Algorithm**

**Input:** \( q \leftarrow 0, \delta_0, M, \) initial, \( \epsilon, \theta_{ij}, c_1^{(q)}, p_i^{(q)}, |p_i^{(q+1)} - p_i^{(q)}| > \epsilon, \)

**Output:** \( \Omega_{C}, \Omega_{P} \)

for \( i \in \mathcal{I} \) do

1. Train data according to (15);
2. Deliver estimated \( \theta_{ij} \) to (5) according to (6);
end

while \( |p_i^{(q+1)} - p_i^{(q)}| > \epsilon \) do

for \( i \in \mathcal{I} \) do

1. Given \( c_i^{(q)} \), DRA \( D_i \) calculate \( p_i^{(q+1)} \) according to (22a);
2. Update the dual variable by using (22b);
end
end

Given \( \tau_i^{(q+1)} \), \( i \in \mathcal{I} \), the DMO updates \( \Omega_{iC}^{(q+1)} \) according to (24a);

1. Deliver \( \Omega_{C}^{(q+1)} \) to DRAs;
2. Update the dual variable by using (24b), where \( \delta_2 > 0 \) is sufficiently small;
end
16 return Output.

3) Combined Distributed Algorithm: Pseudo code of the combined algorithm for the two-stage framework is shown in **Algorithm 1**, where step numbers are shown on the left side.

The proposed algorithm can be implemented in parallel. Fig. 2 shows the detailed operation and implementation of **Algorithm 1** in a market management system.

In a DRA, the server applies multi-string processing to steps 2–3 in the data-driven stage to speed up calculation. In the Stackelberg-pricing stage, single-program multiple-data (SPMD)-based parallel computing can be used since (16) can be decomposed into multiple independent subproblems through step 9 [35]. The workload of (16) is distributed to different cores of the CPU. The parameters of the decomposed problems, such as \( c_k^* \) and \( P \), are stored in different data blocks, where the cores run different decomposed problems in parallel.
In the DMO, the price strategy $\Omega_C$ can only be calculated using $\Omega_P$ through step 12 since the market rule does not allow the DMO to access the full information of DRAs, followed by the DMO revealing $\Omega_P$ to the DRAs. Each DRA can calculate its $P_i$ given the revealed $C_i$ through steps 8–9. The process repeats until the game converges to a unique SE.

VI. CASE STUDIES

The proposed method was tested on a realistic DR project in China using the IEEE 33-bus test feeder that included 4 DRAs and a larger distribution system. All calculations were performed on the Iowa State University Condo cluster with two 2.6 GHz 8-core Intel E5v3 processors, 128 GB RAM, and CPLEX 12.6 under GAMS.

A. Data-Mining Methods

To test the predictive accuracy of the proposed data-driven method, experiments were implemented on the Changdao project in Shandong China, where 15-minute day-ahead electricity prices were sent to three similar DRAs, each with 157 households, through the proposed pricing strategy. Each DRA’s households then consumed electricity based on the given prices on the next day. The price-sensitive smart controllers installed in each house controlled appliances and plug-in electric vehicles (PEV) based on the house owner’s preferences. Appliances in the home included controllable (space cooling/heating, water heating, and clothes washing) and critical (cooking, lighting, refrigerator, freezer, and others) systems. The total penetration of the PEVs was 13.2%.

Fig. 3 depicts a sample training dataset of a DRA’s practical price-load data. Fig. 4(a) identifies the 3 DRA-managed regions on a geographical map. To validate the performance of the proposed data-driven method, the following cases were compared: 1) ARX [32] DR modeling; 2) the proposed inverse optimization-based DR modeling solved by an off-the-shelf CPLEX solver (InvC); and 3) the proposed inverse optimization-based modeling with the newly developed M-penalty algorithm (Inv). Note that all of the above cases used the same pricing strategy as proposed in Section IV.

Before the tests, we first determined the values of the parameters $M$ and $F$ for the proposed load modeling method. A combination of these parameters were searched to minimize validation errors. We utilized cross-validation to perform a sensitivity analysis of $M$ and $F$ by mean absolute percentage error (MAPE). Fig. 4(b) illustrates this MAPE with respect to different combinations of $M$ and $F$. From the figure, we can see that $M = 0.2$ and $F = 1$ result in the best prediction performance that minimizes out-of-sample errors.

As illustrated in Fig. 5, the test was conducted in three steps. First, the above 3 cases were simultaneously implemented on 3 DRAs as the data-driven stage of the market strategy, where each case produced a set of day-ahead prices and consumption predictions. Second, each DRA’s households resulted in a set of actual consumptions on the next day. Finally, each case’s prediction performance was calculated by comparing the predicted and actual consumption of the corresponding DRA.

Results from two consecutive days over the March 24 to 27, 2017 period are shown in Fig. 6. The ARX method provides good performance, but can not follow sudden load changes. The InvC method has an overfitting problem. Biased prices were decided under this overfitting, leading to a worse DR performance. The Inv method has the best performance with reasonable out-of-sample errors. In addition, we used four metrics to measure the prediction performance in March in Table I: MAPE, root mean square error (RMSE), mean absolute error (MAE), and computational time (CT). In terms of prediction-error minimization, the Inv method is better than ARX and InvC. Although Inv takes more calculation time than ARX due to higher dimensions in the explanatory variable vector, it is still acceptable for day-ahead markets.
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Table I

PERFORMANCE OF DIFFERENT DATA-DRIVEN METHODS IN MARCH

<table>
<thead>
<tr>
<th>Method</th>
<th>MAPE</th>
<th>RMSE</th>
<th>MAE</th>
<th>CT(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARX</td>
<td>19.178</td>
<td>23.421</td>
<td>0.1869</td>
<td>503</td>
</tr>
<tr>
<td>InvC</td>
<td>57.639</td>
<td>69.473</td>
<td>0.6371</td>
<td>3076</td>
</tr>
<tr>
<td>Inv</td>
<td>12.102</td>
<td>15.602</td>
<td>0.0963</td>
<td>897</td>
</tr>
</tbody>
</table>

Table II

DG AND CONSUMER DATA OF DRA S

<table>
<thead>
<tr>
<th>DRA</th>
<th>Output Limit (kW)</th>
<th>Range Up/Down Rate (kWh)</th>
<th>Operation Cost ($/kWh)</th>
<th>PV Generator</th>
<th>Number of Users</th>
<th>Number in IEEE 33-Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>430/470</td>
<td>0.15</td>
<td>1250</td>
<td>5.1%</td>
<td>127</td>
</tr>
<tr>
<td>2</td>
<td>575</td>
<td>280/300</td>
<td>0.2</td>
<td>650</td>
<td>5.3%</td>
<td>98</td>
</tr>
<tr>
<td>3</td>
<td>375</td>
<td>280/300</td>
<td>0.2</td>
<td>1300</td>
<td>5.0%</td>
<td>109</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>200/230</td>
<td>0.25</td>
<td>375</td>
<td>5.1%</td>
<td>75</td>
</tr>
</tbody>
</table>

B. Pricing Strategies

Numerical simulations were performed on an IEEE 33-bus distribution system [31] with 4 connected DRAs. Each DRA had different groups of end users, whose training data sets were pulled from the Changdao project database. For example, data for 127 users was randomly selected from this database to form an aggregated training set for DRA 1. Table II presents the setups for the DRAs obtained from the Changdao project, which were mapped to the IEEE 33-bus system. The simulation parameters were set as follows: penalty factor $\gamma_t = $9.00/kWh [36]; $\varepsilon = 0.02$ p.u.; $\mu = $0.17/kWh; $M = 0.2$; $\tilde{c} = $3.60/kWh; $\omega_t$ was obtained from historical real-time market prices; $c_t$ and $\tilde{p}_t$ can be found in [24]. For Algorithm 1: $\varepsilon = 0.01$; $\delta_1 = [0.05, 0.05]$, $\delta_2 = 0.03$; and $T = 24h$.

1) Load Peak Shaving: The two benchmarks used for comparing the results from the proposed method in this work were flat rate (FR) and time-of-use (TOU) pricing schemes. For illustrative purposes, the mean and standard deviation of the consumptions of the four DRAs are depicted in Fig. 7. Under the FR strategy, each aggregator’s end users had no incentive to minimize their power usage and resulting costs. Although TOU shaves the original peak between 10:00-21:00 by a lower price, undesirable load pickup occurs between 22:00-4:00 with a large standard deviation. Note that the FR and TOU strategies are predefined, and cannot represent the interactions between DRAs and the DMO. Under the proposed strategy, the overall load profile was smoothed, since the pricing scheme was dynamic. When a lower price promoted a DRA to consume more power, the DMO raised the price to make more profit, leading to a lower consumption.

In Fig. 8, the proposed pricing strategy shaves the total peak load of the distribution system. When some DRAs consume more power, the DMO raises the prices of these DRAs to make more profit, leading to their lower consumption.

2) Operational Security: We compare our method to the game model proposed in [19], which does not consider operational constraints. Fig. 9 shows 24 h mean voltage values at representative nodes. We can see that the model in [19] may lead to voltage violations.

3) Economic Performance: We compare the economic performances of the proposed Stackelberg strategy and the TOU, with results shown in Table III.
In Table III, PS outperforms TOU in several economic aspects such as DAR costs, DMO costs, and total costs under the same conditions. Since end users in the day-ahead market are well satisfied through DR management of PS, total costs of the real-time market are less than with TOU pricing. In addition, the proposed game model for day-ahead and real-time markets converges at the SE with iteration numbers of 17 and 8, respectively.

C. Test on a Larger System

The proposed strategy was then applied to a real distribution system in the Changdao project with 128 nodes and 7 DRAs. Due to data confidentiality, only details important to understand the results are provided. The single line diagram of the system is shown in Fig. 10. Table IV lists the DRA information. The appliance types are the same as described in the system in Fig. 4, and the total penetration of EVs was 11.3%.

The following test cases (TC) were considered:

- TC 1: the distribution system included DRA 1 and DRA 4
- TC 2: the distribution system included DRA 2 and DRA 4
- TC 3: the distribution system included DRA 3 and DRA 4
- TC 4: the distribution system included DRA 1-6

Information pertaining to the local renewable energy generators, load outputs, and training data set can be found in the historical data of the project. The remaining parameters were derived from the project and the previous case study conducted in this paper.

To analyze the impact of renewable integration, the results from the simulations for TC 1–4 are found in Table V and Fig. 11.

In Table V, PS outperforms TOU under different renewable penetration levels, since PS considers the uncertainty of renewables and the dynamic pricing. The operational costs increase as the renewable penetration level increases from TC 1 to TC 3, since more costs are taken to mitigate the fluctuation introduced by the higher renewable penetration. From TC 1 to TC 2, there is a 3.1% increase in PS operational costs caused by a 4.1% increase in renewable integration, while the TOU operational cost percentage increase is 4.3%. From TC 2 to TC 3, a 2.7% increase in PS operational costs is caused by a 3.2% increased renewable integration, while the TOU operational costs increase by 5.1%.

Fig. 11(a) shows that a higher variance in renewable generation leads to a higher operational cost. Fig. 11(b) demonstrates that the operational costs converge as the number of scenarios increases.

To illustrate the effectiveness of the PS method, 60 independent simulations were performed in TC 4, with the results shown in Table VI.
It can be seen that in a large system with 6 different DRAs, PS can still converge in a reasonable time.

VII. CONCLUSION

This paper proposed a data-driven Stackelberg market strategy for DR-enabled distribution systems, which coordinates multiple profit-pursuing entities (DRAs or VPPs) and bridges the regulation gap between the ISO and distribution systems. For the data-driven stage, an innovative inverse method was developed to train the DR model, which achieves good prediction performance and presents a generalized computationally light modeling approach. Considering operational practice in retail markets, a Stackelberg game-based pricing strategy was designed to maximize each market participant's profit and guarantee operational security. An efficient M-penalty algorithm was developed for the data-driven stage to minimize out-of-sample errors and save computational cost. A distributed HDDGD algorithm was proposed for the Stackelberg-pricing stage to obtain an n-follower time-series-based SE within a reasonable calculation period.

Through real-life experiment-based comparisons with two groups of benchmarks, i.e., data-driven models and pricing strategies, and simulations on different distribution systems, we found that our data-driven load modeling method can realize fast computation and accurate prediction, and that our pricing strategy can achieve peak load shaving, operational security, and economic profits.

REFERENCES


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![Tianguang Lu](image)

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