Cooperative Peak Shaving and Voltage Regulation in Unbalanced Distribution Feeders

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Abstract—This paper considers the co-operation of distributed generators (DGs), battery energy storage systems (BESSs) and voltage regulating devices for integrated peak shaving and voltage regulation in distribution grids through a co-optimization framework, which aims to minimize the operational costs while fulfilling the operational constraints of network and devices. To account for the uncertainties of load demand and generation, we then convert the co-optimization model into a two-stage stochastic program where state-of-charge (SoC) trajectories of BESSs and operation of voltage regulating devices are optimized at the first stage for day-ahead scheduling, while the reactive powers of DGs and BESSs are left at the second stage for potential intra-day scheduling to handle short-term voltage issues. The proposed co-optimization scheme is validated on the IEEE 37-node test feeder and compared with other practices.

Index Terms—Battery energy storage system (BESS), co-optimization, distributed generator (DG), peak shaving, two-stage stochastic programming, voltage regulation.

NOMENCLATURE

A. Sets

\( N := \{0, 1, \ldots, n\} \) \hspace{1cm} \text{Set of buses}

\( N^+ \) \hspace{1cm} \text{Set of children buses of bus } i

\( E \subseteq N \times N \) \hspace{1cm} \text{Set of branches}

\( T := \{1, \ldots, 24\} \) \hspace{1cm} \text{Set of time intervals}

\( \Phi_i, \Phi_{ij} \) \hspace{1cm} \text{Phase sets of bus } i \text{ and branch } (i,j)

\( \Xi \) \hspace{1cm} \text{Set of scenarios}

B. Parameters

\( \Delta T \) \hspace{1cm} \text{Time resolution [h]}

\( V_n \) \hspace{1cm} \text{Nominal bus voltage}

\( \lambda_{ele} \) \hspace{1cm} \text{Predicted electricity price [$/kWh]}

\( \lambda_{bat} \) \hspace{1cm} \text{Battery degradation cost [$/kWh]}

\( \lambda_{cell} \) \hspace{1cm} \text{Battery cell price [$/kWh]}

\( \lambda_{tap} \) \hspace{1cm} \text{Adjustment cost of on-load tap changer [$/time]}

\( \lambda_{cap} \) \hspace{1cm} \text{Switching cost of capacitor bank [$/time]}

\( V_{\min}, V_{\max} \) \hspace{1cm} \text{Min./max. voltage magnitude limits}

\( I_{\max} \) \hspace{1cm} \text{Max. current limit of branch } (i,j)

\( \Delta T_{ap,ij} \) \hspace{1cm} \text{Tap change limit per time step}

\( \Delta q^u_{ij}, \Delta q^d_{ij} \) \hspace{1cm} \text{Capacity per bank at bus } i \text{, phase } \varphi

\( \Delta K_{ij}^s, K_{ij}^s \) \hspace{1cm} \text{Peak load at bus } i \text{, phase } \varphi

\( B_{i,\varphi}^{\text{max}}, B_{i,\varphi}^{\text{min}} \) \hspace{1cm} \text{Available power of DG}

\( P_{\text{Peak}} \) \hspace{1cm} \text{Transformer capacity}

\( S_i, \delta_i, \eta_{dc} \) \hspace{1cm} \text{Min./max. operation limits of SoC}

\( \text{Det.} \) \hspace{1cm} \text{Peak load at bus } i \text{, phase } \varphi

C. Variables

\( s_{i,\varphi} \) \hspace{1cm} \text{Complex power injection from BESS at bus } i \text{, phase } \varphi \text{, } s_{b} := [s_{b}^{s}, s_{b}^{d}] \in \Phi_s \text{, phase } \varphi

\( s_{i,\varphi}^{e} \) \hspace{1cm} \text{Complex power injection from capacitor banks at bus } i \text{, phase } \varphi \text{, } s_{b}^{e} := [s_{b}^{e,s}, s_{b}^{e,d}] \in \Phi_s \text{, phase } \varphi

\( s_{i,\varphi}^{g} \) \hspace{1cm} \text{Complex DG power injection at bus } i \text{, phase } \varphi \text{, } s_{b}^{g} := [s_{b}^{g,s}, s_{b}^{g,d}] \in \Phi_s \text{, phase } \varphi

\( s_{i,\varphi}^{d} \) \hspace{1cm} \text{Complex load consumption at bus } i \text{, phase } \varphi \text{, } s_{b}^{d} := [s_{b}^{d,s}, s_{b}^{d,d}] \in \Phi_s \text{, phase } \varphi

\( v_i \) \hspace{1cm} \text{Complex voltage matrix at bus } i \text{, phase } \varphi

\( l_{ij} \in \mathbb{C}^{(\Phi_i \times |\Phi_j|)} \) \hspace{1cm} \text{Complex line current from buses } i \text{ to } j \text{, phase } \varphi

\( K_{ij,\varphi} \) \hspace{1cm} \text{Tap change limit per time step}

\( B_{i,\varphi}^{\text{max}}, B_{i,\varphi}^{\text{min}} \) \hspace{1cm} \text{Max. solar irradiance level

\( \zeta_{ij} \in \mathbb{C}^{(\Phi_i \times |\Phi_j|)} \) \hspace{1cm} \text{Impedance matrix of branch } (i,j)

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A. Background and Motivation

In recent decades, a variety of government policy-based incentives have supported the growth of distributed generators (DGs) such as wind, photovoltaic (PV), fuel cells, biomass, etc. Indeed, DGs bring technical, economic and environmental benefits; however, they may in turn incur new operational stress, e.g., power quality and network congestion issues [1]. Battery energy storage system (BESS) is arguably the most promising solution to aid the integration of renewables since it can be deployed in a modular and distributed fashion [2], [3]. Clearly, with a high penetration of renewable-based DGs, the real load profile may significantly deviate from the forecast, which will affect the utility companies’ bidding behaviors in the wholesale electricity market. Correspondingly, the feeder voltage profile will vary with the net load. Hence, in a nutshell, while the ongoing deployment of renewables and BESSs poses challenges to energy management of distribution systems, it facilitates the revolution to exploit renewables in a cost-effective way at the same time.

Peak shaving and voltage/reactive power (volt/var) regulation are the two fundamental functionalities in distribution management systems. Peak shaving is a process of flattening the load profile by shifting peak load demand to off-peak periods via energy storage and/or demand side management [7]. It benefits the entire power systems including power plants, system operators as well as end-users. Particularly, for system operators, effective peak shaving can postpone the expensive upgrades for transmission and distribution systems. The primary goal of volt/var regulation is, as the name suggests, maintaining the feeder voltages within a feasible range (e.g., 0.95–1.05 p.u. in ANSI Standard C84.1 [8]) by scheduling the voltage regulating devices, e.g., on-load tap changers (OLTCs), step-voltage regulators (SVRs) and capacitor banks [9]. Moreover, the advanced four-quadrant inverter-interfaced DGs and BESSs are capable of providing fast and continuous volt/var support locally [4], [5], which can alleviate the work loads on the legacy devices [6].

Thanks to the conventional separate operation of peak shaving and volt/var regulation [9], a substantial body of studies have solely investigated either peak shaving or volt/var regulation for a long time; see [7] and [10], [11] for surveys on these two isolate topics, respectively. However, the practical operation reveals the fact that they interact with each other due to the physical nature of power network: i) reshaping the load profile also reshapes the voltage profile, especially for some low-voltage feeders with high R/X ratios; and ii) regulating voltages can lower the peak load via reducing line losses and load demand [12].

In light of this, the co-operation of peak shaving and voltage regulation becomes appealing since it can maximize the usage of DGs and storage, thereby unlocking additional benefits in terms of operational cost, power quality, supply reliability as well as network reinforcement, which cannot be well accomplished by the traditional separate architectures.

B. Literature Review

A few studies have addressed the co-operation between peak shaving and volt/var regulation, especially for the planning of DGs and BESSs considering the operation conditions. Several rule-based control algorithms have been proposed in [13]–[15]. However, they rely on the heuristic design without providing system-wide optimality guarantees.

Several studies have bridged the methodology gap by developing optimization frameworks. In [16], the authors investigate the potential of BESSs in deferring upgrades needed to host a higher penetration of PV, where an optimal power flow (OPF) problem is formulated with the aim of mitigating voltage deviation and reducing peak load restricted by limited capital and operation and maintenance costs of BESSs. In [17], an optimization model that minimizes BESS cost, voltage deviation, voltage unbalance and peak demand charge together, is built. It should be noted that the weight allocation on multiple heterogeneous objectives as in [16], [17] is usually tricky. A short-term scheduling scheme of BESSs is proposed in [18] to address peak shaving, volt/var regulation and reliability enhancement simultaneously, by solving an OPF program using Tabu search. In [19], a bi-level scheduling strategy is developed, consisting of the bidding in day-ahead market (DAM) to minimize the overall costs in supplying the net load and a real-time dispatch to compensate for the energy gap. However, [16]–[19] mainly focus on the operation of BESSs, neglecting the coordination with voltage regulating devices.

To address such issue, [20]–[22] further have the legacy voltage regulating devices participate in the co-operation. In [20], a two-stage optimal dispatch framework is proposed for distribution grids with distributed wind, where the peak shaving and volt/var regulation are implemented in a successive coordinated fashion instead of the so-called co-optimization in a strict sense. The authors in [21] develop an integrated framework for conservation voltage reduction and demand response to reduce the energy bills of customers. In [22], a model predictive control scheme is proposed to minimize network losses or energy purchase cost whilst maintaining voltages within limits by...
co-optimizing the operation of OLTCs, PV inverters and BESSs in two different timescales (1 h and 15-min). Besides, [20]–[22] address the prediction uncertainties of DGs and load by leveraging the scenario-based stochastic programming techniques with one-stage [20], [21] or two-stage models [22]. However, only balanced feeders are considered.

C. Contributions

In spirit, this work is close to [19]–[21] which consider a day-ahead multi-step scheduling of DGs and BESSs to enhance utilities’ bidding strategies in the DAM. However, we contribute in the following distinct ways:

1) Firstly, we, for the first time, propose a comprehensive co-optimization framework for an integrated peak shaving and volt/var regulation by scheduling DGs, BESSs and voltage regulating devices. This framework aims to minimize the overall operational costs including energy purchase, battery degradation, as well as wear and tear of tap changers and capacitor banks, while satisfying the operational constraints. The unbalanced case is especially addressed by generalizing the linear multi-phase branch flow model to incorporate tap changers, rendering the problem computationally tractable.

2) Secondly, to account for the forecast uncertainties of renewables and load while relieving the conservative behavior of a robust decision, we propose to reformulate the problem into a two-stage stochastic program. It is noteworthy that, with this two-stage model, only the SoC trajectories of BESSs and voltage regulating devices will be actually implemented in day-ahead operation whereas the reactive powers of DGs and BESSs are left for a re-scheduling.

3) Lastly, we demonstrate the proposed co-optimization unlocks additional revenue in comparison to the successive optimization method and also demonstrate that only relying on cost reduction does not necessarily lower the peak load. This implies that an explicit peak load limit should be imposed in the co-optimization.

The rest of this paper is organized as follows: Section II presents the deterministic formulation of the co-optimization problem. In Section III, the optimization problem is reformulated as a two-stage stochastic program accounting for uncertainties. Section IV presents the numerical results, followed by conclusions.

II. PROBLEM FORMULATION

This section presents the problem formulation of the co-optimization framework for day-ahead cooperative peak shaving and volt/var regulation over the time horizon of 24 h with 1-h time resolution compatible with the DAM. Fig. 1 presents the overview of the proposed framework.

A. Objective Function

The co-optimization framework aims to minimize the overall operational costs including energy purchase, battery degradation, as well as wear-and-tear of tap changers and capacitor banks during $T$, which is mathematically given as follows:

1) Electricity Purchase Cost:

$$C_{\text{ele}} := \sum_{t \in T} \lambda_{\text{ele},t} \left( \text{Re} \left\{ \text{Tr}(S_{01,t}) \right\} \right) + \sum_{(i,j) \in E} \text{Re} \left\{ \text{Tr}(z_{ij}l_{ij,t}) \right\} \Delta T$$  \hspace{1cm} (1)

where the first part is the feed-in power flow from the substation (that does not include the line losses) and the second term represents the total line losses.

2) Battery Degradation Cost:

$$C_{\text{bat}} := \sum_{t \in T} \sum_{i \in N} \sum_{\varphi \in \Phi_i} \lambda_{\text{bat}} \left| \text{Re} \left\{ s_{i,\varphi,t} \right\} \right| \Delta T.$$  \hspace{1cm} (2)

3) Operational Cost of Tap Changer:

$$C_{\text{tap}} := \sum_{t \in T} \sum_{(i,j) \in E} \sum_{\varphi \in \Phi_i} \lambda_{\text{tap}} \left| K_{ij,\varphi,t} - K_{ij,\varphi,t-1} \right|.$$  \hspace{1cm} (3)

4) Operational Cost of Capacitor Bank:

$$C_{\text{cap}} := \sum_{t \in T} \sum_{i \in N} \sum_{\varphi \in \Phi_i} \lambda_{\text{cap}} \left| B_{i,\varphi,t} - B_{i,\varphi,t-1} \right|.$$  \hspace{1cm} (4)

Accordingly, the overall cost function is given by,

$$C := C_{\text{ele}} + C_{\text{bat}} + C_{\text{tap}} + C_{\text{cap}}.$$  \hspace{1cm} (5)

B. Constraints

1) Multi-Phase Power Flow: The convex relaxation techniques, e.g., second-order cone programming (SOCP) relaxation [24], [25] and semidefinite programming (SDP) relaxation [26], [27], are usually leveraged to convexify the non-linear power flow equations. Some applications can be observed in the existing works related to our topic. For example, the works [21] and [22] use the SOCP relaxation to convexify the OPF programs but it cannot be easily extended to unbalanced cases due to the mutual impedance of feeders [28]. The SDP relaxation is applicable to unbalanced systems; however, it may be computationally expensive, especially in the presence of...
5) **Tap Change:** The operational constraints of tap changer over branch are given by, for any \((i, j) \in E\) and \(\varphi \in \Phi_{ij}\),

\[
K^\text{min}_{ij, \varphi} \leq K_{ij, \varphi, t} \leq K^\text{max}_{ij, \varphi}, \quad K_{ij, \varphi, t} \in \mathbb{Z}, t \in T
\]

\[
|K_{ij, \varphi, t} - K_{ij, \varphi, t-1}| \leq \Delta K^\text{max}_{ij, \varphi}, \quad t \in T
\]

\[
\sum_{t \in T} |K_{ij, \varphi, t} - K_{ij, \varphi, t-1}| \leq \Delta K^\text{tot}_{ij, \varphi}
\]

where (16) denotes the tap position limits; (17) constrains the tap change during a sampling time interval; and (18) constrains the total operation times of tap changers during \(T\).

6) **Capacitor Bank:** The operational constraints of capacitor banks are given as, for any bus \(i \in N\) and \(\varphi \in \Phi_i\),

\[
\Re\{s^c_{i, \varphi, t}\} = 0, t \in T
\]

\[
\Im\{s^c_{i, \varphi, t}\} = B_{i, \varphi, t} \Delta t^c_{i, \varphi, t}, t \in T
\]

\[
0 \leq B_{i, \varphi, t} \leq B^\text{max}_{i, \varphi}, \quad B_{i, \varphi, t} \in \mathbb{Z}, t \in T
\]

\[
\sum_{t \in T} |B_{i, \varphi, t} - B_{i, \varphi, t-1}| \leq \Delta B^\text{tot}_{i, \varphi}
\]

where (20) denotes the total reactive power injected by capacitor banks; (21) constrains the maximum number of capacitor banks; (22) constrains the maximum switching times of capacitor banks during \(T\).

7) **Battery Energy Storage:** In this paper, we consider the lithium-ion battery—one of the most popular options today. If we limit the battery operation within certain depth of discharge region to avoid the overcharge and over-discharge, there is a constant marginal cost for the cycle depth increase. In this way, the battery degradation cost can be prorated with respect to charged and discharged energy into a per-kWh cost [29],

\[
\lambda_{\text{bat}} = \frac{\lambda_{\text{cell}}}{2M(Soc_{\text{max}} - Soc_{\text{min}})}
\]

where \(M\) is the number of cycles that the battery could be operated within \([Soc_{\text{min}}, Soc_{\text{max}}]\).

The model and operational constraints of a BESS at \(\varphi \in \Phi_i\) of bus \(i \in N\) can be expressed as,

\[
\Re\{s^b_{i, \varphi, t}\} = b^b_{i, \varphi, t} \Delta t^b_{i, \varphi, t}, t \in T
\]

\[
0 \leq b^b_{i, \varphi, t} \leq \mu_{i, \varphi, t} \cdot s^b_{i, \varphi, t}, t \in T
\]

\[
0 \leq b^b_{i, \varphi, t} \leq (1 - \mu_{i, \varphi, t}) \cdot s^b_{i, \varphi, t}, t \in T
\]

\[
\mu_{i, \varphi, t} \in \{0, 1\}, \quad t \in T
\]

\[
SoC_{i, \varphi, t} = SoC_{i, \varphi, t-1} + \left(b^b_{i, \varphi, t} \eta^b_{i, \varphi} - \frac{b^b_{i, \varphi, t} \eta^b_{i, \varphi}}{\eta^b_{i, \varphi}}\right) \Delta T_{i, \varphi}, t \in T
\]

\[
SoC_{\text{min}} \leq SoC_{i, \varphi, t} \leq SoC_{\text{max}}, \quad t \in T
\]

\[
SoC_{i, \varphi, 0} = SoC_{i, \varphi, 24}
\]

\[
\left|\frac{\Re\{s^b_{i, \varphi, t}\}}{\Im\{s^b_{i, \varphi, t}\}}\right| \leq S_{i, \varphi, t}, \quad t \in T
\]

Constraints (24)–(27) represent the real power model of a BESS. Constraint (28) represents the physical model of SoC of...
a BESS while (29)–(30) represent its operational constraints. As shown in (30), the SoC levels at the beginning and the end of a day should be equal so that the framework can periodically operate. (31) constrains the apparent power of BESS converter that restricts the real and reactive power in a coupling way.

8) Inverter-Based DG: A four-quadrant inverter-interfaced DG at $\varphi \in \Phi_i$ of bus $i \in N$ is modeled by,

$$\text{Re} \{s^q_{i,\varphi,t}\} = p^q_{i,\varphi,t}, t \in T$$ (32)

$$\left\| \text{Re} \{s^q_{i,\varphi,t}\} \right\|_2 \leq \bar{S}^q_{i,\varphi}, t \in T$$ (33)

where it is assumed that the PV system operates with the maximum power tracking mode (track $K_{\varphi}$ and capacitor banks, which are not tractable for off-the-shelf BESS or DG installation, we have $s^c_{i,t} = 0, s^q_{i,t} = 0$ or $s^r_{i,t} = 0$, respectively.

C. Reformulation and Compact Expression

The objectives (2)–(4) and constraints (18) and (22) contain the sum of absolute terms with respect to the tap position and capacitor banks, which are not tractable for off-the-shelf solvers. Thus, we reformulate them by introducing the auxiliary variables $K^+_{ij,\varphi}, K^-_{ij,\varphi}$, $B^+_{ij,\varphi}$ and $B^-_{ij,\varphi}$ such reformulation has been given in (24)–(27) for BESSs). Then, constraint (18) can be equivalently rewritten as,

$$K^+_{ij,\varphi,t} - K^-_{ij,\varphi,t} = K^+_{ij,\varphi,t} - K^-_{ij,\varphi,t}, t \in T$$ (34)

$$\sum_{t \in T} (K^+_{ij,\varphi,t} + K^-_{ij,\varphi,t}) \leq \Delta K^T_{ij,\varphi}$$ (35)

$$K^+_{ij,\varphi,t} \geq 0, K^-_{ij,\varphi,t} \geq 0, K^+_{ij,\varphi,t}, K^-_{ij,\varphi,t} \in Z_t, t \in T.$$ (36)

Similarly, constraint (22) becomes,

$$B^+_{ij,\varphi,t} - B^-_{ij,\varphi,t} = B^+_{ij,\varphi,t} - B^-_{ij,\varphi,t}, t \in T$$ (37)

$$\sum_{t \in T} (B^+_{ij,\varphi,t} + B^-_{ij,\varphi,t}) \leq \Delta B^T_{ij,\varphi}$$ (38)

$$B^+_{ij,\varphi,t} \geq 0, B^-_{ij,\varphi,t} \geq 0, B^+_{ij,\varphi,t}, B^-_{ij,\varphi,t} \in Z_t, t \in T.$$ (39)

Correspondingly, the cost functions $C_{\text{tap}}, C_{\text{cap}}$ as well as $C_{\text{bat}}$ can be rewritten as,

$$C_{\text{tap}} = \sum_{t \in T} \sum_{(i,j) \in E} \sum_{\varphi \in \Phi_j} \lambda_{\text{tap}} (K^+_{ij,\varphi,t} + K^-_{ij,\varphi,t})$$ (40)

$$C_{\text{cap}} = \sum_{t \in T} \sum_{i \in N} \sum_{\varphi \in \Phi_i} \lambda_{\text{cap}} (B^+_{ij,\varphi,t} + B^-_{ij,\varphi,t})$$ (41)

$$C_{\text{bat}} = \sum_{t \in T} \sum_{i \in N} \lambda_{\text{bat}} (B^+_{ij,\varphi,t} + B^-_{ij,\varphi,t}) \Delta T.$$ (42)

Finally, the optimization problem is abstractly expressed as,

$$(\text{DP}): \text{minimize } C(u)$$ (43a)

where $u$ is the compact decision vector of all the decisions; $U$ is the Cartesian product of real, complex and integer number sets, which characterizes $u$ in an element-wise manner.

So far, the deterministic problem formulation (DP) has been given in (43), which is inherently a mixed-integer second-order cone program (MISOCP) that can be handled by off-the-shelf solvers, e.g., CPLEX, MOSEK, etc.

III. STOCHASTIC PROGRAMMING FORMULATION

The day-ahead operation scheduling establishes on the load, renewable generation and electricity price, etc. However, due to various uncertainties, e.g. stochastic nature of the load and renewables, it is difficult to forecast them with very high accuracy. Therefore, we consider the forecast uncertainties of load and renewables by converting the deterministic optimization program DP into a two-stage stochastic program, while allowing for re-dispatching reactive power resources.

A. Scenario Generation and Reduction

The load consumption prediction error is calculated based on a truncated normal distribution [30]. The solar power generation is dependent on the incident solar irradiance, while the irradiance significantly depends on the cloud coverage condition. Therefore, the solar irradiance prediction error is modeled by introducing a correction factor to the prediction $U$ with a clear weather, following a normal distribution that depends on the given cloud coverage level [31],

$$\iota = U(1 - \varepsilon), \varepsilon = \text{Norm}(\mu_\iota, \sigma_\iota)$$ (44)

where $[\cdot]_0^1$ denotes the projection onto the set $[0,1]$.

Based on the known probability distributions, Monte-Carlo simulation is conducted to create a required number of scenarios for solar irradiance and load. They are then reduced to a given number of scenarios by the backward reduction method, of which more details can be referred to [32].

B. Two-Stage Stochastic Programming Formulation

Firstly, we split $u \in U$ into two groups, i.e., $u := \{x, y\}$ and $U := X \times Y$ where

- $x$ represents the decision variables associated with the charging/discharging of BESSs, operation of tap changers and operation of capacitor banks themselves.
- $y$ consists of all the remaining variables.

Correspondingly, the cost function and constraints in DP can be reconstructed as,

$$C(u) \Rightarrow C_1(x) + C_2(y)$$ (45)

$$h(u) \Rightarrow h_1(x) \cap h_2(x, y) = 0$$ (46)
which is inherently an extensive MISOCP program that can be directly handled by conic programming solvers. (SP): minimize $C_1(x) + \mathbb{E}_\xi \left\{ \min_{y \in \mathcal{Y}} C_2(y; \xi) \right\}$

subject to $h_1(x) = 0$
$g_1(x) \leq 0$
$h_2(x; y; \xi) = 0$
$g_2(x; y; \xi) = 0$

where $x$ corresponds to the first-stage (here-and-now) decisions before the realization of $\xi$ and $y$ corresponds to the second-stage (wait-and-see) corrective actions under a given realization of $\xi$. The independent control variables at the first stage include the charging/discharging power of BESSs $\Re \{p^b_{i,\phi,t}\}$, the operation trajectories of tap changers $K_{i,j,t}$ and the operation trajectories of capacitor banks $s^c_{i,t}$. The second-stage control variables are the reactive powers of BESSs and DGs, i.e., $\Im \{s_{i,\phi,t}\}$ and $\Im \{s^c_{i,t}\}$.

C. Deterministic Equivalent

Representing the uncertainties through a finite scenario set $\Xi := \{\xi_1, \ldots, \xi_{|\Xi|}\}$ with the probability distribution $\rho_1, \ldots, \rho_{|\Xi|}$, the approximate deterministic equivalent problem of SP in the extensive form can be given as,

$$(\text{SP-d}): \text{minimize } C_1(x) + \sum_{k=1}^{|\Xi|} \rho_k C_2(y_k; \xi_k)$$

subject to $h_1(x) = 0$
$g_1(x) \leq 0$
$h_2(x, y_k; \xi_k) = 0, k = 1, \ldots, |\Xi|$
$g_2(x, y_k; \xi_k) = 0, k = 1, \ldots, |\Xi|$

which is inherently an extensive MISOCP program that can be also directly handled by conic programming solvers.

D. Robustness

In practice, the prior probability distributions regarding load and solar may be inaccurate. Interestingly, the proposed two-stage stochastic co-optimization framework is robust against it in the sense that the reactive power capabilities of DGs are considered in the day-ahead co-optimization, but the reactive powers of DGs are not directly dispatched in a day-ahead manner. Instead, DGs will be re-dispatched according to the intra-day operational status of distribution networks (with more accurate load and PV data). In this way, the undesired operational status (e.g., voltage violations) induced by the inaccurate modeling of uncertainties can be corrected/compensated.

IV. NUMERICAL RESULTS

The proposed co-optimization methodologies are tested on the modified IEEE 37-node test feeder (see Fig. 2) [33]. The SVR has an operation range of $[0.9,1.1]$ p.u. with $\pm 16$ tap positions (i.e., $K^{\text{min}} = -16$, $K^{\text{max}} = 16$ and $\Delta T\,\text{ap} = 0.2/32$). The Lithium Manganese Oxide battery is considered for the simulation with a cell price of 0.5$/time and 0.24$/time, which can be adjusted as per the switching risk assessment of utilities [34]. The daily load profile of a real distribution feeder in Iowa, U.S. and a solar generation time series generated by a testbed [35] are used as...
Fig. 3. Load and solar generation profiles (1-h resolution). The thick lines represent the predicted profiles while others are generated stochastic scenarios.

Fig. 4. Day-ahead locational marginal price in central Iowa at July 3rd 2017 obtained from historical MISO market dataset.

Fig. 5. Operational cost with different operation strategies.

A. Co-Optimization v.s. Successive Optimization

In this section, we perform a comparison between the proposed co-optimization (cooperative peak shaving and volt/var regulation) and the successive coordinated optimization proposed in [20] to demonstrate the unlocked additional benefits by the proposed co-operation. For the successive optimization, the peak shaving and the volt/var optimization are performed in a successive way; for the benchmark, the distribution system operates without peak shaving and volt/var regulation. For the sake of clarity, this comparison is performed on a deterministic case. To better illustrate the effectiveness of the proposed method, the benchmark load demand in [33] is scaled up by four. As shown in Fig. 5, the operational costs with different optimization strategies are compared. It shows that the co-optimization strategy reduces the operational cost compared to the successive optimization one with the same peak load and voltage limits. As seen from Fig. 6, to achieve peak shaving, the load during peak times will be shifted to 12:00 AM–06:00 AM with relatively low prices by scheduling the BESSs. Utilities will thus purchase more electricity for this period. Besides, as shown in Figs. 7 (b) and (c), the voltage profiles with the two optimization methods are effectively regulated within the limits [0.95,1.05] p.u. But by comparison, the co-optimization results in smoother voltage variations. The benchmark has the lower operational costs because it does not include any operational costs of BESSs and
voltage regulating devices but most bus voltages significantly violate the lower limit while the peak load stays high.

B. Merit of an Explicit Peak Load Constraint

In this subsection, we examine the necessity of a hard and explicit peak load limit constraint in the co-optimization. As shown in Fig. 8, only relying on the cost reduction (Case B) does not effectively lower the peak load because the imposed operational cost of BESSs is more expensive than cost savings by leveraging the ToU price, though it does reduce the overall operational costs of the system. Without considering BESS costs in the optimization, it is observed that the peak load can be slightly reduced. But, consider that if we have sufficient available load shifting capability, there will be a trend that all the load will be shifted/aggregated to the periods with the lowest price. Therefore, there will be a new (and higher) peak at 04:00 AM. This demonstrates the necessity of an explicit constraint on peak load in the optimization problem.

C. Deterministic Optimization v.s. Stochastic Optimization

The comparison between the deterministic co-optimization and (single-stage and two-stage) stochastic co-optimization methods is carried out to demonstrate the value of stochastic programming. 1000 random scenarios of load and solar power time-series are generated as shown in Fig. 3 and are then reduced to 15 representative scenarios, which strives for a balance between performance and computational complexity. 100 new random scenarios are generated to test the performance of different methods under uncertainties. The deterministic co-optimization solves (43). The single-stage stochastic co-optimization solves $x$ and $y$ in one stage (same solution for all scenarios) based on the reduced scenario set. The two-stage stochastic co-optimization solves (50) based on the reduced scenario set, which only yields $x$; and then in the tests, it allows solving $y$ again with fixed $x$ to simulate the intra-day re-dispatch under a given test scenario. Fig. 9 compares the voltage performance among different optimization methods. We record the highest and lowest value voltage magnitude of all buses after 100 random Monte-Carlo simulations. It can be observed that some voltage buses (especially for Phase C) with the DP violate the lower limit under some scenarios since it does not consider the uncertainties from load and solar in the optimization. The single-stage stochastic optimization strategy schedules all the controllable devices in one stage together considering the uncertain prediction errors and thus, it alleviates the voltage violations in Phase C but there are still several bus voltages lower than 0.95 p.u. In comparison, the two-stage stochastic optimization framework regulates all the bus voltages within the ANSI limit since it considers the uncertainties and allows a re-scheduling of reactive powers of BESSs and solar inverters, thereby exhibiting better robustness. This justifies the necessity of the intra-day re-scheduling of available controllable devices. Fig. 10 gives the comparison in terms of peak shaving performance. It can be observed that, with the deterministic optimization, the peak load violates 6000 kW in most of scenarios with the highest peak of 6856.4 kW; the single-stage stochastic optimization alleviates the violation with the highest peak of 6450.1 kW. By contrast, the two-stage optimization can effectively regulate the peak load (maximum peak load 6087.5 kW). The rationale behind this can be elaborated as follows. Obviously, since the deterministic optimization does not take into account any uncertainties in the decision making, it has no robustness against it. The stochastic optimization also violates the peak limit because the reduced scenario space cannot cover all the possible scenarios but it performs better than the deterministic optimization. The single-stage stochastic optimization model considers the uncertainties but does not allow different reactive power outputs from DGs and BESSs under different scenarios. In comparison, given the two-stage model allows for re-dispatching reactive power outputs of DGs and BESSs at the second stage (so-called “wait-and-see”), the network losses can be further reduced under different scenarios.
so that a lower peak load can be achieved. This validates the merit
of stochastic optimization and the necessity of re-dispatch.

V. CONCLUSION

This paper addresses the day-ahead cooperative operation of
peak shaving and voltage regulation in an unbalanced dis-
tribution through a joint optimization framework. We then
consider the uncertainties of load and solar by converting the
coop-optimization model into a two-stage stochastic program. The
numerical results show that the proposed co-optimization frame-
work brings more cost benefits than the successive optimization
method while effectively regulating the voltages and peak load
within the limits. Furthermore, due to the consideration of un-
certainties and the enabled re-dispatch, the proposed two-stage
stochastic programming method facilitates robust operations.
Besides, we also verify the necessity of an explicit peak load
constraint in the optimization for effective peak shaving.

For large-scale networks with a number of stochastic scenar-
ios, the efficiency of centralized solution may be challenged.
Therefore, the distributed solution framework is required for
better scalability. We leave it for future work.

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