A Sequential Black-Start Restoration Model for Resilient Active Distribution Networks

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***Abstract*—This letter presents a novel sequential black-start restoration model to improve the system resilience after a natural disaster in an active distribution network. Two challenges have been addressed: First, a new set of radiality constraints is designed as the network topology is changing along the restoration paths; Second, a black-start restoration model is proposed where multiple backup black-start units in the system are coordinated to find the best restoration paths. Case studies on two IEEE test systems verify the effectiveness of the proposed model.**

***Index Terms*—Active distribution system, black-start restoration, resilience, radiality constraint, sequential restoration.**

I. Introduction

THE ongoing occurrences of severe natural disasters have exposed power systems, especially distribution systems, to large-scale blackouts. After the blackout in power distribution systems, black-start restoration enables the generating units with the self-starting capability to gradually energize the available power supply and realize the system recovery. Therefore, a fast black-start restoration scheme can effectively reduce the power outage duration, accelerate the restoration process, and reduce load curtailments.

In the existing literature, several sequential restoration strategies have been proposed to support the power system. Reference [1] proposed a restoration model to utilize the sequential dispatch of maintenance and restoration crews. Nevertheless, only some of the branches and buses were out of service during the natural disaster, and the situation where all lines are out of service and the whole system is in a blackout state has not been considered in this model. To address large-scale cascading outages defected in [1], result-oriented and resource-oriented restoration approaches were proposed in [2] to restore critical loads sequentially. Similarly, [3] proposed a sequential recovery model that took potential cascading outages into consideration. Reference [4] proposed a sequential service restoration framework to generate restoration solutions for distribution systems and microgrids (MGs) in large-scale power outages. However, references [2]–[4] did not consider the network topology changing along the restoration paths and lacked the explicit formulation of radiality constraints, which may result in infeasible sequential restoration paths.

Since there is a wide range of potential challenges during a disaster in power distribution systems, several models have been proposed to address the blackouts in the distribution network. A sequential restoration model based on the Petri net inference methodology was proposed in [5] to recover loads effectively in a black-out, and [6] presented a methodology for generating restoration sequences in distribution systems. In addition, based on frequency dynamic constraints in power distribution systems, [7] proposed a service restoration model for unbalanced distribution systems to optimize the amount of load restoration and ensure the dynamic performance of system frequency after natural disasters. A generator start-up optimization strategy was proposed in [8], which utilized MGs as BSUs. Reference [9] integrated the BSU procurement decision with a restoration planning model to produce a minimum cost procurement plan in a black-start process. Similarly, [10] formulated an optimization problem to optimally allocate BSUs in the power grid and simultaneously optimize the operation of BSUs during the black-start process.

However, the previous papers have not addressed the sequence in a black-start restoration path for a power distribution network. In fact, restoration paths and corresponding sequences will influence the restoration process and solution, leading to two challenges: i) since the power network topology is changing along restoration paths, the explicit formulation of radiality constraints should be considered for a power distribution network; ii) since there may be several backup BSUs in the distribution system (such as micro-gas turbine units, small hydro units, etc.) the BSUs with their designated paths should be coordinated to find the best restoration path.

To address the above challenges, we propose a sequential black-start restoration model for a resilient active distribution network that applies a mixed-integer second-order cone programming. A novel topology constraint is constructed dynamically concerning the sequence of recovered lines and buses.

II. Mathematical Formulation

Assume a power distribution network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V}$ and $\mathcal{E}$ representing sets of buses and lines, and N_B and N_L identifying the numbers of buses and lines, respectively. After a natural disaster, all lines and buses are out of service. Considering that

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there are N_G backup black-start units in the active distribution network, they should be optimally scheduled to fast restore critical loads and improve the system resilience. However, recovering the system from black to normal conditions would need to address line and bus sequences in the restoration path.

Along a restoration path, if a line is repaired, the corresponding bus will also be returned to the normal condition and the load connecting to this bus will be restored. The sequential black-start restoration strategy for a resilient active distribution network would find an optimal restoration path, where network lines and buses will be sequentially repaired and the corresponding loads will be sequentially restored. The proposed optimization model aims to maximize the total value of restored loads over *T* periods while satisfying the prevailing constraints. In particular, the N_G backup black-start units should be coordinated during the restoration process.

Assuming that K_t lines at period *t* are repaired, the objective function with *T* time periods can be expressed as

$$\max \sum_{t=1}^{T} \sum_{\forall j \in \mathcal{V}} w_j P_{L,jt} \tag{1}$$

where w_j is the load value at bus *j*; $P_{L,jt}$ is the restored active load at bus *j* in period *t*.

The constraints of the proposed model, including operation, logic, and topology, are discussed as follows.

1) Operation Constraints: The operation constraints include power flow and other physical limits, stated as

$$\begin{cases} P_{BS,jt} - P_{L,jt} = \sum\limits_{\forall s \in \delta(j)} H_{jst} - \sum\limits_{\forall i \in \pi(j)} H_{ijt} \\ Q_{BS,jt} - Q_{L,jt} = \sum\limits_{\forall s \in \delta(j)} G_{jst} - \sum\limits_{\forall i \in \pi(j)} G_{ijt} \end{cases},$$

$$\forall j \in \{BS\}, \forall t = 1, \ldots, T \tag{2a}$$

$$\begin{cases} -P_{L,jt} = \sum\limits_{\forall s \in \delta(j)} H_{jst} - \sum\limits_{\forall i \in \pi(j)} H_{ijt} \\ -Q_{L,jt} = \sum\limits_{\forall s \in \delta(j)} G_{jst} - \sum\limits_{\forall i \in \pi(j)} G_{ijt} \end{cases},$$

$$\forall j \in \mathcal{V} \backslash \{BS\}, \forall t = 1, \ldots, T \tag{2b}$$

$$Q_{L,jt} = \tan \theta_j P_{L,jt}, \forall j \in \mathcal{V}, \forall t = 1, \ldots, T \tag{3}$$

$$U_j^{\min} y_{jt} \le U_{jt} \le U_j^{\max} y_{jt}, \forall j \in \mathcal{V} \backslash \{BS\}, \forall t = 1, \ldots, T \tag{5}$$

$$U_{jt} = U_j^0, \forall j \in \{BS\}, \forall t = 1, \ldots, T \tag{6}$$

$$\begin{cases} P_{BS,j}^{\min} y_{jt} \le P_{BS,jt} \le P_{BS,j}^{\max} y_{jt} \\ Q_{BS,j}^{\min} y_{jt} \le Q_{BS,jt} \le Q_{BS,j}^{\max} y_{jt} \end{cases}, \forall j \in \{BS\}, \forall t = 1, \ldots, T \tag{7}$$

$$P_{L,j}^{\min} y_{jt} \le P_{L,jt} \le P_{L,j}^0 y_{jt}, \forall j \in \mathcal{V}, \forall t = 1, \ldots, T \tag{8}$$

where $P_{BS,it}$ and $Q_{BS,jt}$ are the active and reactive powers of *j*-th BSU at period *t*; $Q_{L,jt}$ is the restored reactive load of bus *j* at period *t*; H_{ijt} and G_{ijt} are the active and reactive power flows from bus *i* to bus *j* in period *t*; θ_j is the power factor of the load at but *j*; U_{jt} is the voltage magnitude of bus *j* in period *t*; z_{ijt} is the branch status of the line *ij* in period *t*: if $z_{ijt} = 1$, the branch *ij* is in service and otherwise, the branch is on outage; y_{jt} is the bus *j* status in period *t*: if $y_{jt} = 1$, bus *j* is returned to normal, otherwise, the bus is unrecovered; r_{ij} and x_{ij} are the resistance and the reactance of branch *ij*; ${U^0}_j$ is the specified voltage magnitude of the BSU at bus *j*; {*BS*} is the set of BSUs; ${P_{BS,j}}^{\min}$ and ${P_{BS,j}}^{\max}$ are the minimum and the maximum active power limits of BSU at bus *j*; ${Q_{BS,j}}^{\min}$ and ${Q_{BS,j}}^{\max}$ are the minimum and the maximum reactive power limits of BSU at bus *j*; $U_j^{\min}$ and $U_j^{\max}$ are the lower and the upper bounds of voltage magnitude at bus *j*; ${P^0}_{L,j}$ is the load demand at normal condition; $P_{L,j}^{\min}$ is the minimum load demand of bus *j* in the black-start process;$\pi(j)$ and $\delta(j)$ are the sets of buses with flow inflowing to and outflowing from bus *j*; ${S_{ij}}^{\max}$ is the branch *ij* flow limit; *M* is a large number.

Moreover, (2) denotes the power balance constraint at each bus; (3) indicates that the bus power factor is fixed; neglecting distribution system losses, (4) is the linearized distribution branch flow [11]–[15], and (4) characterizes the voltage drop along a line: if $z_{ijt} = 1$, the line is in service and the voltage drop is given as $r_{ij} H_{ijt} + x_{ij} G_{ijt}$; otherwise, the line is out of service; (5) gives the limits on bus voltage magnitudes; (6) specifies the voltage for each BSU; (7) refers to active and reactive power supply limits of BSUs; (8) gives the load shedding limit.

2) Logic Constraints: The logic constraints present the sequence of lines and buses, and their relations, using the following expressions

$$\sum_{\forall ij \in \mathcal{E}} (z_{ijt} - z_{ijt-1}) \le K_t, \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{9}$$

$$y_{it} \ge z_{ijt}, y_{jt} \ge z_{ijt}, \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{10}$$

$$z_{ijt} \ge z_{ijt-1}, \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{11}$$

$$\begin{cases} y_{j0} = 0 \ \forall j \in \mathcal{V} \backslash \{BS\} \\ y_{j0} = 1 \quad \forall j \in \{BS\} \end{cases}, \forall t = 1, \ldots, T \tag{12}$$

$$z_{ij0} = 0, \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{13}$$

where (9) indicates that at most K_t lines are repaired at each period. Please note that this number would be a boundary condition which can be either a given value or a variable. Here, in this paper, we take it as a given value. Constraint (10) shows that the two end-buses of a repaired line will be returned to normal; however, if the line is not repaired, the two end-buses would have an unknown status. Constraint (11) gives the sequence of lines and buses. Once a bus or a line is returned to normal, it will be available in subsequent periods. (12) and (13) give the initial status of lines and buses, where all lines and buses (except those with BSUs) will be on outage during a blackout.

3) Topology Constraints: For a distribution network, network radiality should be guaranteed along restoration paths. Since there are multiple islands along restoration paths, the distribution network is termed as a forest structure in graph theory where each path has a tree structure. According to [16], [17], two conditions should be satisfied for the network radiality of a forest: (i) each island is connected; (ii) the number of branches is equal to the number of buses minus the number of islands. However, the number of islands is dynamically changed during the restoration process. Therefore, the above two conditions are modified. At

$$\begin{cases} -M(1 - z_{ijt}) \le U_{jt} - U_{it} - (r_{ij} H_{ijt} + x_{ij} G_{ijt}) \le M(1 - z_{ijt}), \\ H_{ijt}^2 + G_{ijt}^2 \le S_{ij}^{\max} z_{ijt} \end{cases} \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{4}$$

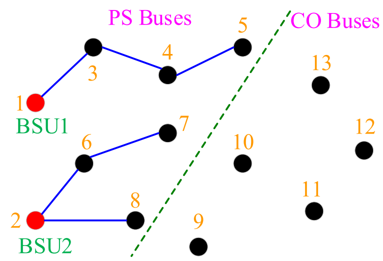

Fig. 1. Classification of buses in a distribution network.

each period, buses are classified into two types: power supported buses (PS) which are connected to one BSU and cut-off buses (CO) which are not supported by any BSUs. CO buses should be connected to PS buses in a restoration path. For example, there are 7 islands in Fig. 1. If the CO bus 13 is connected to one PS bus, the number of islands becomes 6. Moreover, in a restoration path, we have the third condition (iii) which states that CO buses should not be connected with each other because the connection without BSUs would not help restoration. To achieve conditions (i) and (iii), the single commodity flow is employed using the following constraints

$$\sum_{\forall s\in\delta(j)} F_{jst} - \sum_{\forall i\in\pi(j)} F_{ijt} = -y_{jt},\ \forall j\in\mathcal{V}\backslash\{BS\},\forall t=1,\ldots,T \tag{14}$$

$$\sum_{\forall s\in\delta(j)} F_{jst} - \sum_{\forall i\in\pi(j)} F_{ijt} = W_{jt},\ \forall j\in\{BS\},\ \forall t=1,\ldots,T \tag{15}$$

$$W_{jt} \geq 0,\ \forall j\in\{BS\},\ \forall t=1,\ldots,T \tag{16}$$

$$-Mz_{ijt} \leq F_{ijt} \leq Mz_{ijt},\ \forall ij\in\mathcal{E},\ \forall t=1,\ldots,T \tag{17}$$

where F_{ijt} is the power flow on the line *ij* in the fictitious network at period *t*; and W_{jt} is the power supplied by BSUs in the fictitious network. Constraint (14) implies that the flow is balanced at each bus. Moreover, if the bus is recovered, the injected power is -1; otherwise, it is 0. Also, there must be one path that connects a recovered bus with a balanced flow to one BSU. Constraints (15) and (16) show that the BSUs in the fictitious network can have any positive power output. Constraint (17) implies that a recovered line could have any flow value; otherwise, line flow is zero.

Using (11), if the two end buses of a repaired line are recovered. According to (14), these two buses must be connected to one BSU. Therefore, condition (iii) must be satisfied and a CO bus is a single bus with 0 power injection. For each PS bus, there must be one path that connects the PS bus to one BSU; so the network connectivity, i.e., condition (i), would be satisfied.

To achieve condition (ii), we have

$$\sum_{\forall ij\in\mathcal{E}} z_{ijt} = N_B - \left(\sum_{\forall j\in\mathcal{V}} (1-y_{jt}) + N_{BS}\right),\ \forall t=1,\ldots,T \tag{18}$$

where N_{BS} is the number of BSUs. In (18), the left-hand side provides the number of repaired branches. At the initial time, there are N_{BS} islands. When one bus is restored, it will be connected to a certain BSU, and the number of islands will be reduced by one. Therefore, there are $\sum_{\forall j\in\mathcal{V}} (1-y_{jt}) + N_{BS}$ islands at time *t*. Constraint (18) shows that the number of branches is equal to the number of buses minus the number of islands.

Using condition (ii), we derive that the number of lines in service should not exceed N_B-N_G. Thus, the total number of optimization periods, i.e., *T*, is stated as

$$\sum_{t=1}^{T-1} K_t \leq N_B - N_G \leq \sum_{t=1}^{T} K_t \tag{19}$$

Finally, the objective function (1) is subject to (2)–(18), which presents a mixed-integer second-order cone programming.

Discussions: It is desired to note that three situations are not included in the feasible set defined by the proposed model:

1) The proposed model can provide a black-start resilience strategy for a distribution system operating in a balance condition. The unbalanced operations of the power distribution system are not included in the feasible set. We will analyze in our future studies unbalanced operations of power distribution networks during a black-start process according to [4], [18] and [19];
2) The black-start units in our paper do not include renewable energy sources, because their outputs are variable, and they cannot provide a stable power supply for the black-start process. Therefore, the feasible set of the proposed model doesn't give the modeling of renewable energy resources. In future work, the virtual power plant with distributed renewable energy resources and energy storage systems can be considered as BSUs in a black-start process [20], [21].
3) In our paper, the number of BSUs and microgrids would remain unchanged during the black-start process. And the microgrid interconnection and BSU merging have not been considered in our model.

III. Numerical Results

The proposed model is tested on several IEEE standard systems which are carried on a computer with an Intel Core i7 Duo Processor (2.4 GHz) and 8 GB RAM by GUROBI solver. For the 33-bus system [19], we choose 3 BSUs on buses {1, 15, 29}, and K_t is the same at each period. We prioritize all loads into five levels with parameters provided in [18]. The restoration paths for $K_t = 5$ and $K_t = 10$ are shown in Figs. 2 and 3, respectively, where there are 6 periods for $K_t = 5$ and 3 periods for $K_t = 10$. The final topologies for the two cases are the same but the restoration process is different. At each period, there are K_t branches, buses are repaired, and corresponding bus loads are restored by BSUs. Since load values at buses {12, 13, 14, 20, 21} are the largest, they have the highest restoration priority. Branches {19-20, 12-13, 7-8,} are sequentially reconfigured to reduce the network loss and pick up additional loads during the black-start process. Besides, the radial topology can be strictly guaranteed during the restoration process.

Finally, the proposed model is tested on an IEEE 906-bus system [22], [23] with different K_t and N_G values, and the computational time is shown in Table I. Equation (1) suggests that either increasing K_t or decreasing N_G will increase the number of periods, so decision variables, constraints, and computation time will increase accordingly. Overall, the largest computation time for this case is about 30 minutes.

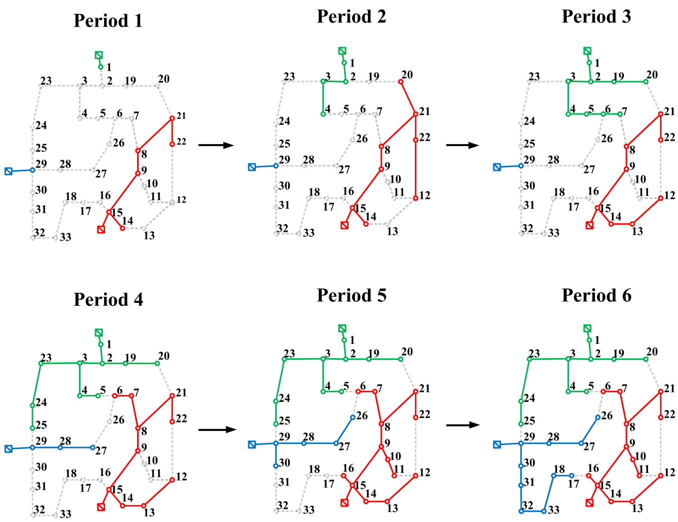

Fig. 2. Restoration paths in the 33-bus distribution system ($K = 5$).

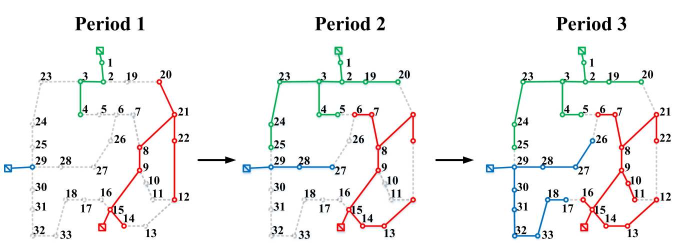

Fig. 3. Restoration paths in the 33-bus distribution system ($K = 10$).

TABLE I
COMPUTATIONAL TIME OF THE PROPOSED METHOD (SEC.)

N_G	K_t					
	50	100	150	200	250	300
10	1252	996	842	759	694	635
20	1339	1176	997	806	755	676
30	1499	1302	1088	913	836	722
40	1611	1444	1209	1153	963	842
50	1880	1620	1453	1216	1022	903

IV. Conclusion

A sequential black-start restoration model is proposed for a resilient power distribution network, which is formulated as a mixed-integer second-order cone programming to address the restoration sequence. Simulation results suggest that the proposed model can strictly guarantee the required constraints in restoration paths. Moreover, the computational time is mainly related to the numbers of black-start units and repaired lines at each period.

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