# Distribution Grid Modeling Using Smart Meter Data

Yifei Guo, *Member, IEEE*, Yuxuan Yuan, *Graduate Student Member, IEEE*, and Zhaoyu Wang, *Senior Member, IEEE*



***Abstract*—The knowledge of distribution grid models, including topologies and line impedances, is essential for grid monitoring, control and protection. However, such information is often unavailable, incomplete or outdated. The increasing deployment of smart meters (SMs) provides a unique opportunity to tackle this issue. This paper proposes a two-stage framework for distribution grid modeling using SM data. In the first stage, the network topology is identified by reconstructing a weighted Laplacian matrix of distribution networks. In the second stage, a least absolute deviations (LAD) regression model is developed for estimating line impedance of a single branch based on the nonlinear (inverse) power flow model, wherein a conductor library is leveraged to narrow down the solution space. The LAD regression model is originally a mixed-integer nonlinear program whose continuous relaxation is still non-convex. Thus, we specially address its convex relaxation and discuss the exactness. The modified regression model is then embedded within a bottom-up sweep algorithm to achieve the identification across the network in a branch-wise manner. Numerical results on the IEEE 13-bus, 37-bus and 69-bus test feeders validate the effectiveness of the proposed methods.**

***Index Terms*—Distribution grid, inverse power flow, line impedance estimation, topology identification, smart meter, convex relaxation.**

## I. Introduction

With the increasing penetration of distributed energy resources (DERs), grid monitoring and energy management are imperative to distribution system operation [1]. However, such functionalities require complete and accurate knowledge of distribution grid models, including network topologies and line parameters. Unlike transmission systems that enjoy a high level of data redundancy, distribution grid models could be inaccurate or even unavailable [2]. Some utilities only have simple one-line diagrams of their systems without detailed line parameters; other utilities may have system models, but they are often incomplete or outdated due to the frequent system expansion and reconfiguration. Field inspection is a conventional approach to draw the model information, which is laborious, costly, and time-consuming, especially for large-scale systems [3]. This suggests an urgent need of efficient and tractable approaches for distribution grid modeling.

In recent years, the deployment of advanced monitoring and metering infrastructures, e.g., smart meters (SMs) and micro-phasor measurement units ($\mu$PMUs), provides an opportunity to extract the distribution grid models from field measurements [4]. Some studies extend the classical state estimation tools [5]–[8] to infer the status of switches, shunt capacitors/reactors, etc. [9]–[12]. However, this paper considers a different problem: the general distribution grid modeling consisting of a full topology and parameter identification of the whole network from scratch instead of only detecting the status of switchable devices. This cannot be easily handled by generalizing the state estimation tools.

Recently, data-driven approaches for network topology and parameter identification have attracted a lot of attention. These methods can be roughly classified into two categories according to whether they require complex voltage and current measurements (i.e., phase angle information). The studies of the first category rely on high-granularity synchrophasor measurements [13]–[16]. In [13], a multi-run optimization method was proposed to estimate line parameters of a three-phase distribution feeder based on the synchronized voltage phasors and line flow measurements. The authors in [14] proposed to identify network topology based on both fundamental and harmonic synchrophasor data by solving a mixed-integer linear program. With the help of phase angle information, the work of [15] jointly estimates the network topology and parameters by directly reconstructing the admittance matrix. In [16], a similar joint estimation was achieved by carrying out the topology and parameter identification alternately. Note that these phasor-based methods require a high or even full coverage of $\mu$PMUs, which is cost-prohibitive, especially for low-voltage (LV) grids. In addition, the existing joint topology and parameter estimation methods need to solve a large-scale centralized optimization program and may require iterations between topology and parameter identification; thus, the computational complexity significantly grows with the network size.

Rather than using synchrophasor measurements, another line of research managed to identify topology or parameters using voltage magnitude and power measurements [17]–[23]. In [17], a mixed-integer quadratic programming (QP) model was developed to identify network topology with the known line impedance information. In [18], a structure learning method was developed to estimate the grid topology by assuming the nodal power injections are uncorrelated or with non-negative covariances. In [19] and [20], correlation

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The authors are with the Department of Electrical and Computer Engineering, Iowa State University, Ames, IA 50011 USA (e-mail: yifeig@iastate.edu; yuanyx@iastate.edu; wzy@iastate.edu).

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analysis-based algorithms were proposed to identify the grid topology using SM data, under the assumption that the correlation/similarity between customers' voltage profiles increases as the electrical distance decreases. In [21], a Markov random field-based algorithm was proposed to detect the topology based on uncorrelated power loads. Notice that such statistical assumptions may be challenged by the high penetration of behind-the-meter DERs. The authors in [22] formulated the parameter identification problem as a maximum likelihood estimation model based on the linearized power flow. In [23], an error compensation model was developed to achieve a robust estimation of distribution line parameters.

It is observed that the existing methods in this category either conduct topology identification based on some prior line parameter information (e.g., impedance or R/X ratio), or perform parameter identification with a known topology. A joint network topology and parameter estimation is still challenging in the sense that such prior information is usually unavailable in practice. Statistical methods usually need massive measurement streams, composed of several hours or even many days of recorded data. This may hinder them from detecting topology changes in real time.

In this context, we propose a novel two-stage framework to identify network topology and parameters in this paper. In the first stage, we develop a novel topology identification method, which consists of a linear least squares (LS) model for estimating a weighted Laplacian matrix (WLM) and a density-based clustering method for recovering topology from the estimated WLM. Different with existing methods, the proposed topology identification initially builds on distribution grids with a homogeneous R/X ratio, which yields a tractable fitting model. Then, its robustness against heterogeneous R/X ratios is also analyzed and demonstrated.

In the second stage, a nonlinear least absolute deviations (LAD) regression model is developed for parameter estimation of a single branch based on the branch flow model [24]. To improve the accuracy of parameter estimation, the LAD regression model establishes on the nonlinear power flow, which is therefore nonlinear and nonconvex. Then, we propose a convex relaxation method of the LAD model. A conductor library is exploited to significantly narrow the solution space of parameter estimation. Finally, a bottom-up sweep algorithm is proposed to accomplish the parameter estimation across the entire system by carrying out the estimation of line impedance and line flow, alternatively.

The topology identification solves an unconstrained convex QP program and the density-based clustering method needs to scan the whole network only once. The parameter estimation is performed in a branch-wise manner, so that the computational complexity is approximately *linear* with the network size. Therefore, the proposed method enjoys good computational efficiency and scalability.

The rest of this paper is organized as follows. Section II gives the preliminaries including the power flow model, some facts and basic assumptions, used for developing the proposed method. Sections III and IV present the details of topology identification and line impedance estimation methods, respectively. Numerical test results are provided in Section V. Some discussions in terms of robustness and scalability is given in Section VI, followed by conclusions.

## II. Preliminaries

Regarding notation, for a column vector $\mathbf{v}$, let $v_i$ denote its $i$th entry; and $\|\mathbf{v}\|_1$ and $\|\mathbf{v}\|_2$ denote its $\mathcal{L}_1$-norm and $\mathcal{L}_2$-norm, respectively. Given a matrix $\mathbf{M}$, let $m_{ij}$ denote its entry at $i$-th row and $j$-th column and $[\mathbf{M}]_i$ denotes its $i$th row; $\mathbf{M}^{-1}$, $\mathbf{M}^T$ and $\mathbf{M}^{-T}$ denote its inverse, transpose and inverse transpose, respectively. Let $\mathbf{1}_n$ be the $n \times 1$ column vector with all entries being 1 and $\mathbf{I}_n$ be the $n \times n$ identity matrix. The superscript $\widehat{(\bullet)}$ denotes the *estimation* and $(\bullet)^\star$ means the *optimum*.

Consider a radial distribution grid comprised of $n+1$ buses. Let $\mathcal{N} \cup \{0\}$ be the set of buses where the secondary side of substation transformer is indexed by 0 (the unique slack bus in the distribution grid) and $\mathcal{N} := \{1, \ldots, n\}$ denotes the set of other buses. For any $j \in \mathcal{N}$, $\mathcal{C}_j \subseteq \mathcal{N}$ denotes its children bus set. $\mathcal{P}_j$ denotes the set of buses in the *unique* path from bus $j$ to bus 0 (including bus $j$ itself). Without loss of generality, we uniquely label a branch by its downstream end bus (i.e., branch $j$'s downstream end is bus $j$). In this way, we are able to characterize the network only by bus labels.

The proposed topology and parameter identification methods both build on the branch flow model [24] that relaxes the voltage angle. For notation convenience throughout this paper, we modify the original version by *splitting* the power balance equations as:

$$P_j = \sum_{k \in \mathcal{C}_j} \bar{P}_k - p_j, \;\; \bar{P}_j = P_j + r_j \cdot \frac{P_j^2 + Q_j^2}{v_j} \tag{1a}$$

$$Q_j = \sum_{k \in \mathcal{C}_j} \bar{Q}_k - q_j, \;\; \bar{Q}_j = Q_j + x_j \cdot \frac{P_j^2 + Q_j^2}{v_j} \tag{1b}$$

$$v_i - v_j = 2\left(r_j P_j + x_j Q_j\right) + \left(r_j^2 + x_j^2\right) \cdot \frac{P_j^2 + Q_j^2}{v_j} \tag{1c}$$

for any $j \in \mathcal{N}$, where $p_j, q_j$ denote the *net* real/reactive power injection at bus $j$; $\bar{P}_j, \bar{Q}_j$ denote the real and reactive power flowing from the upstream bus $i$; $P_j, Q_j$ denote the real and reactive power flowing to the downstream bus $j$; $r_j, x_j > 0$ are the line resistance and reactance; $v_i$ and $v_j$ are the *squared* voltage magnitude at buses $i$ and $j$.

Different than the power flow analysis that solves voltages and line flows, the line impedance and topology information will be extracted based on the known voltage and line flows, which is referred to as *inverse power flow* [25].

Some facts and assumptions throughout this paper are clarified as below:

- The proposed method only considers balanced distribution networks. This is due to the invisibility of the grid-edge phase angle information. In this work, the available data source required only consists of voltage magnitude and power measurements recorded by SMs. In practice, this method can be applied for balanced medium-voltage (MV) systems or single-phase LV grids.

- It is assumed that there is a full coverage of SM with nodal net load and voltage magnitude measurements. This assumption is consistent with the recent expansion of smart grid monitoring devices. By the end of 2020, about 107 million SMs had been deployed, covering about 75% of U.S. households [26]. Further, although reactive power measurement is rarely collected in practice, SMs are, in fact, able to measure reactive power in real time. Utilities do not activate this function due to financial and data storage concerns.
- Typically, SMs are installed on the LV customer side. So, SM data should be aggregated if they are used for MV grids. For example, the customer power measurements are aggregated at secondary transformer level.
- The SM data used for identification is assumed to be perfect since data quality problems are not the primary focus of this work. Though this may not be the case in practice, a number of advanced data pre-processing [27]–[30] and pseudo measurement generation methods [31]–[33] can be implemented first to mitigate the impact of and data quality issues (e.g., asynchrony, bad data, missing data).
- A library of line conductor types is assumed to be known, which is typically available in practice, in the sense that the conductor types are often well recorded by utilities.

## III. Network Topology Identification Based on Weighted Laplacian Matrix

In this section, we firstly develop an optimization-based topology identification method for distribution grids with the homogeneous R/X ratio, and then discuss its robustness against the variability of R/X ratios.

### A. Link Between Grid Topology and Power Flow

Assuming the line loss is negligible compared to line flow[1], a linear approximation of (1) that neglects the nonlinear terms in (1) is conducted as,

$$\mathbf{v} \simeq 2\mathbf{A}^{-T}\mathbf{R}\mathbf{A}^{-1}\mathbf{p} + 2\mathbf{A}^{-T}\mathbf{X}\mathbf{A}^{-1}\mathbf{q} - v_0\mathbf{A}^{-T}\mathbf{a}_0 \qquad (2)$$

where $\mathbf{v} := [v_1, \ldots, v_n]^T$, $\mathbf{p} := [p_1, \ldots, p_n]^T$, and $\mathbf{q} := [q_1, \ldots, q_n]^T$ denote the vectors collecting squared bus voltage magnitudes, real power and reactive power injections at buses $1, \ldots, n$, respectively; $[\mathbf{a}_0, \mathbf{A}^T]^T \in \{0, \pm 1\}^{(n+1)\times n}$ is the incidence matrix of the radial-topology graph where $\mathbf{a}_0^T$ denotes the first row of the incidence matrix; $\mathbf{R} := \mathrm{diag}(r_1, \ldots, r_n)$ and $\mathbf{X} := \mathrm{diag}(x_1, \ldots, x_n)$ are diagonal matrices with $j$-th diagonal entry being the resistance and reactance of $j$-th branch.

Regarding the sort order within $\mathbf{p}, \mathbf{q}$ and $\mathbf{v}$, it should be clarified that the entries within vectors $\mathbf{p}, \mathbf{q}$ and $\mathbf{v}$ can be sorted without any prior restriction. To be more clear, buses $1, \ldots, n$ can be arbitrarily labelled regardless of the actual bus position in the network. The *only* requirement is that they should be organized in a coherent way, meaning $p_j, q_j$ and $v_j$ that characterize bus $j$ should come from the same SM.

[1] Since line losses are usually much smaller than power flows, the approximation error is relatively small, typically at the order of 1% [34].

For a radial distribution network, the reduced incidence matrix $\mathbf{A} := [a_{ij}]_{n\times n}$ is non-singular [35] and $\mathbf{A}^{-T}\mathbf{a}_0 = -\mathbf{1}_n$. Therefore, a variant of (2) reads,

$$\frac{1}{2}\underbrace{\mathbf{A}\mathbf{X}^{-1}\mathbf{A}^T}_{\mathbf{Y}}(\mathbf{v} - v_0\mathbf{1}_n) = \underbrace{\mathbf{A}\mathbf{X}^{-1}\mathbf{R}\mathbf{A}^{-1}}_{\mathbf{\Phi}}\mathbf{p} + \mathbf{q} \qquad (3)$$

where $\mathbf{Y} := [y_{ij}]_{n\times n}$ is a weighted *Laplacian* matrix of the network with the entries being:

$$y_{ij} = y_{ji} = \begin{cases} -1/x_j, & \text{if } j \in \mathcal{C}_i \\ \sum\limits_{k\in\{j\}\cup\mathcal{C}_j} 1/x_k, & \text{if } i = j \\ 0, & \text{otherwise.} \end{cases} \qquad (4)$$

Mathematically, the rationale behind $\mathbf{Y}$ is: for any two distinct buses $i$ and $j$, $y_{ij} < 0$ if they are (directly) physically connected and otherwise, $y_{ij} = 0$. In a physical sense, $\mathbf{Y}$ is structurally close to the admittance matrix but without considering the line resistance. Therefore, if one can (approximately) identify $\mathbf{Y}$ that uniquely characterizes the connectivity, the topology can be extracted accordingly. This inspires a $\mathbf{Y}$-based topology identification method.

### B. Identification Model

We thus attempt to develop a regression model of $\mathbf{Y}$ based on (3) and the measurements of $\mathbf{p}, \mathbf{q}, \mathbf{v}$ and $v_0$ that can be obtained from SM data. It minimizes the mismatch between both sides of (3). Unfortunately, $\mathbf{\Phi}$ involves the network topology and parameters that are unknown yet.

But interestingly, suppose the network has a homogeneous R/X ratio, i.e.,

$$\frac{r_1}{x_1} = \cdots = \frac{r_n}{x_n} = \lambda, \qquad (5)$$

$\mathbf{\Phi}$ reduces to

$$\mathbf{\Phi} = \mathbf{A}\begin{bmatrix} r_1/x_1 & & \\ & \ldots & \\ & & r_n/x_n \end{bmatrix}\mathbf{A}^{-1} = \lambda\mathbf{I}_n. \qquad (6)$$

Accordingly, (3) becomes,

$$\mathbf{Y}(\mathbf{v} - v_0\mathbf{1}_n) = 2(\lambda\mathbf{p} + \mathbf{q}). \qquad (7)$$

This exactly eliminates the requirement of prior information regarding $\mathbf{A}$, $\mathbf{R}$ and $\mathbf{X}$, and relies on $\mathbf{p}, \mathbf{q}, \mathbf{v}$ and $v_0$.

Then, defining the mismatch vector regarding $k$-th sample,

$$\mathbf{e}^{(k)} := \widehat{\mathbf{Y}}\left(\mathbf{v}^{(k)} - v_0^{(k)}\mathbf{1}_n\right) - 2\lambda\mathbf{p}^{(k)} - 2\mathbf{q}^{(k)}, \ \forall k \qquad (8)$$

and $\mathbf{e} := [(\mathbf{e}^{(1)})^T, \ldots, (\mathbf{e}^{(K)})^T]^T$ where $K \gg n$ is the total number of samples, a linear LS regression model of $\mathbf{Y}$ reads,

$$\underset{\widehat{\mathbf{Y}},\lambda}{\text{minimize}} \ ||\mathbf{e}||_2^2. \qquad (9)$$

Clearly, this fitting model is an unconstrained convex QP program that can be efficiently solved.

**Algorithm 1:** Recovering Topology From $\widehat{\mathbf{Y}}$ by Clustering.

**Initialization**: Initialize $i \leftarrow 1, j \leftarrow 1, \gamma, \xi$
**repeat**
  [**S1**]: Select the $i$th row of $\widehat{\mathbf{Y}}$.
  **repeat**
    [**S2**]: Pick $\widehat{y}_{ij}$ and retrieve all direct density-reachable points using $\xi$.
    [**S3**]: Based on $\gamma$, if $\widehat{y}_{ij}$ is a core point, a cluster is formed; otherwise, update $j \leftarrow j+1$.
  **until** $j = n$ or no new point can be added to any cluster
  [**S4**]: Update $i \leftarrow i+1$.
**until** $i = n$.

### C. Recovering Topology From Weighted Laplacian Matrix

Recovering the topology from $\widehat{\mathbf{Y}}$ can be cast as an *anomaly detection* problem based on the property in (4). Considering the sparsity of the distribution grid topology, a density-based spatial clustering of applications with noise method [36] is utilized here, which is tabulated as Algorithm 1. The rationale behind our task is that most of the entries in $\widehat{\mathbf{Y}}_i$ for all $i$, are concentrated on a small range, which can be grouped into several clusters that represent the unconnected buses; the non-diagonal entries that do not belong to these clusters are declared as anomalies which indicate the connectivity. To achieve this, the method uses a minimum density level estimation based on two user-defined hyperparameters, a threshold for the minimum number of neighbors, $\gamma$, and the radius, $\xi$. $\widehat{y}_{ij}$ with more than $\gamma$ neighbors within $\xi$ distance are considered to be a core point. All neighbors within the $\xi$ radius of a core point are considered to be part of the same cluster as the core point. Based on multiple core points, all entries in $\widehat{\mathbf{Y}}$ can be separated by clusters of lower density. The cluster with the minimum entries is considered to contain the connected buses. Overall, our method leverages the density drop between the unconnected and the connected entries in $\widehat{\mathbf{Y}}$ to detect the cluster boundaries for recovering topology from estimated weighted Laplacian matrix. Unlike other clustering algorithms that assume normally shaped clusters, this method is capable of finding clusters with arbitrary shapes and sizes. Moreover, it does not require *a priori* specification on the number of clusters, therefore ensuring the robustness and practicality [37]. Note that it does not enforce radiality.

### D. Robustness Analysis on Heterogeneous Networks

As mentioned above, the proposed regression model is derived on the assumption of a homogeneous R/X ratio, which may not be true in practical networks. However, a distribution grid at a given voltage level has the *relatively heterogeneous* R/X ratios [38], which is widely believed to hold in many practical cases (see [39] for some examples). In what follows, we will show that our proposed method has some robustness against the heterogeneous R/X ratios.

Let $\overline{\lambda} := (\lambda_1 + \cdots + \lambda_n)/n$ be the mean of R/X ratios and accordingly, let $\lambda_j := \overline{\lambda} + \Delta\lambda_j, \forall j$, where $\Delta\lambda_j$ denotes the deviation to $\lambda$. Therefore, we have,

$$\mathbf{\Phi} = \overline{\lambda}\mathbf{I}_n + \mathbf{\Delta}_\lambda \tag{10}$$

where $\mathbf{\Delta}_\lambda := \mathbf{A}\mathrm{diag}(\Delta\lambda_1, \ldots, \Delta\lambda_n)\mathbf{A}^{-1}$.

*Proposition 1:* Matrix $\mathbf{\Delta}_\lambda := [\Delta_{ij}]_{n\times n}$ is a matrix with the entries being,

$$\Delta_{ij} = \begin{cases} \Delta\lambda_i - \Delta\lambda_{\mathcal{C}_i \cap \mathcal{P}_j}, & \text{if } \ i \in \mathcal{P}_j \\ \Delta\lambda_i, & \text{if } \ i = j \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

The proof is provided in the appendix. Observe (11), $\mathbf{\Delta}_\lambda$ can be used to quantify the heterogeneity of R/X across the whole network. For a netwrok with relatively homogeneous R/X ratios, $\lambda_j \simeq \overline{\lambda}, \forall j$ and consequently, $|\Delta\lambda_i| \simeq 0$ and $|\Delta\lambda_i - \Delta\lambda_j| \simeq 0, \forall i, j$. Therefore, $\mathbf{\Delta}_\lambda$ will not significantly affect the solution of (9) provided the program is numerically stable. And for a strictly homogeneous network, (10) completely reduces to (6).

## IV. Line Impedance Estimation: LAD Regression Model and Bottom-Up Sweep Framework

Here, we develop a regression model for line impedance estimation of a *single* branch–a LAD model with mixed-integer semidefinite programming (MISDP) formulation. It is then embedded with a *bottom-up* sweep algorithm to accomplish the parameter estimation across the entire network.

Keep in mind that the proposed regression model is built on full nonlinear inverse power flow instead of its linearized counterpart, in the sense that the latter may be unable to accurately recover the parameters, especially when the regression problem is ill-posed [40]; see Fig. 1 for a numerical example on the IEEE 13-bus feeder. Note that, the non-convexity of the nonlinear inverse power flow model makes the regression problems NP-hard even after continuous relaxation. This motivates us to specially address its convexification.

### A. Regression Model for a Single Branch

The line impedance estimation establishes on the voltage drop relationship (1 e). Define the vector of model mismatch $\mathbf{e}_j := [e_j^{(1)}, \ldots, e_j^{(K)}]^T$ for all $j \in \mathcal{N}$ with

$$\begin{aligned} e_j^{(k)} :=\ & v_i^{(k)} - v_j^{(k)} - 2\left(\widehat{r}_j P_j^{(k)} + \widehat{x}_j Q_j^{(k)}\right) \\ & - \left(\widehat{R}_j + \widehat{X}_j\right) \cdot \frac{\left(P_j^{(k)}\right)^2 + \left(Q_j^{(k)}\right)^2}{v_j^{(k)}}, \ \forall k \end{aligned} \tag{12}$$

where $x_j$ and $r_j$ denote the estimation of $r_j$ and $x_j$; $\widehat{R}_j := \widehat{r}_j^2$ and $\widehat{X}_j := \widehat{x}_j^2$.

The impedance estimation minimizes the $\mathcal{L}_1$-norm (LAD) of $\mathbf{e}_j$, which is expected to hold the following features. On the one hand, the nonlinearity of the inverse power flow is well-captured to guarantee estimation accuracy. On the other hand, the library of R/X ratios (obtained from the line conductor library) is exploited to significantly narrow the solution space; otherwise, the solution may easily fall into some remote local optimum. Therefore, the line impedance estimation, which is inherently

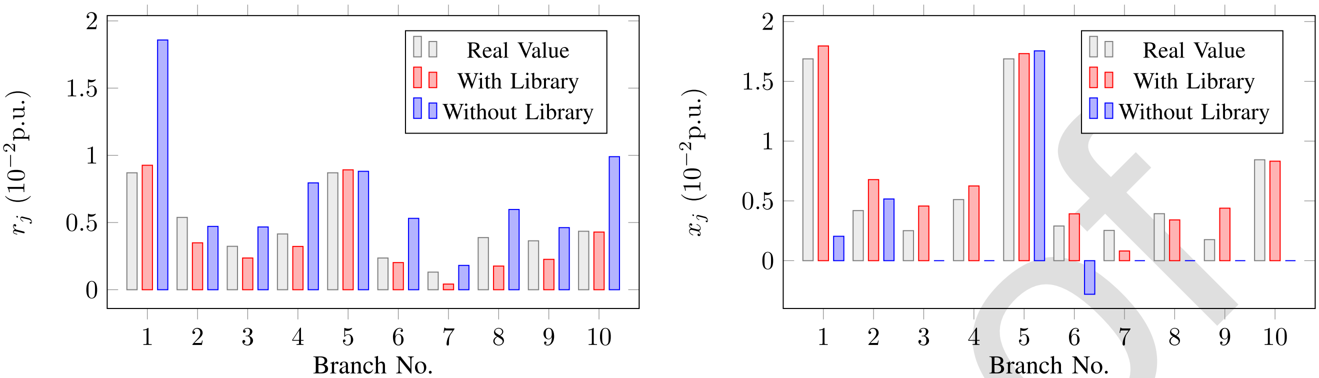
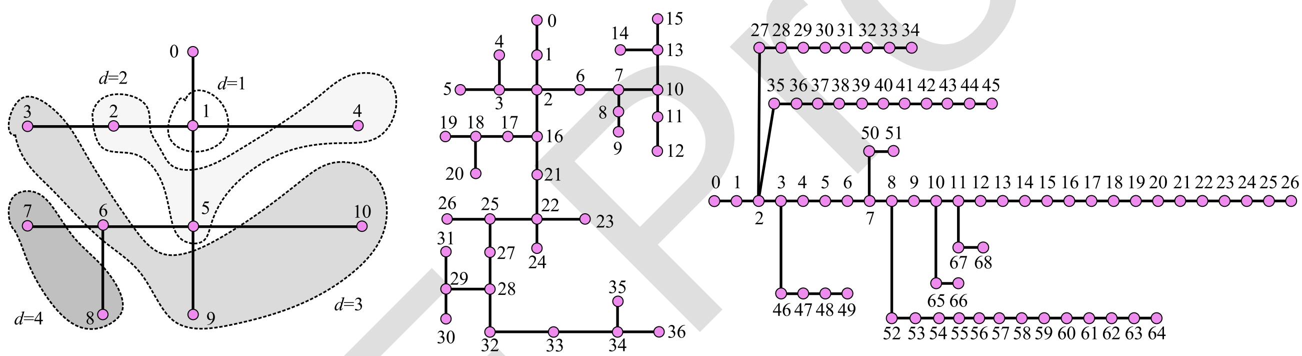



Fig. 1. Least-squares-based line parameter estimation results of the modified IEEE 13-bus test feeder (see Section V for details) based on the linearized inverse power flow model with and without the help of a R/X ratio library.




(a) 13-bus Test Feeder (b) 37-bus Test Feeder (c) 69-bus Test Feeder

Fig. 2. One-line diagrams of the modified IEEE (a) 13-bus, (b) 37-bus and (c) 69-bus test feeders (balanced) where the original 13-bus test feeder is modified to a 11-bus test feeder by removing the dummy buses 634 and 692 of the original case and the line impedance of 69-bus feeder are slightly modified to achieve several typical R/X ratios. The resultant R/X ratio libraries are $\{0.5153, 1.2840, 0.8124, 0.8112, 0.9864, 2.0655\}$, $\{1.4536, 1.6222, 2.7482, 1.9691\}$, and $\{0.4000, 0.8000, 0.9000, 2.0000, 2.9000, 3.0000, 3.1000, 3.3000, 3.4000\}$ in the three cases, respectively.

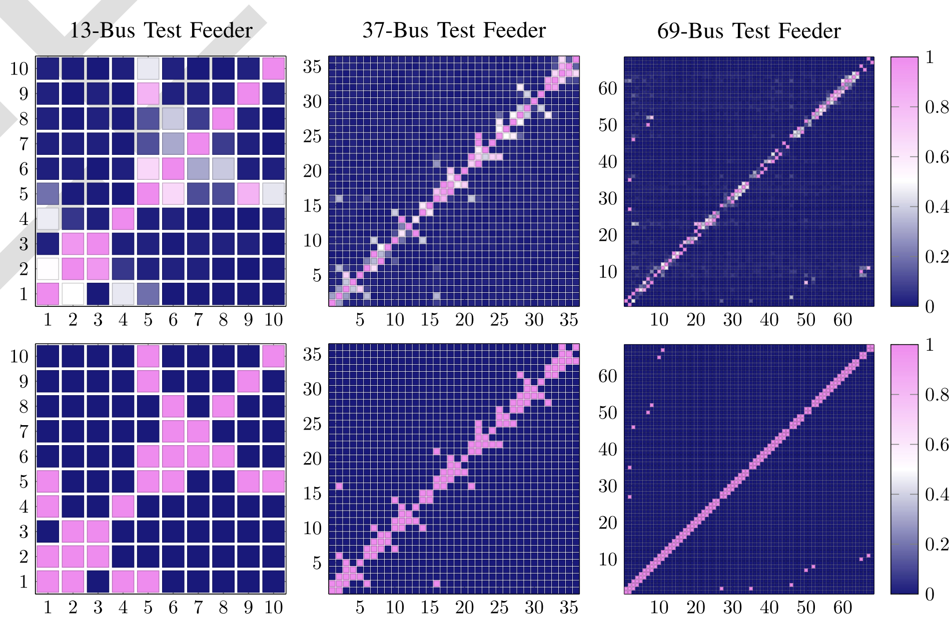



Fig. 3. Results of topology identification of the modified IEEE 13-bus (left), 37-bus (middle), and 69-bus (right) test feeders. The top part shows the normalized counterpart of $\widehat{\mathbf{Y}}$. The bottom part shows the output of Algorithm 1, which represents the connectivity.

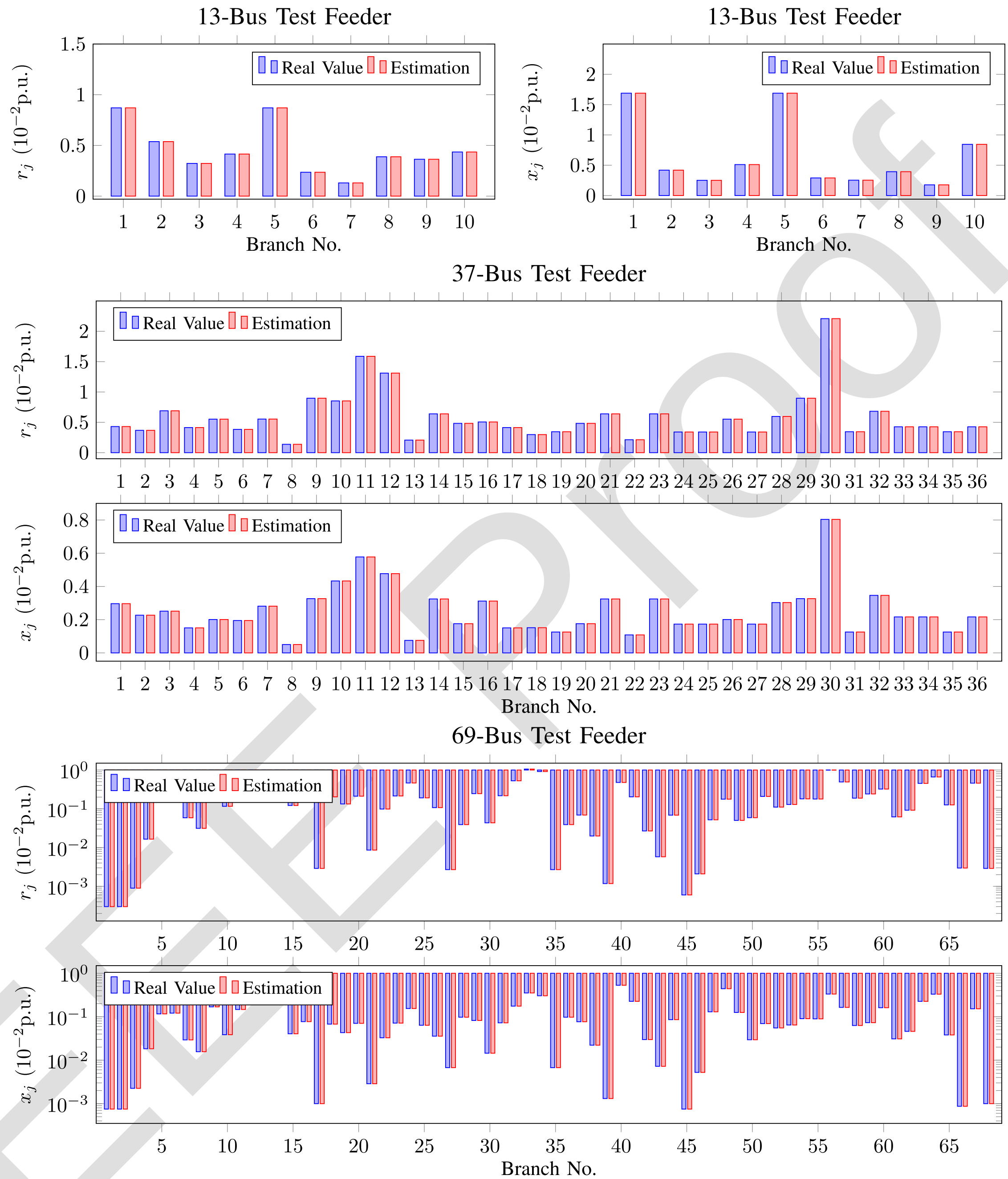



Fig. 4. Results of line parameter estimation of 13-, 37- and 69-bus test feeders.

a combinatorial optimization problem, can be cast as a mixed-integer nonlinear programming (MINLP) model by introducing the binary variables $\alpha_1, .., \alpha_H$,

$$\underset{\alpha_h, r_j, x_j, R_j, X_j}{\text{minimize}} \quad f(\mathbf{e}_j) := ||\mathbf{e}_j||_1 \tag{13a}$$

$$\text{subject to} \quad \widehat{R}_j = \widehat{r}_j^2 \tag{13b}$$

$$\widehat{X}_j = \widehat{x}_j^2 \tag{13c}$$

$$\widehat{r}_j = \sum_{h=1}^{H} \lambda_h \alpha_h \widehat{x}_j \tag{13d}$$

$$\sum_{h=1}^{H} \alpha_h = 1, \ \alpha_h \in \{0, 1\}, \ \forall h. \tag{13e}$$

The Big-M technique is exploited to linearize the bilinear term $\alpha_h x_j$ as,

$$-M_j(1 - \alpha_h) \leq \widehat{r}_j - \lambda_h \widehat{x}_j \leq M_j(1 - \alpha_h), \ \forall z \tag{14}$$

where $M_j$ is a large real number.

While (13) can be handled by some general MINLP solvers, there is no guarantee of global optimality since its continuous relaxation counterpart is still non-convex due to the quadratic constraints (13 b) and (13 c). Therefore, in what follows, we will discuss the convexification.

To make the optimization model tractable, we first rewrite the cost function in an equivalent form without $\mathcal{L}_1$-norm operator by introducing the auxiliary variables $\theta_j^{(1)},\ldots,\theta_j^{(K)}$:

$$f(\theta_j^{(1)},\ldots,\theta_j^{(K)})=\sum_{k=1}^{K}\theta_j^{(k)} \tag{15}$$

with the additional constraints,

$$\theta_j^{(k)}\geq e_j^{(k)},\ -\theta_j^{(k)}\leq e_j^{(k)},\ \forall k. \tag{16}$$

To tackle the non-convex quadratic equalities (13 b) and (13 c), we propose to convexify them via SDP relaxation. We first rewrite (13 b) and (13 c) as,

$$\mathbf{W}_j^r:=\begin{bmatrix}1 & \widehat{r}_j\\ \widehat{r}_j & \widehat{R}_j\end{bmatrix}\succeq 0,\ \mathrm{rank}\left\{\mathbf{W}_j^r\right\}=1,\ \forall j \tag{17a}$$

$$\mathbf{W}_j^x:=\begin{bmatrix}1 & \widehat{x}_j\\ \widehat{x}_j & \widehat{X}_j\end{bmatrix}\succeq 0,\ \mathrm{rank}\left\{\mathbf{W}_j^x\right\}=1,\ \forall j. \tag{17b}$$

Then, removing the rank-1 constraints in (17), a MISDP model whose continuous relaxation is a convex SDP, is given by,

$$\underset{\alpha_h,r_j,x_j,R_j,X_j,\theta_j^{(k)}}{\text{minimize}}\quad \sum_{k=1}^{K}\theta_j^{(k)} \tag{18a}$$

$$\text{subject to}\quad \theta_j^{(k)}\geq e_j^{(k)},\forall k \tag{18b}$$

$$-\theta_j^{(k)}\leq e_j^{(k)},\forall k \tag{18c}$$

$$\mathbf{W}_j^r\succeq 0 \tag{18d}$$

$$\mathbf{W}_j^x\succeq 0 \tag{18e}$$

$$\text{(13e) and (14).} \tag{18f}$$

In this way, it can be handled by MISDP solvers. The following proposition provides a sufficient condition that guarantees the SDP relaxation is exact while the estimation is error-free.

*Proposition 2:* Let $\mu:=[r_j,x_j,r_j^2,x_j^2]^T$. If $\mu$ is the optimal solution of (18), then the SDP relaxation is exact and the estimation is exact.

The proof is provided in the appendix. Proposition 2 implies that if the measurements are error-free, such sufficient condition naturally holds because $f(\mu)=\inf f=0$. If the measurements are erroneous but the errors do not affect the optimal solution (i.e., such sufficient condition still holds), the relaxation is still exact. Furthermore, if the errors are so large that the resultant optimal solution of (18) is no longer equal to $\mu$, it is still possible that such relaxation is exact but it depends on the properties of samples.

### B. Bottom-Up Sweep Algorithm

Clearly, developing (18) requires the knowledge of voltage magnitude and line flow values. Unfortunately, due to the low coverage of line flow sensors, there are few line flow measurements available. Exceptions are the *tail* branches since they have

**Algorithm 2:** Bottom-Up Sweep Algorithm.

**Initialization**: Initialize $d\leftarrow D$.
**repeat**
[**S1**]: Update the $P_j^{(k)}$ and $Q_j^{(k)}$ by (1 a) and (1 b) for all $k=1,\ldots,K$ and $j$ in layer $d$.
[**S2**]: Calculate $r_j,x_j$ of each line segment by solving (18) for all $j$ in layer $d$.
[**S3**]: Calculate $\bar{P}_j^{(k)}$ and $\bar{Q}_j^{(k)}$ as per (1 c) and (1 d) for all $k=1,\ldots,K$ and $j$ in layer $d$.
[**S4**]: Update $d\leftarrow d-1$.
**until** $d=0$.

no further downstream neighbors, and thus the line flows physically equal the power injections at the leaf buses, which can be measured by SMs. Moreover, as per (1 a)–(1 b), *the line flow over a given branch can be calculated, provided all of its neighboring downstream line flows have been known.* These facts motivate the design of a bottom-up (a.k.a. leaf-to-root) sweep algorithm that manipulates the line flow and line impedance estimation in an alternating way.

We first partition a radial distribution network into multiple layers which are labeled as $1,\ldots,D$ where $D$ is the maximum *depth* [see Fig. 2(a) for an example with $D=4$]. Physically, bus $j$ belongs to layer $d$" means there are $d$ intermediate line segments in the path from bus $j$ to the root bus 0. As stated in Section II, for a radial network, there is a unique path from any bus $j$ to the root bus 0. Therefore, the partition of layers is unique as well. The bottom-up sweep algorithm with the breadth-first search is tabulated as Algorithm 2.

## V. Numerical Results

In this section, the proposed topology and parameter identification methods are verified on the modified IEEE 13, 37 and 69-bus test feeders, which are depicted in Fig. 2. We have utilized the real SM data from our utility partners in Midwest to replace the load data of these benchmark systems. More precisely, the available customer power measurements with 1-h resolution are aggregated at secondary transformer level by summing them at different times. The power flow analysis takes as input these distribution system models and the nodal load demand time-series. The computed nodal voltages are treated as the voltage measurements, along with the load time-series, used for topology and parameter identification. In this work, the length of the time window is 200 samples. Following the previous works [37], the minimum number of neighbors, $\gamma$, and the radius $\xi$ in topology recovery are assigned as 2 and 3, respectively. The optimization programs are solved by YALMIP Toolbox in MATLAB, along with the solver `MOSEK` [41].

### A. Results of Topology Identification

The topology identification results of the modified IEEE 13-bus, 37-bus and 69-bus test feeders are depicted in Fig. 3. For data visualization, the min-max normalization [42] is utilized to rescale the entries of $[\widehat{\mathbf{Y}}]_i$ to be within [0,1] for all $i$.

The upper part of Fig. 3 illustrates the *rescaled* variant of matrices $\widehat{\mathbf{Y}}$ of each test feeder. Then, by performing Algorithm 1, the estimated connectivity between any two buses of each test feeder is obtained (see the bottom part of Fig. 3 where "0" denotes "unconnected" and "1" denotes "connected"). The performance is validated by comparing the estimated connectivity and the real connectivity. In this work, the proposed method achieves a 100% accurate topology recovery for all the three distribution feeders. Note that, this verifies our method's robustness against heterogeneous R/X ratios.

### B. Results of Line Parameter Estimation

The line parameter estimation results of the modified IEEE 13-bus, 37-bus and 69-bus test feeders are depicted in Fig. 4. As can be seen from Fig. 4, the SDP-based LAD model precisely recovers the line impedance of each branch, under all the three test cases. In terms of $r_j$, the largest relative errors (among all branches) are $3.33 \times 10^{-5}\%$, $3.40 \times 10^{-4}\%$ and $1.44 \times 10^{-4}\%$ for the modified IEEE 13-bus, 37-bus and 69-bus test feeders, respectively; and as for $x_j$, the largest relative errors are $3.33 \times 10^{-5}\%$, $3.40 \times 10^{-4}\%$ and $7.06 \times 10^{-5}\%$, respectively.

To quantify the exactness of SDP relaxation in (18), that is how close are the matrices $\mathbf{W}_j^r$ and $\mathbf{W}_j^x$ to rank one, one can compute the ratio between their largest two eigenvalues, i.e., $\sigma_2(\mathbf{W})/\sigma_1(\mathbf{W})$. The maximum values of $\sigma_2(\mathbf{W}^r)/\sigma_1(\mathbf{W}^r)$ among all branches are $6.77 \times 10^{-11}$, $6.33 \times 10^{-10}$ and $1.49 \times 10^{-10}$ for the 13-bus, 37-bus and 69-bus test feeders; and the maximum values of $\sigma_2(\mathbf{W}^x)/\sigma_1(\mathbf{W}^x)$ among all branches are $6.20 \times 10^{-11}$, $9.44 \times 10^{-10}$ and $1.47 \times 10^{-10}$, respectively. It is demonstrated that the SDP relaxation is exact.

## VI. Discussions

### A. Robustness

For topology identification, given that the nonlinearity of power flow is dropped in the regression model, it is unlikely to solve $\widehat{\mathbf{Y}}$ to be exactly equal to the true $\mathbf{Y}$. Yet, in fact, it does *not* require an exact estimation, because the topology is only sensitive to the structural feature of the matrix rather than its exact value. Such a feature makes the proposed method robust against imperfect data to a certain extent.

For line parameter estimation, on the one hand, the conductor library can reduce the solution space. This may enhance the numerical stability of the regression model, and reduces the impact of data quality issues. On the other hand, the proposed bottom-up sweep algorithm is inherently robust because the estimation errors regarding downstream branches only affect the line losses, which slightly contributes to the upstream-end line flows. It is therefore expected that the effects of estimation errors can asymptotically diminish.

### B. Efficiency and Scalability

Besides the tractable fitting model of $\mathbf{Y}$, the density-based clustering method used for extracting topology from $\widehat{\mathbf{Y}}$ is also efficient since this method scans the whole dataset only one time. Further, we have applied an indexing structure that executes a neighboring query in $O(\log n)$. Consequently, the computational complexity of this anomaly detection is $O(n \log n)$ [37]. In our tests, recovering the topology from the estimated weighted Laplacian matrix can be done in a few seconds.

The line impedance estimation method has good scalability. Via SDP relaxation, the LAD regression model (18) can be easily handled by off-the-shelf solvers (solved in milliseconds in our tests). The optimization model is designed and solved in a branch-wise manner whose computation burden does not grow with network size [the computation burden of (18) is only related to the number of samples and the size of library]. Besides, the sweep algorithm only requires very simple algebraic operations for line flow computation, which scales well with the network size as well. Therefore, the total computation burden for parameter estimation is approximately linear with the network size.

The high computational efficiency and good scalability enable a real-time application of the proposed method after some system changes (e.g., network reconfiguration).

## VII. Conclusion

In this paper, we propose a data-driven framework to accurately and efficiently find the connectivity of different nodes in entire or partial networks using SM data. The proposed topology identification establishes on reconstructing the weighted Laplacian matrix of a homogeneous distribution circuit, which also exhibits provable robustness against heterogeneous R/X ratios. The mixed-integer nonlinear LAD regression model for parameter identification is developed and convexified. We then embed it in a bottom-up sweep algorithm to achieve the line parameter estimation across the whole network. The test results validate the effectiveness and accuracy of the proposed methods.

At present, this work only focuses on balanced radial distribution grids. In future studies, the proposed method will be generalized to unbalanced and/or meshed grids, with the help of limited available $\mu$PMU data on a few critical nodes. Moreover, the proposed method will be enhanced for better robustness against heterogeneous R/X ratios and various data quality problems.

## Appendix

### A. Proof of Proposition 1

Let $\mathbf{B} := \mathbf{A}^{-1}$. First, as per the linear algebra theory, $\mathbf{I}_n \to \mathbf{B}$ can be accomplished via the elementary row operations, by which, in turn, one can exactly achieve $\mathbf{A} \to \mathbf{I}_n$. Second, given that $\mathbf{A}$ is the reduced incidence matrix of a tree-topology network, we have $a_{ij} = -1$ if $i = j$, $a_{ij} = 1$ if $i \in \mathcal{P}_j \backslash \{j\}$ and otherwise, $a_{ij} = 0$. To achieve $\mathbf{A} \to \mathbf{I}_n$, one has to add the rows $j$ for all $j \in \{j | i \in \mathcal{P}_j\}$ onto the row $i$, and then multiply row $i$ by $-1$. Accordingly, one can obtain $\mathbf{B} := [b_{ij}]_{n \times n}$ with

$$b_{ij} = \begin{cases} -1, & \text{if } i \in \mathcal{P}_j \text{ or } i = j \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

And then, we have

$$a_{ik} \cdot b_{kj} = \begin{cases} 1, & \text{if } k \in \{i\} \cap \mathcal{P}_j \\ -1, & \text{if } k \in \mathcal{C}_i \cap \mathcal{P}_j \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

which indicates it is non-zero if and only if $i \in \mathcal{P}_j$. Therefore,

$$\begin{aligned} \delta_{ij} &= \mathbf{A}_i \mathrm{diag}(\Delta\lambda_1, \ldots, \Delta\lambda_n)\mathbf{B}^j = \sum_{k=1}^{n} a_{ik}\Delta\lambda_k b_{kj} \\ &= \begin{cases} \Delta\lambda_i - \Delta\lambda_{\mathcal{C}_i \cap \mathcal{P}_j}, & \text{if } i \in \mathcal{P}_j \backslash \{j\} \\ \Delta\lambda_i, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{21}$$

for any $i, j \in \mathcal{N}$. ■

### B. Proof of Proposition 2

Let $f^\star_{\mathrm{NLP}}$ and $f^\star_{\mathrm{SDP}}$ be the optimal cost function value of (13) and (18). Obviously, it follows that $f^\star_{\mathrm{NLP}} \geq f^\star_{\mathrm{SDP}}$.

It is observed that $\mu$ is a feasible solution of (13). If $f^\star_{\mathrm{SDP}} = f(\mu)$ holds, we have

$$f(\mu) \geq f^\star_{\mathrm{NLP}} \geq f^\star_{\mathrm{SDP}} = f(\mu). \tag{22}$$

This yields $f^\star_{\mathrm{NLP}} = f^\star_{\mathrm{SDP}} = f(\mu)$, and therefore, the relaxation is exact while the estimation is exact. ■

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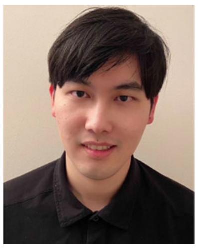
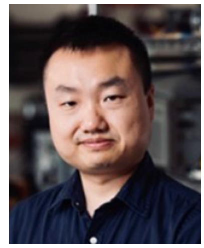


**Yifei Guo** (Member, IEEE) received the B.E. and Ph.D. degrees in electrical engineering from Shandong University, Jinan, China, in 2014 and 2019, respectively. He is currently a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering, Iowa State University, Ames, IA, USA. During 2017–2018, he was a Visiting Student with the Department of Electrical Engineering, Technical University of Denmark, Lyngby, Denmark.

His research interests include distribution system modeling, optimization, and control.

**Yuxuan Yuan** (Graduate Student Member, IEEE) received the B.S. degree in 2017 in electrical & computer engineering from Iowa State University, Ames, IA, USA, where he is currently working toward the Ph.D. degree. His research interests include distribution system state estimation, synthetic networks, data analytics, and machine learning.



**Zhaoyu Wang** (Senior Member, IEEE) received the B.S. and M.S. degrees in electrical engineering from Shanghai Jiao Tong University, Shanghai, China, and the M.S. and Ph.D. degrees in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, GA, USA. He is currently the Northrop Grumman Endowed Associate Professor with Iowa State University. His research interests include optimization and data analytics in power distribution systems and microgrids. He was the recipient of the National Science Foundation CAREER Award, the Society-Level Outstanding Young Engineer Award from IEEE Power and Energy Society (PES), the Northrop Grumman Endowment, College of Engineering's Early Achievement in Research Award, and the Harpole-Pentair Young Faculty Award Endowment. He is the Principal Investigator for a multitude of projects funded by the National Science Foundation, the Department of Energy, National Laboratories, PSERC, and Iowa Economic Development Authority. He is the Chair of IEEE PES PSOPE Award Subcommittee, the Co-Vice Chair of PES Distribution System Operation and Planning Subcommittee, and the Vice Chair of PES Task Force on Advances in Natural Disaster Mitigation Methods. He is an Associate Editor for the IEEE Transactions on Power Systems, IEEE Transactions on Smart Grid, IEEE Open Access Journal of Power and Energy, IEEE Power Engineering Letters, and *IET Smart Grid*.