EE 303 Energy Systems and Power Electronics

Introduction to Power Basics

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Phasor

Complex Number: $\boldsymbol{Z = a + jb}$

- Exponential Form: $Z = |Z|\,.\,e^{j\theta}$;

  where, $|Z| = \sqrt{a^2 + b^2}$,

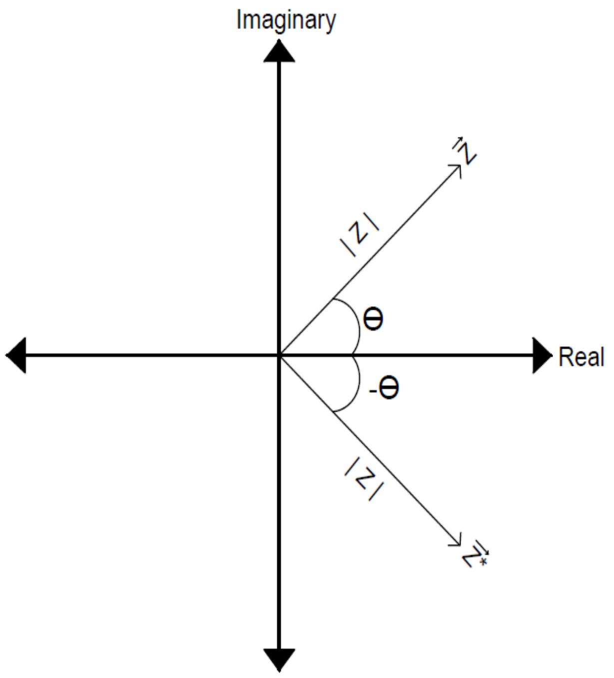
  $\theta = \arctan\frac{b}{a}$,

  $e^{j\theta} = \cos\theta + \sin\theta$

- Polar Form: $Z = |Z|\measuredangle\theta$

- Rectangular Form: $Z = |Z|\cos\theta + j|Z|\sin\theta$

Conjugation: $Z^* = a - jb$

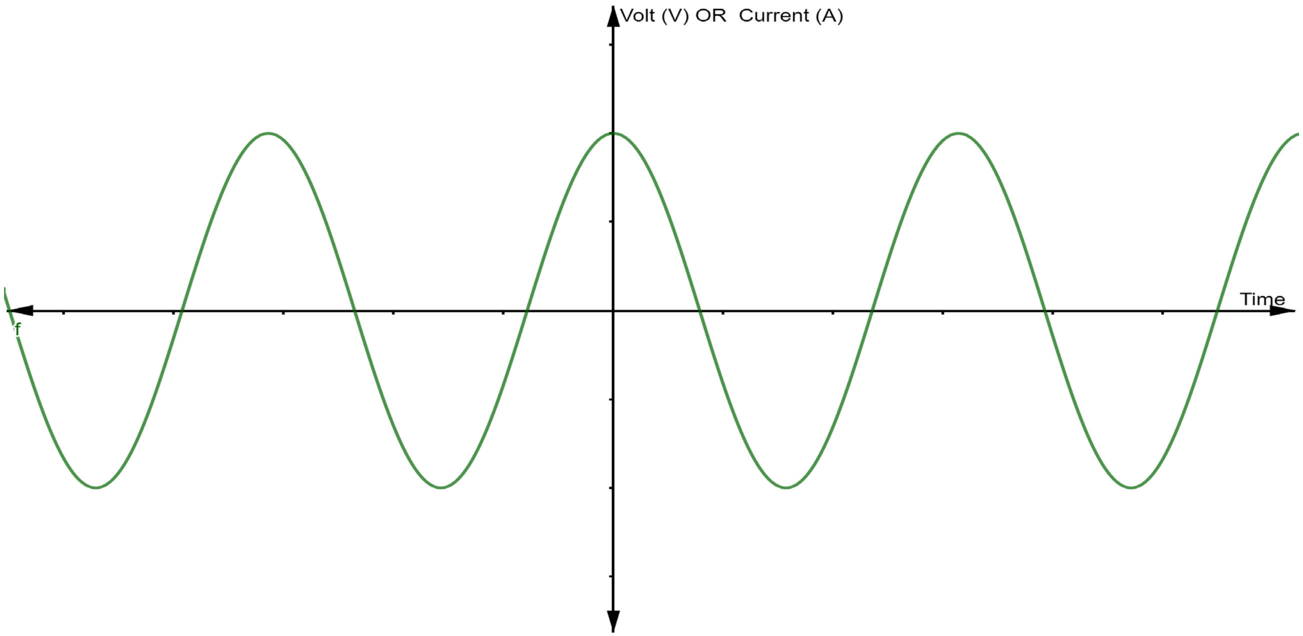


Mathematical Operations

$$Z_1 + Z_2 = a_1 + jb_1 + a_2 + jb_2$$
$$= (a_1 + a_2) + j(b_1 + b_2)$$

$$Z_1 . Z_2 = (a_1 + jb_1).(a_2 + jb_2)$$
$$= (a_1 a_2 - b_1 b_2) + j\big((a_1 a_2 + b_1 b_2\big)$$
$$= |Z_1|\, e^{j\theta_1} . |Z_2|\, e^{j\theta_2} = |Z_1|\, |Z_2|\, e^{j(\theta_1 + \theta_2)}$$

Time Domain



$$V(t) = V_{max}\cos(\omega t + \theta_V)$$

$I(t) = I_{max}\cos(\omega t + \theta_I)$; where,

$$\omega = 2\pi f; f\ is\ 60Hz$$

Time Domain VS Phasor Domain

Objective: Phasor Domain Analysis for simplifying calculations

Assumption:

- Constant frequency system
- Sine Wave

Time Domain Representation

- We discussed in Lecture 1, the Time domain representation of voltage and current are,

$$V(t) = V_{max}\cos(\omega t + \theta_V)$$
$$\mathrm{I}(t) = I_{max}\cos(\omega t + \theta_I)$$

- Root Mean Square (RMS) of a sine signal is,

$$|V| = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt} = \frac{V_{max}}{\sqrt{2}};\ \ where\ T = \frac{1}{f}$$

Contd…

- $V(t) = \sqrt{2}\,|V|\cos(wt + \theta_V)$
- $I(t) = \sqrt{2}\,|I|\cos(wt + \theta_I)$ ……………………………… (i)

- $V(t) = \sqrt{2}\,|V|\,\mathrm{Re}\,[e^{j(\omega t+\theta_V)}]$
- $I(t) = \sqrt{2}\,|V|\,\mathrm{Re}\,[e^{j(\omega t+\theta_I)}]$ ……………………………… (ii)

Note: [For conversion of equation (i) to (ii)]

As, $e^{j\theta} = \cos\theta + j\sin\theta$

$$\therefore\; e^{j(\omega t+\theta_V)} = \underbrace{\cos(\omega t + \theta_V)}_{\text{Real Part}} + j\,\underbrace{\sin(\omega t + \theta_V)}_{\text{Imaginary Part}}$$

Phasor Domain Representation

Voltage and Current can be represented in phasor domain as,

- $V = |V|e^{j\theta_V} = |V|\angle\theta_V = |V|\cos\theta_V + j\,|V|\sin\theta_V$
- $\mathrm{I} = \underbrace{|I|e^{j\theta_I}}_{\text{Exponential}} = \underbrace{|I|\angle\theta_I}_{\text{Polar}} = \underbrace{|I|\cos\theta_I + j\,|V|\sin\theta_I}_{\text{Rectangular}}$

Representation of Electrical Components

Component	Time Domain	Phasor Domain
Resistor	$V(t) = R.i(t)$	$V = R.I$
Inductor	$V(t) = L.\frac{d\,i(t)}{dt}$	$V = j\omega L.I$
Capacitor	$V(t) = \frac{1}{C}\int_0^t i(t)dt + V(0)$	$V = \frac{1}{j\omega C}.I$

Impedance

'Z' denotes the impedance.

Mathematically,

$$Z = R + jX, \text{ where } R = \textit{resistance and}$$

$$X = \textit{reactance}$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

<u>**Note:**</u>

Here, $X_L = \omega L$ and is called 'Inductive Impedance'

and, $X_C = \frac{1}{\omega C}$ and is called 'Capacitive Impedance'

Contd..

Expression of Z in polar form.

$$\text{As, } Z = R + jX$$

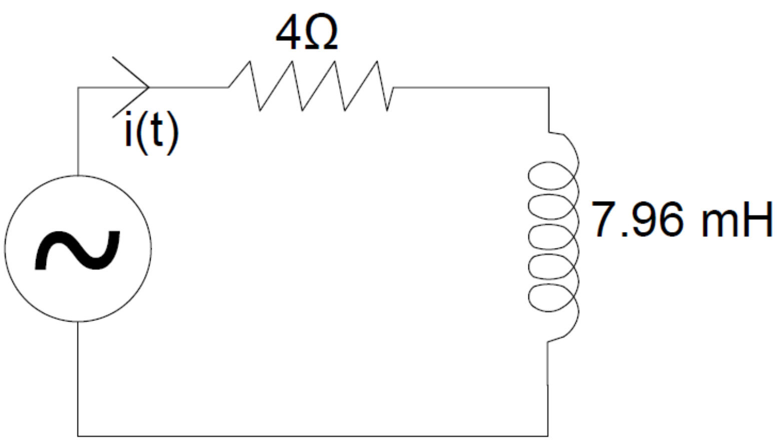
$$\text{In Polar Form, } Z = |Z| \angle \phi \text{ ; where, } |Z| = \sqrt{R^2 + X^2}$$

$$\text{and } \phi = \arctan(\frac{X}{R})$$

- The reciprocal of Impedance is termed as admittance.

$$\text{i.e. } Y = \frac{1}{Z}$$

Example 1: Find i(t)



The voltage source in the figure shown is defined as $V(t)$ where,

$$V(t) = \sqrt{2}\,100\ \cos(\omega t + 30)$$

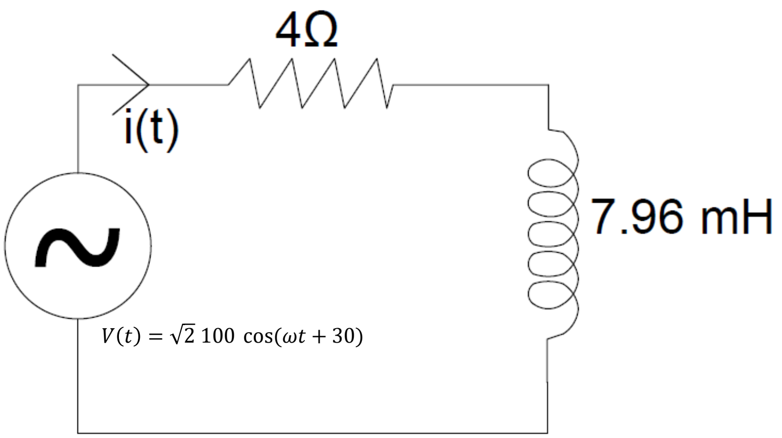
Find i(t).

Solution

Given,

R = 4Ω,
X = $\omega L = 2\Pi f L = 3$

$\therefore Z = 4 + j3$



$|Z| = \sqrt{4^2 + 3^2} = 5$ and

$\phi = \arctan\left(\frac{3}{4}\right) = 36.9^o$

Now, I = $\frac{V}{Z} = \frac{100\angle 30}{5\angle 36.9} = 20\angle -6.9^o$ A

$\therefore i(t) = 20\sqrt{2}\cos(\omega t - 6.9)$

Complex Power

$$Power(t) = V(t).I(t) \quad \text{where, } V(t) = V_{max}\cos(\omega t + \theta_V)$$
$$I(t) = I_{max}\cos(\omega t + \theta_I)$$

$$\left[\because \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]\right]$$

$$\Rightarrow power(t) = \frac{1}{2} V_{max} I_{max}[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

$$\Rightarrow power(t) = \frac{1}{2} V_{max} I_{max}[\cos(\theta_V - \theta_I) + \cos(2(\omega t + \theta_V) - (\theta_V - \theta_I))]$$

$$\Rightarrow power(t) = \frac{1}{2} V_{max} I_{max}\cos(\theta_V - \theta_I)\{1 + \cos[2(\omega t + \theta_V)]\}$$
$$+ \frac{1}{2} V_{max} I_{max}\sin(\theta_V - \theta_I)\sin[2(\omega t + \theta_V)]\}$$

$$\left[\because \cos(\alpha-\beta) = \cos\alpha\, cos\beta + \sin\alpha\sin\beta\right]$$

Contd…

- Average Power $= \frac{1}{2}\int_0^T power(t)\,\text{dt}$

$$= \frac{1}{2} V_{max} I_{max} \cos(\theta_V - \theta_I)$$

$$= |V||I| \cos\underbrace{(\theta_V - \theta_I)}_{\text{Power factor Angle } (\phi)}$$

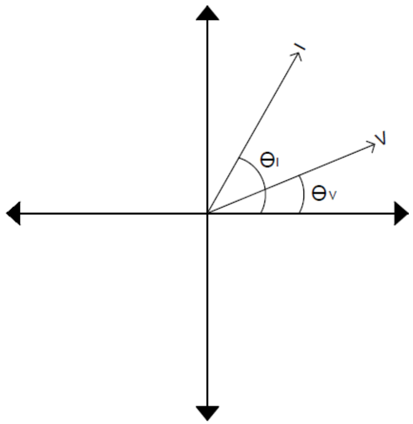
✓ Power factor (pf) = cos ϕ   [pf lies between 0 and 1]

✓ Average power is also referred as active power.

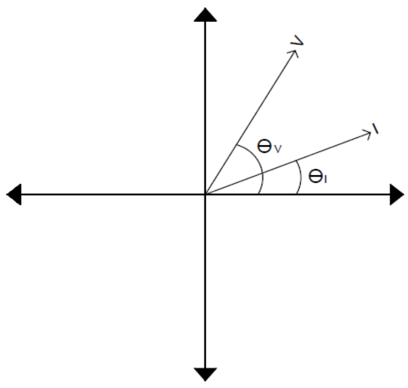
Contd…

Note:

1. If current leads voltage, $\theta_V - \theta_I < 0$ then pf is leading.



2. If current lags voltage, $\theta_V - \theta_I > 0$ then pf is lagging.



Contd…

- Active Power (P)= $|V||I|\cos(\theta_V - \theta_I)$ (Watts)
- Reactive Power (Q))= $|V||I|\sin(\theta_V - \theta_I)$ (VAr)

Complex Power, S = P+jQ (Unit of S is usually VA, kVA)

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$

$$= V.I^*$$

$$= |V|\angle\theta_V . |I|\angle -\theta_I$$

$$= |V||I|\angle(\theta_V - \theta_I)$$

Contd…

- Magnitude of Apparent Power:

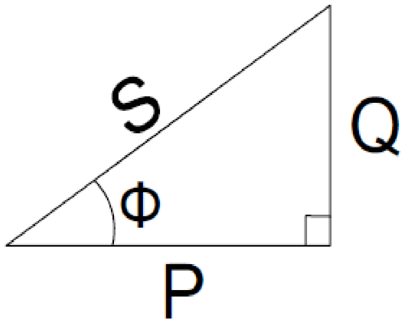
$$|S| = |V|.|I| = \sqrt{P^2 + Q^2}$$

The real power, reactive power and apparent power can be represented as shown in the figure. The figure is called power triangle.

From Power Triangle, we can get,

$$P = S.\cos\phi$$

$$Q = S.\sin\phi = \pm S\sqrt{1 - pf^2}$$



Example 2:

A load draws 100kW with leading pf of 0.85. What is ϕ, Q and S?

Solution:

Contd…

Q. A load draws 100kW with leading pf of 0.85. What is ϕ, Q and S?

Solution:

- $\Phi = -\cos\,(0.85) = -31.8^0$
- Apparent Power (S) = $\frac{P}{\cos\Phi} = 117.6$ Kva
- Reactive Power (Q) = $S.\sin\Phi = -62kVAr$
- Complex Power (S) = $P + jQ = 100 - j62$

Review

- Voltage Phasor: $V = |V|\angle\theta_V = |V|\cos\theta_V + j\,|V|\sin\theta_V$

- Complex Power:

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + jQ$$
$$= |V||I|^*$$

P is Real Power/ Active Power, Units – W, kW, µW

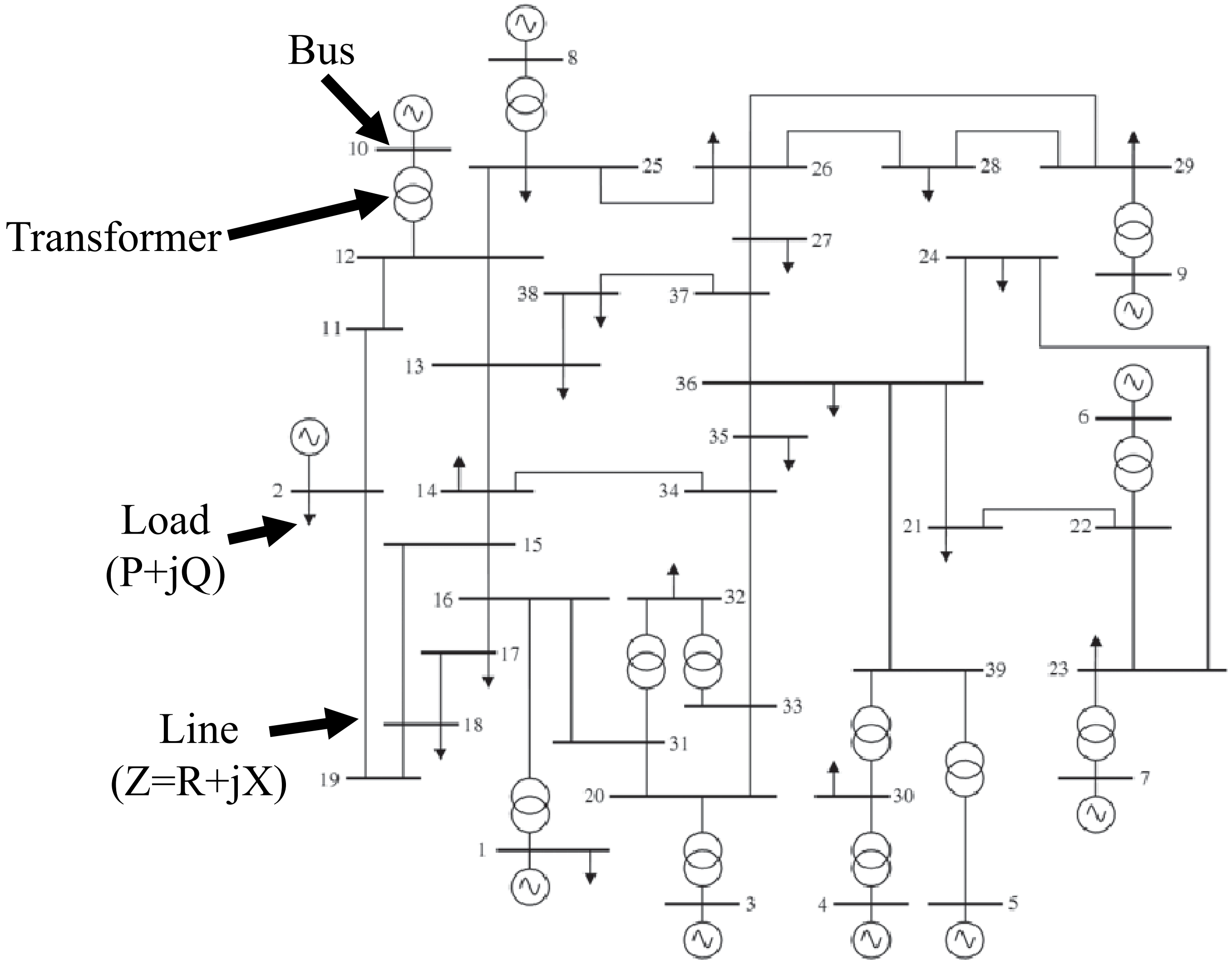
Q is Reactive Power, Units – VAr, kVAr, µVAr

S is Complex Power, Units – VA, kVA, Mva

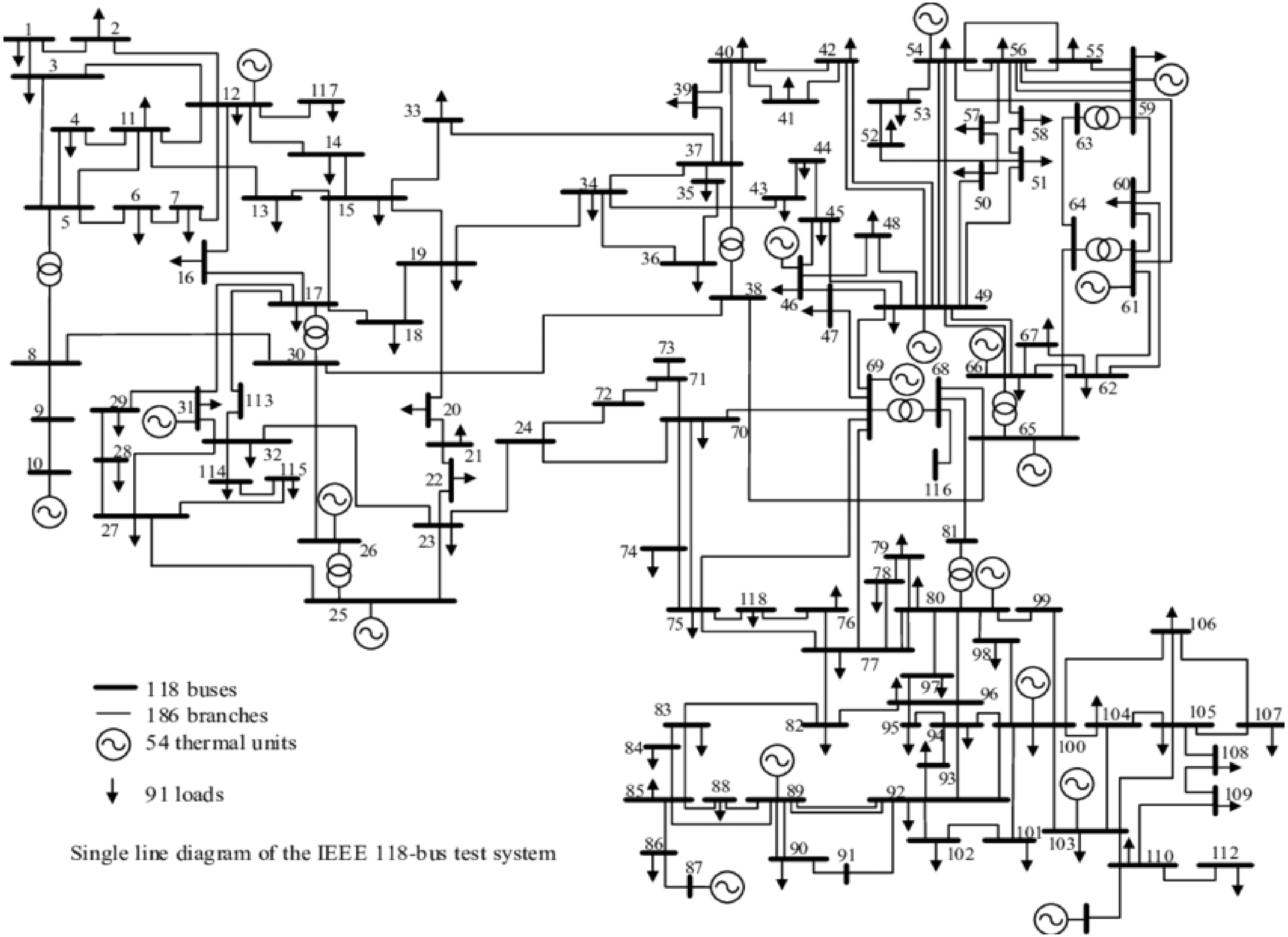
Contd…

- Power factor Angle: $\phi = \theta_V - \theta_I$
- Power factor (pf): $\cos\phi$ (value ranges from 0 to 1)
- Leading pf: current angle leads voltage angle ($\theta_V - \theta_I < 0$).
- Lagging pf: current angle lags voltage angle ($\theta_V - \theta_I > 0$).

39 – Bus Test System



118 – Bus Test System

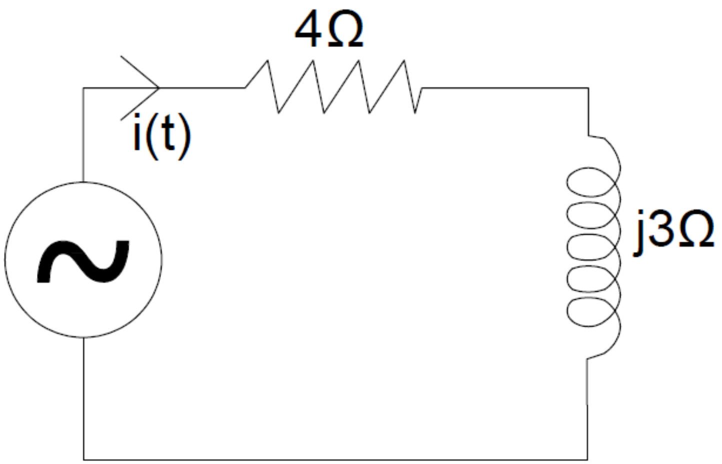


Single line diagram of the IEEE 118-bus test system

Example 1:

The voltage source in the figure shown is defined as $V(t)$ where,

$$V(t) = 100\angle 30^0$$



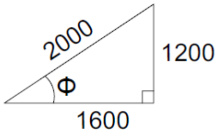
Solution

We know,

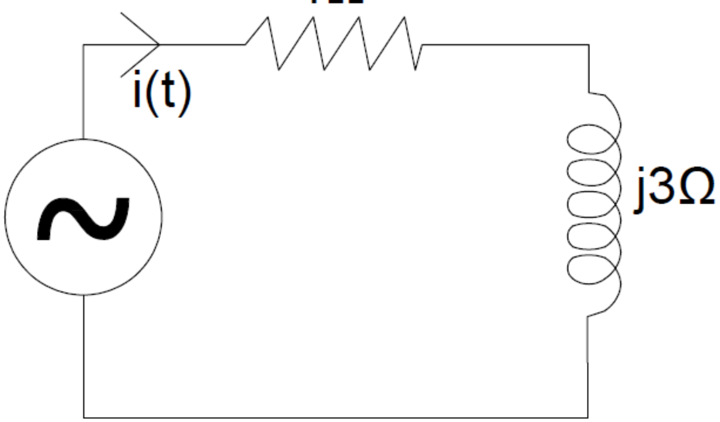
$$I = \frac{V}{Z} = \frac{100\angle 30}{4+j3} = 20\angle -6.9^{o}\ A$$

$$S = V.I^{*} = 100\angle 30^{o} * 20\angle 6.9^{o} = 2000\angle 36.9^{o}$$
$$= 2000\angle 36.9^{o}\ VA = 1600\ W + j1200\ VAr$$

$$\phi = \arctan\left(\frac{1200}{1600}\right) = 36.9^{o} \Rightarrow$$



$$\cos\phi = pf = 0.8\ (\text{lagging})$$



Now,

$$S_R = V_R\,.I^{*} = 4 * 20\angle -6.9^{o} * 20\angle 6.9^{o}$$
$$= 1600\ W \longrightarrow |I^2|.R$$

$$S_X = V_L\,.I^{*} = 3j * 20\angle -6.9^{o} * 20\angle 6.9^{o}$$
$$= 1200\ VAr \longrightarrow |I^2|.X$$

Contd…

Notes:

- Inductors only **consume** reactive power.

$$Q_{inductor} = |I_{inductor}|^2 . X_L$$

where, $X_L = j\omega \text{L}$

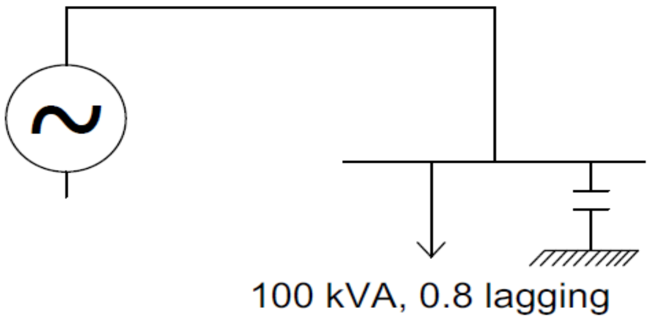
- Capacitors only **generate** reactive power.

$$Q_{capacitor} = \left|I_{capacitor}\right|^2 . X_C$$

where, $X_C = \frac{1}{j\omega C} = -\frac{j}{\omega c}$

Example 2:

Assume a load 100 kVA with pf=0.8 lagging, where as correct pf is 0.95 lagging. Find the value of reactive power that must be injected to maintain the correct pf.



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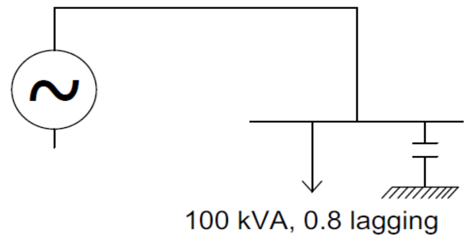
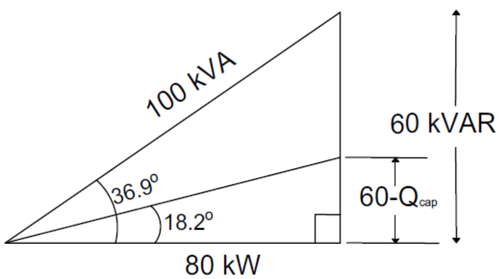


$S = P+jQ = 80+j60$

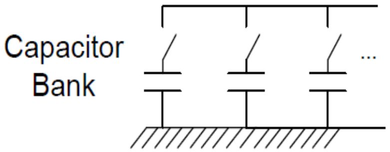
$\phi = arc\cos 0.8 = 36.9^o$

$\phi = arc\cos 0.95 = 18.2^o$

$$\tan 18.2^o = \frac{60 - Qcap}{80}$$



$Q_{cap} = 33.7$ kVAr



Thank You!