Iowa State University

ECpE Department

EE 303 Energy Systems and Power Electronics

Three Phase Systems

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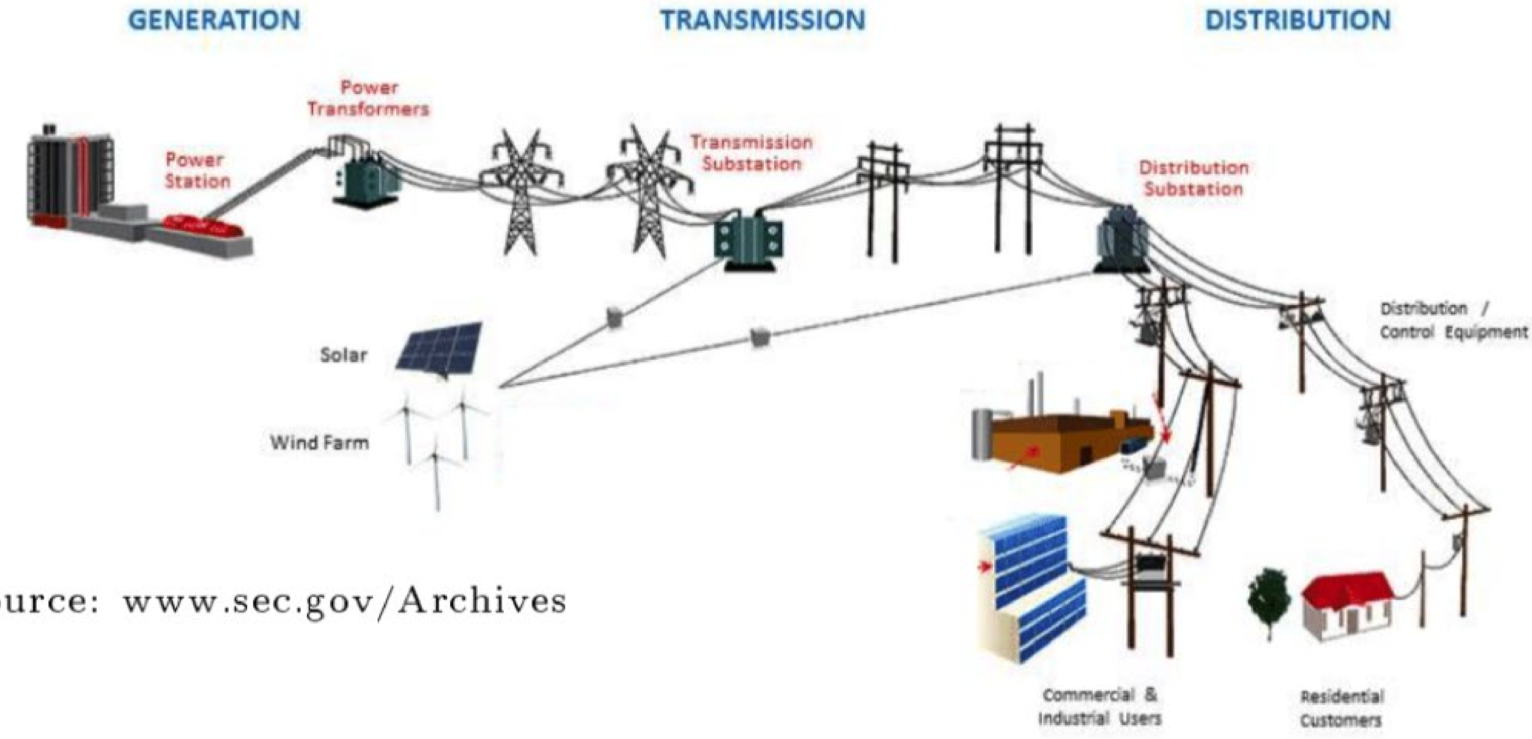
Three Phase Systems

High-power equipment such as generators, transformers, and transmission lines are built as three phase equipment. However, at the distribution level, depending on the voltage/power rating, a mixture of single phase and three-phase systems is used.

The three-phase system has many advantages over the single-phase system.

1. Three-phase systems produce a rotating magnetic field inside the alternating current (ac) motors and, therefore, cause the motors to rotate without the need for extra controls.
2. Three-phase generators produce more power than single-phase generators of equivalent volume.
3. Three-phase transmission lines transmit three times the power of single-phase lines.
4. Three-phase systems are more reliable; when one phase is lost, the other two phases can still deliver some power to the loads



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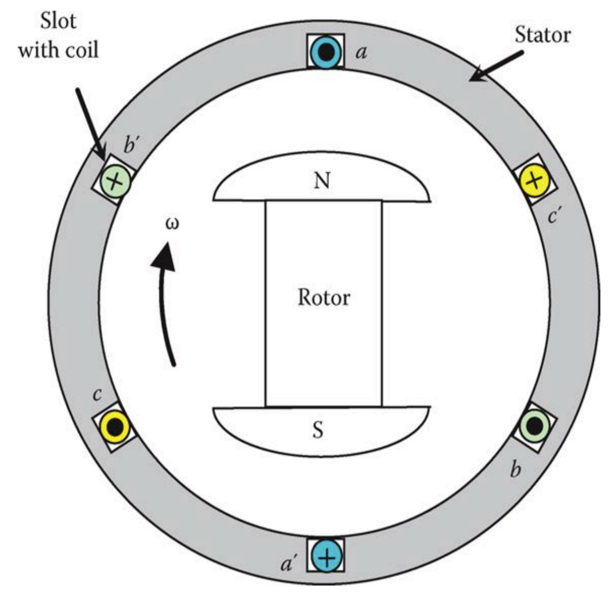
Generation of Three-Phase Voltages

- According to Faraday's law, when a conductor cuts magnetic field lines, a voltage is induced across the conductor. The synchronous generator uses this phenomenon to produce a three-phase voltage.

- The generator consists of an outer frame called **stator** and a rotating magnet called **rotor**.

- At the inner perimeter of the stator, coils are placed inside slots. Each of the two slots separated by 180° houses a single coil (a–a′, b–b′, or c–c′). The coil is built by placing a wire inside a slot in one direction (e.g., a), and winding it back inside the opposite slot (a′).

A three-phase generator.



Generation of Three-Phase Voltages

- When the magnet is spinning clockwise inside the machine by an external prime mover, the magnetic field then cuts all coils and, therefore, induces a voltage across each of them.
- If each coil is connected to a load impedance, a current would flow into the load, and the generator produces electric energy that is consumed by the load.
- The dot inside the coil indicates a current direction toward the reader, and the cross indicates a current in the opposite direction.

The voltage across any of the three coils can be expressed by Faraday's law

$$e = nBl\omega$$

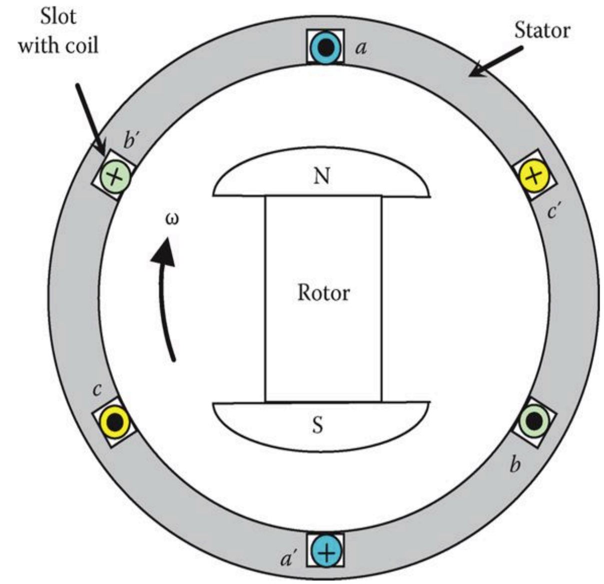
Where,

e is the voltage induced across the coil
n is the number of turns in the coil
B is the flux density of the magnetic field
I is the length of the slot
ω is the angular speed

Generation of Three-Phase Voltages

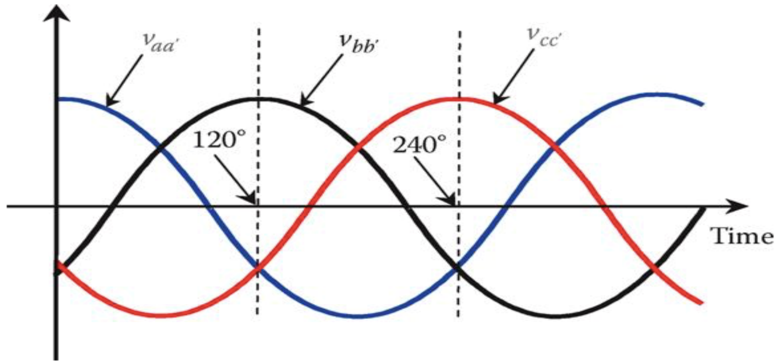
- The voltage induced on a conductor is proportional to the perpendicular component of the magnetic field with respect to the conductor.
- At the rotor position in Figure, coil a–a′ has the maximum perpendicular field and, hence, it has the maximum induced voltage. Coil b–b′ will have its maximum voltage when the rotor moves clockwise by 120°, and coil c–c′ will have its maximum voltage when the rotor is at 240° position.
- If the rotation is continuous, the voltage across each coil is sinusoidal, all coils have the same magnitude of the maximum voltage, and the induced voltages are shifted by 120°.

A three-phase generator.



Generation of Three-Phase Voltages: Mathematical Expression

Waveforms of the three-phases of the generator.



$v_{aa'}$ as reference,

$$v_{aa'} = V_{max}cos(\omega t)$$
$$v_{bb'} = V_{max}cos(\omega t - 120°)$$
$$v_{cc'} = V_{max}cos(\omega t - 240°) = V_{max}cos(\omega t + 120°)$$

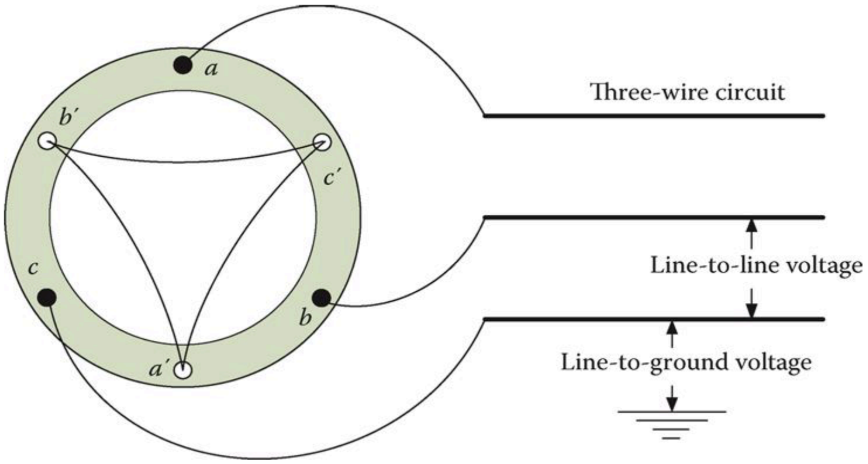
$$\bar{V}_{aa'} = \frac{V_{\max}}{\sqrt{2}}\angle 0° = V\angle 0°$$
$$\bar{V}_{bb'} = \frac{V_{\max}}{\sqrt{2}}\angle -120° = V\angle -120°$$
$$\bar{V}_{cc'} = \frac{V_{\max}}{\sqrt{2}}\angle 120° = V\angle 120°$$

Where,
$\bar{V}$ is the phasor voltage in complex number form
V is the magnitude of the rms voltage

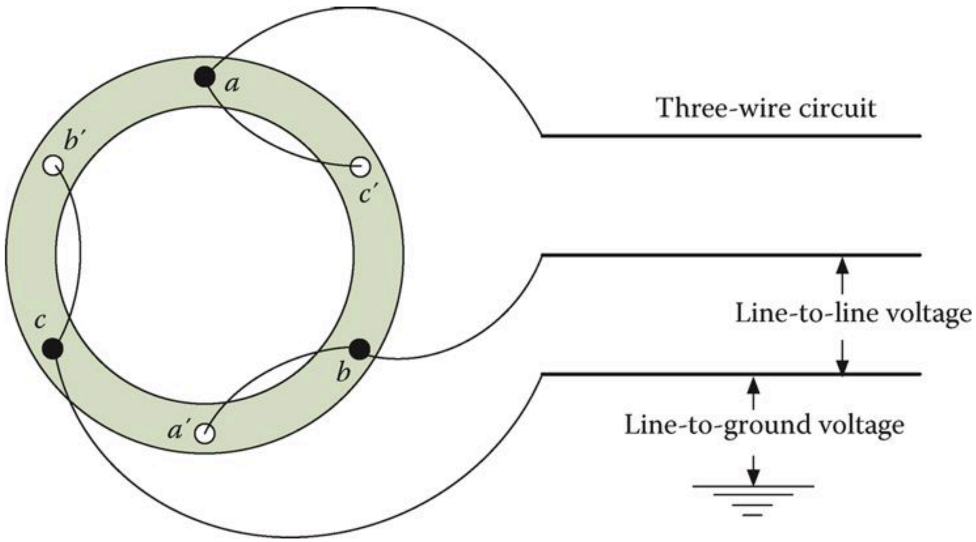
Connection of Three-Phase Circuits

- The three-phase generator has three independent coils and each coil represents a phase.
- Since each coil has two terminals, the generator has six terminals.
- To transmit the generated power from the power plant to the load centers, six wires seem to be needed.
- Since the transmission lines are often very long (hundreds of miles), the cost of the six wires is very high.
- To reduce the cost of the transmission system, the three coils are connected in a wye or delta configuration.. The end of all the coils are bonded to a single point known as neutral. The other ends are connected to the three-wire (three-phase) system.
- In the delta configuration, the end of each coil is connected to the entrance of the adjacent coil. The entrance of each coil is then connected to one of the phases of the transmission line. In each of these configurations, only three wires are needed to transmit the power. These three wires are considered to be one circuit.

Wye-connected generator.

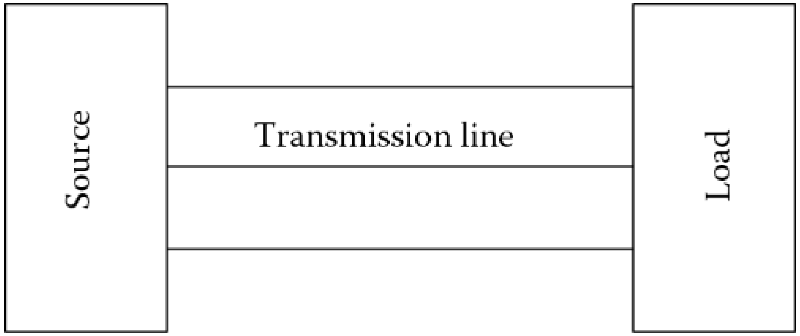


Delta-connected generator.



Connection of Three-Phase Circuits

- The generator is connected to the load via the three-wire transmission circuit.
- This simple power circuit is divided into three distinct parts: source, load, and three-wire circuit. The three-wire circuit is called three-phase transmission line.
- The currents in the transmission line conductors are called line currents. The voltages of the transmission line are classified by two variables: (1) line-to-neutral (line-to-ground) voltage and (2) line-to-line voltage
- The line-to-ground voltage is also called phase voltage or phase-to-ground voltage. The phase voltage is the voltage between any line and the ground (or neutral).
- The voltage between any two lines is the line-to-line voltage.

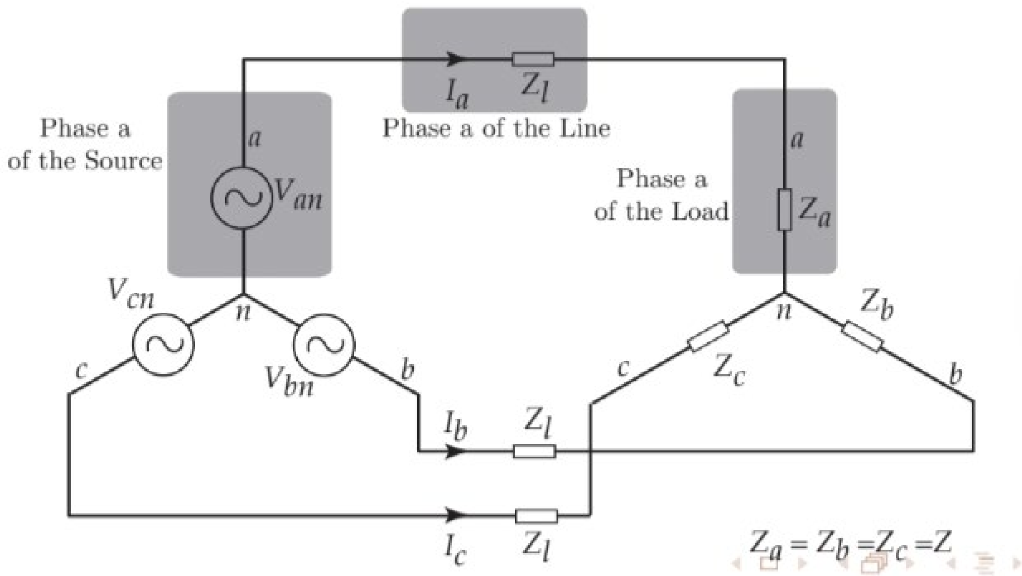


Circuit connection.

Main Components

A balanced 3-phase system has:

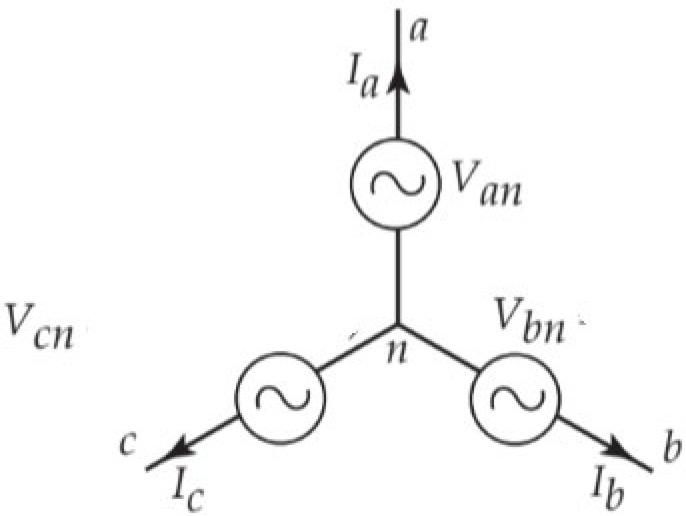
- Three sinusoidal voltage sources with equal magnitude, but with an angle shift of 120°.
- equal loads on each phase.
- equal impedance on the lines connecting the sources to the loads



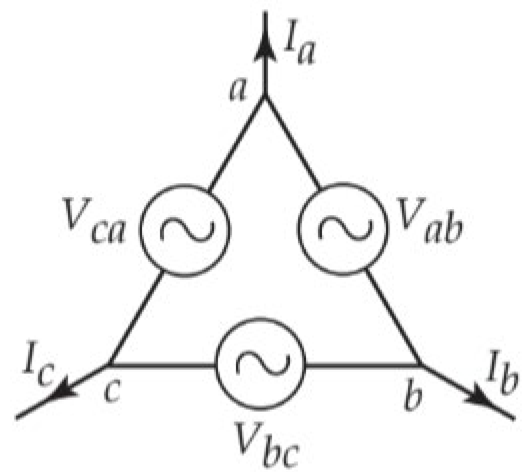
3-Phase Voltage Source

We have two type of 3-Phase voltage source:

- Y- Connected
- **Δ**- Connected



Y-Connected



Δ-Connected

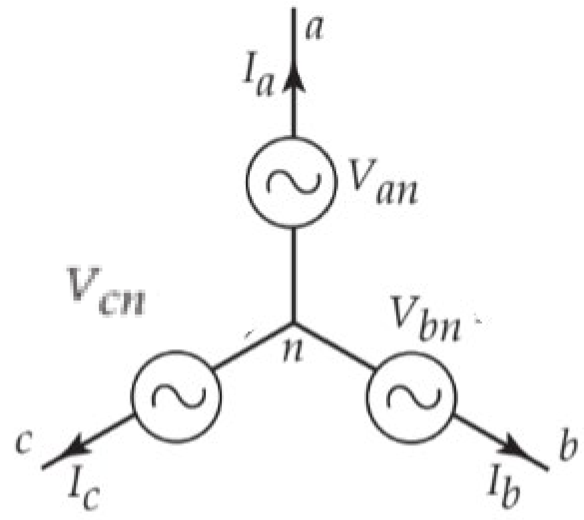
Y- Connected Voltage Source

$$V_{an} = V\angle\alpha$$
$$V_{bm} = V\angle\alpha - 120°$$
$$V_{cn} = V\angle\alpha - 240°$$

Positive Sequence Voltage Source

Note:

$$V = \frac{V_{max}}{\sqrt{2}}$$

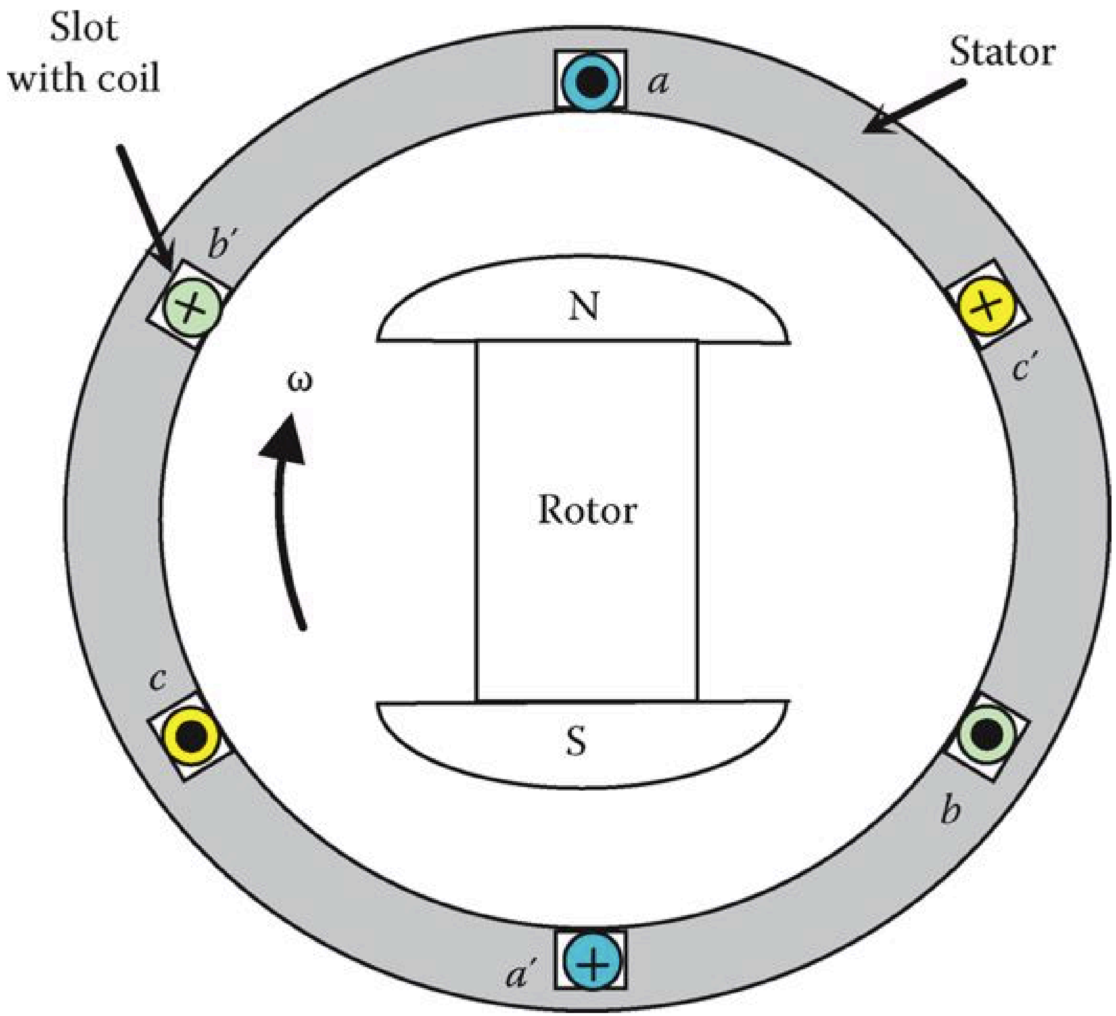


Y-Connected

$$v_{an} = V_{max}cos(\omega t + \alpha)$$
$$v_{bn} = V_{max}cos(\omega t + \alpha - 120^o)$$
$$v_{cn} = V_{max}cos(\omega t + \alpha - 240^o)$$

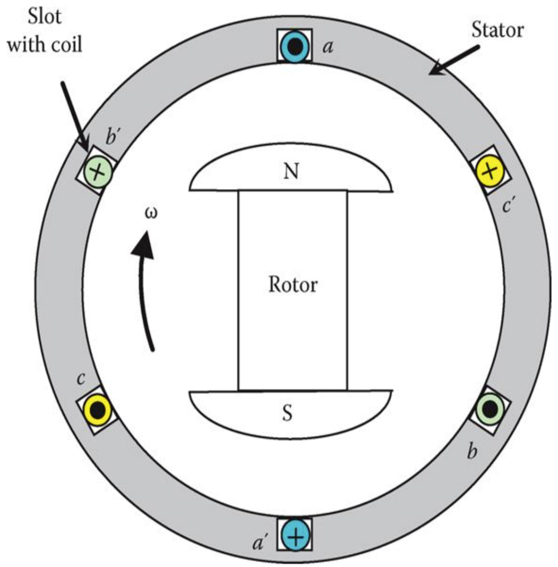
Pictures of Generators and Transformers

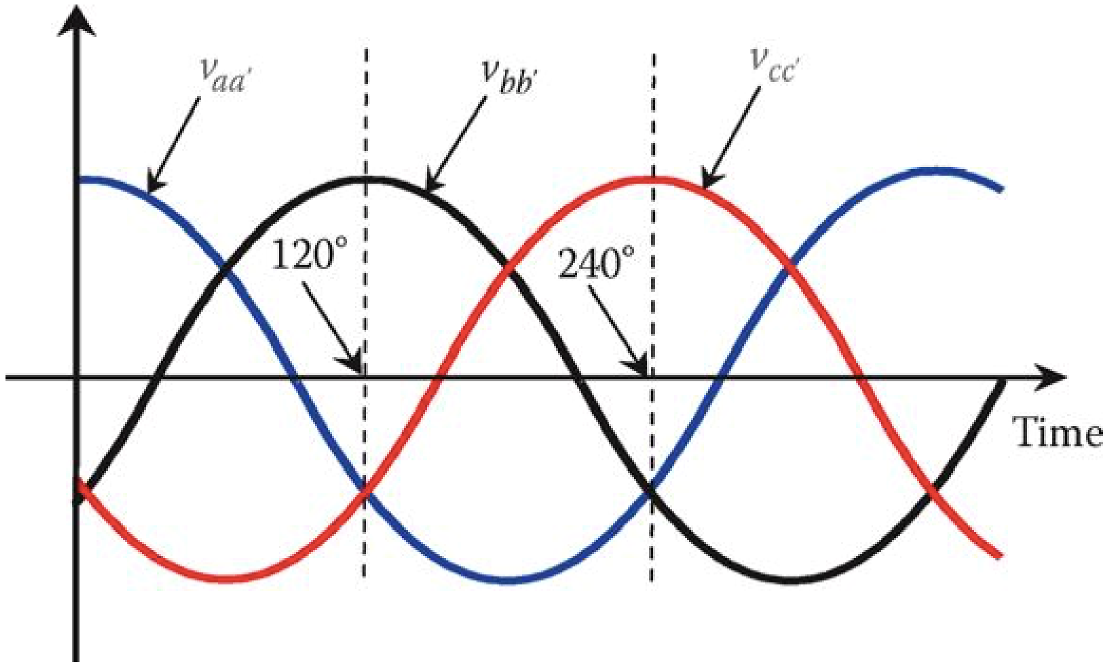
A three-phase generator.



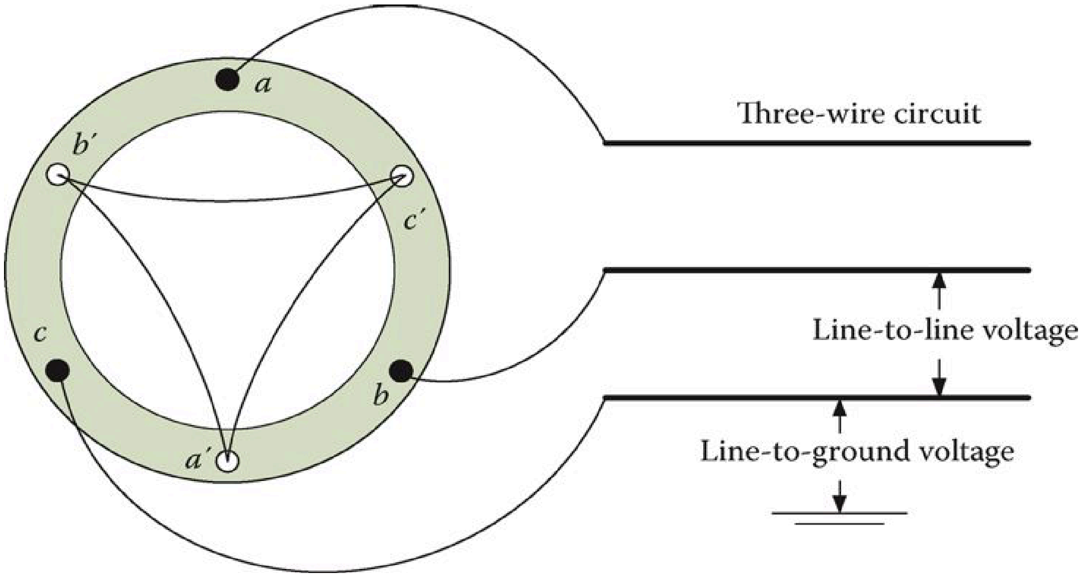
Waveforms of the three-phases of the generator.

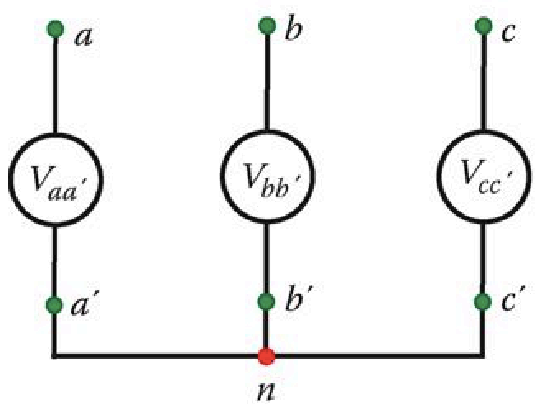
A three-phase generator.

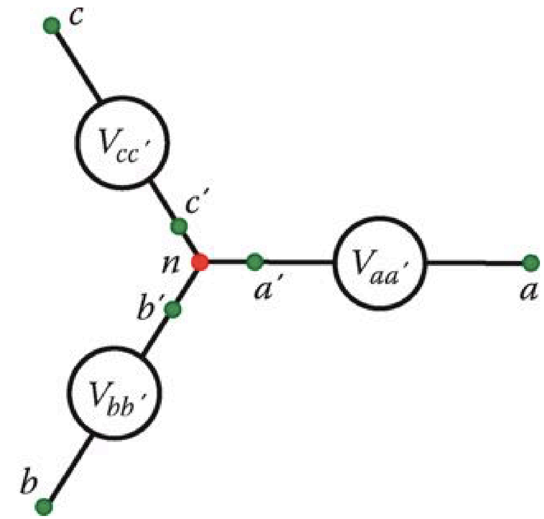




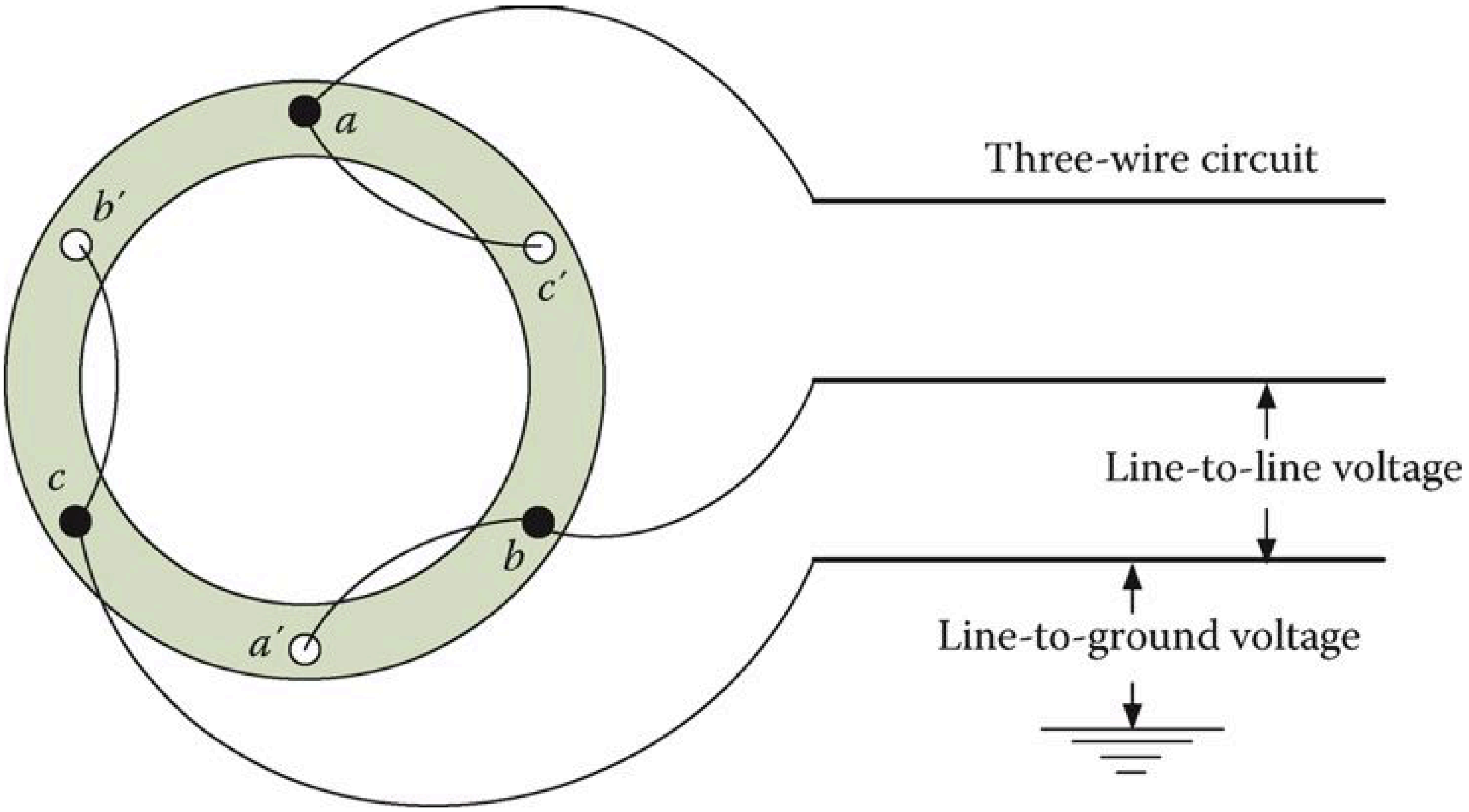
Wye-connected generator.



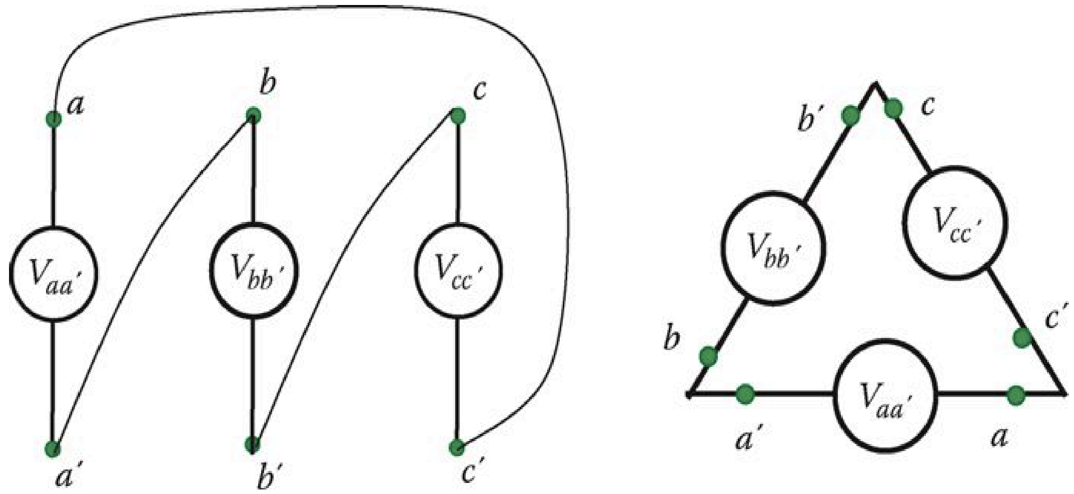




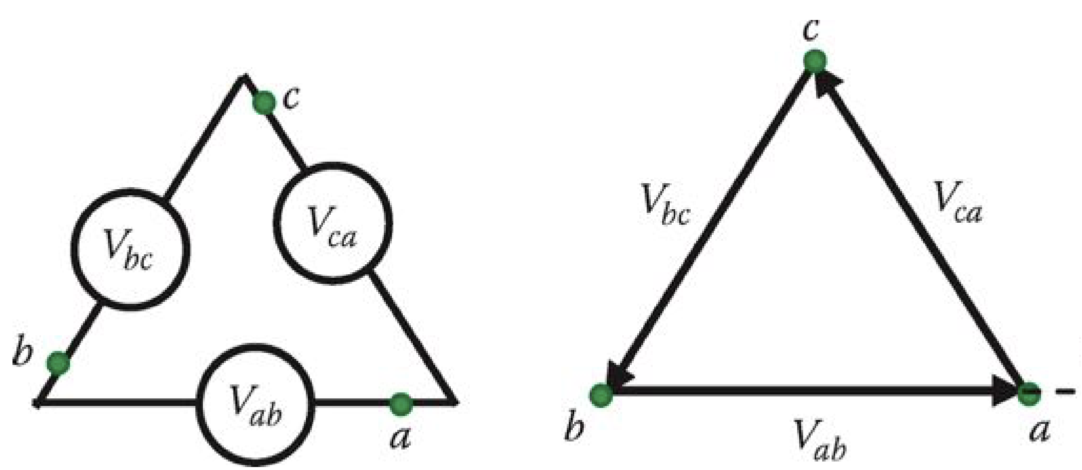
Delta-connected generator.



Delta connection.

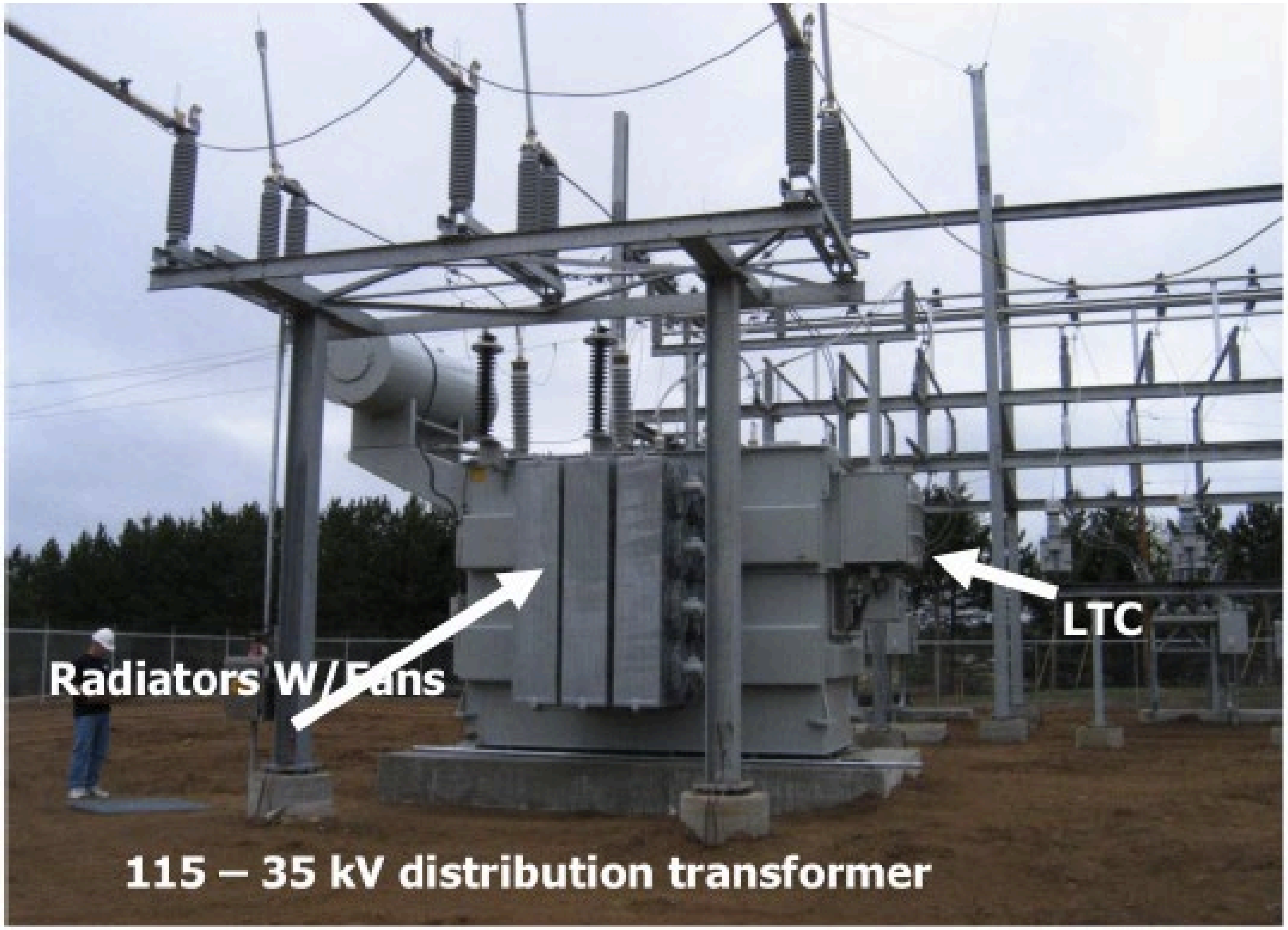


Delta connection and its phasor diagram.

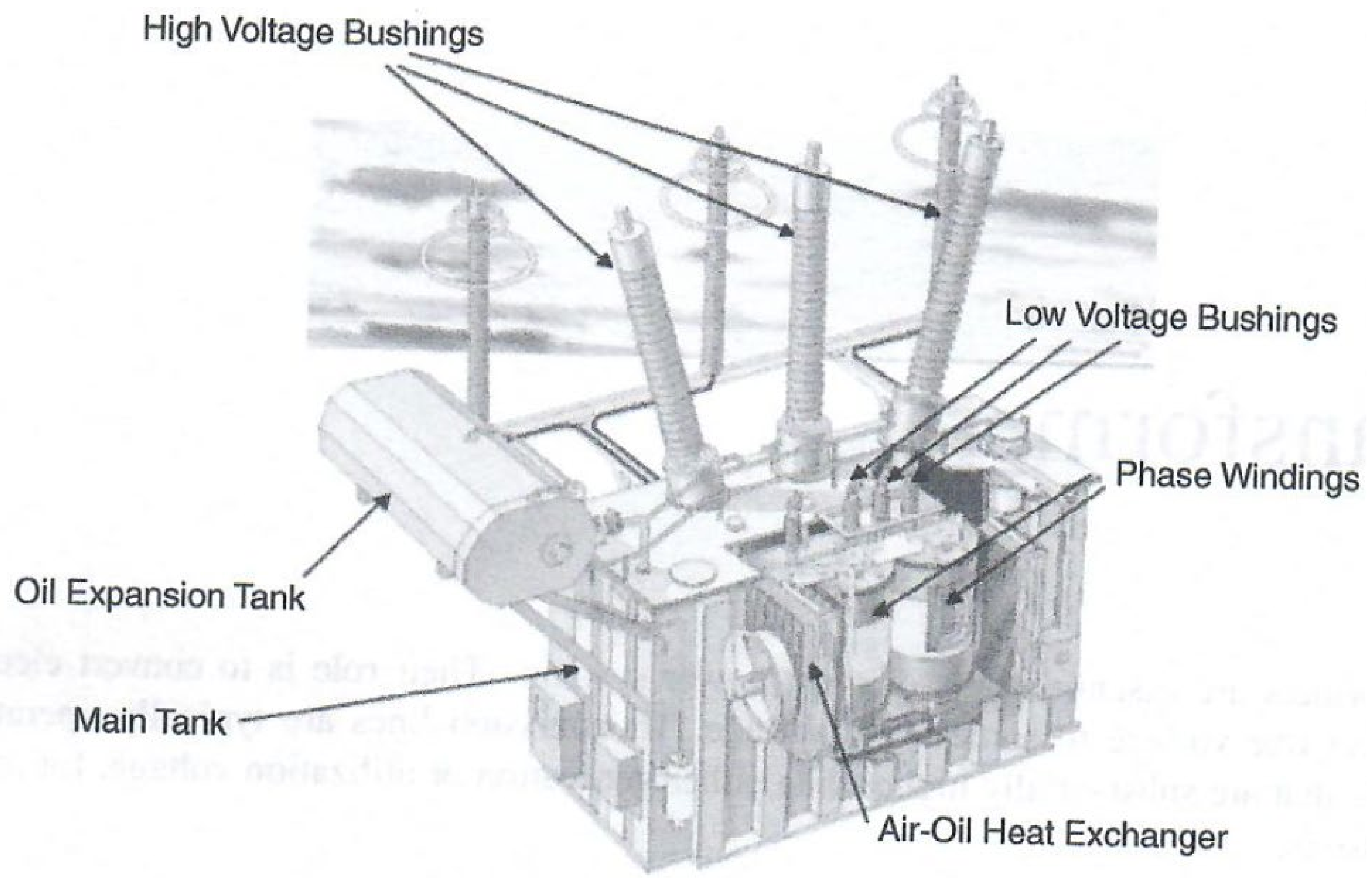


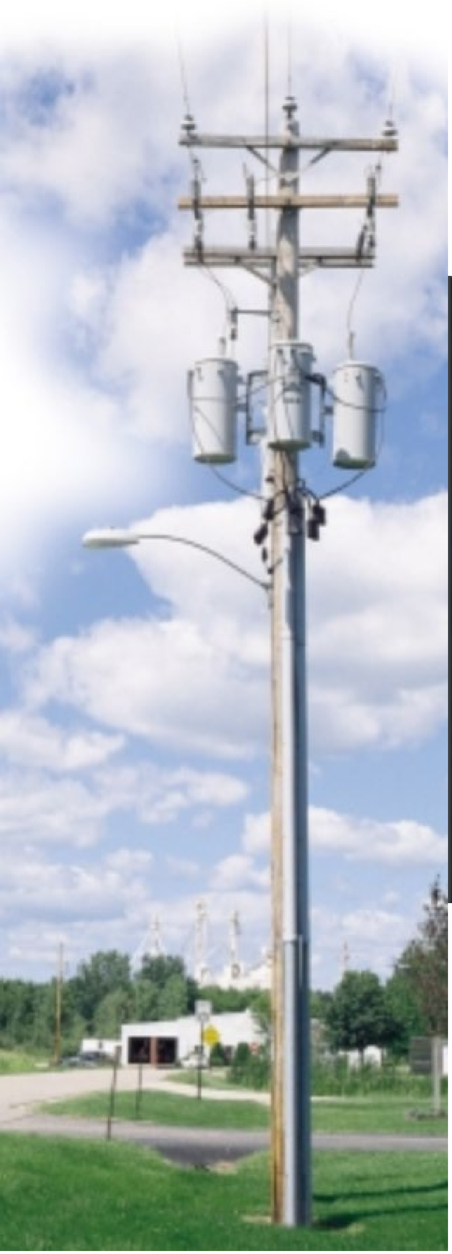
Transmission Level Transformer

Distribution Transformer



Low Power Transformers


High Voltage Bushings
Low Voltage Bushings
Phase Windings
Oil Expansion Tank
Main Tank
Air-Oil Heat Exchanger



Single-Phase Conventional Transformer

TYPE:	Single-Phase, 50 or 60 Hz
LIQUID TYPE:	R-Temp Fluid, Mineral Oil or Envirotemp FR3 Fluid
kVA:	5-500
PRIMARY VOLTAGE:	2400-46,000 V
SECONDARY VOLTAGE:	120-600 V

The single-phase conventional overhead transformer is designed with an interlaced core (5-50 kVA) recommended by *EPRI. Polymer low-voltage bushings are standard at 5-50 kVA. Radiators are included at 250 kVA and above.

Single-Phase Shrubline VFI Transformer

TYPE:	Single-Phase, 60 Hz, Dead-front, with VFI Switch for Loop Protection
LIQUID TYPE:	Mineral Oil
kVA:	25-100
PRIMARY VOLTAGE:	2400-19,920 V
SECONDARY VOLTAGE:	240/120 V

The Shrubline VFI Transformer combines a single-phase dead-front, pad-mounted transformer with a vacuum fault interrupter in one low profile enclosure for resettable fault protection through 25 kV. Ideal for residential areas, the Shrubline is only 36 inches high and is designed to blend with its surroundings. Properly coordinated and strategically located, it can effectively minimize both outage area and outage duration due to a fault.

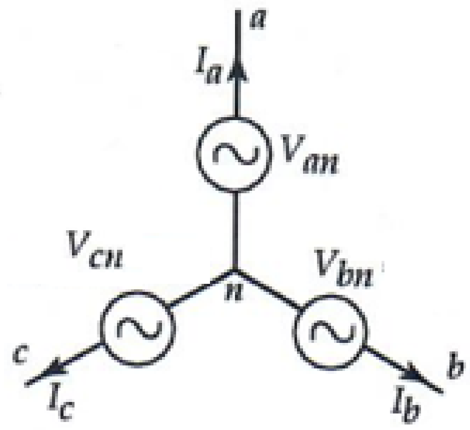
3-Phase Voltage Source – (Y-Connection)

Y-Connected 3-Phase Voltage Source

$$V_{an} = V\angle\alpha$$
$$V_{bn} = V\angle(\alpha - 120°)$$
$$V_{cn} = V\angle(\alpha - 240°)$$

In every balanced three-phase system, the sum of abc variables is ALWAYS zero:

$$V_{an} + V_{bn} + V_{cn} = 0$$



Y-Connected

Contd…

Y-Connected 3-Phase Voltage Source

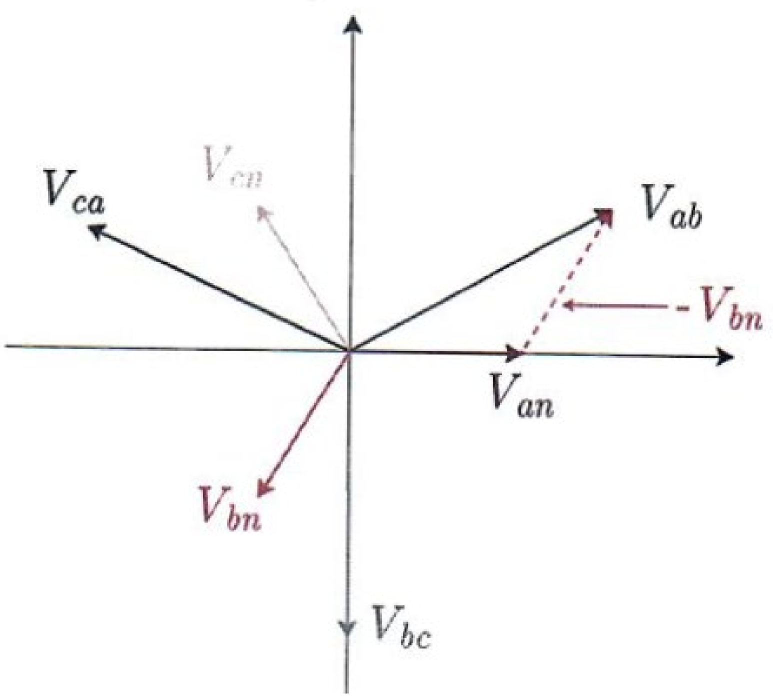
$$V_{an} = V\angle\alpha$$
$$V_{bn} = V\angle\alpha - 120°$$
$$V_{cn} = V\angle\alpha - 240°$$

Therefore,

$$V_{ab} = V_{an} - V_{bn} = V(1\angle\alpha - 1\angle(\alpha - 120°))$$
$$= \sqrt{3}V\angle\alpha + 30°$$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V\angle\alpha - 90°$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V\angle\alpha - 210°$$



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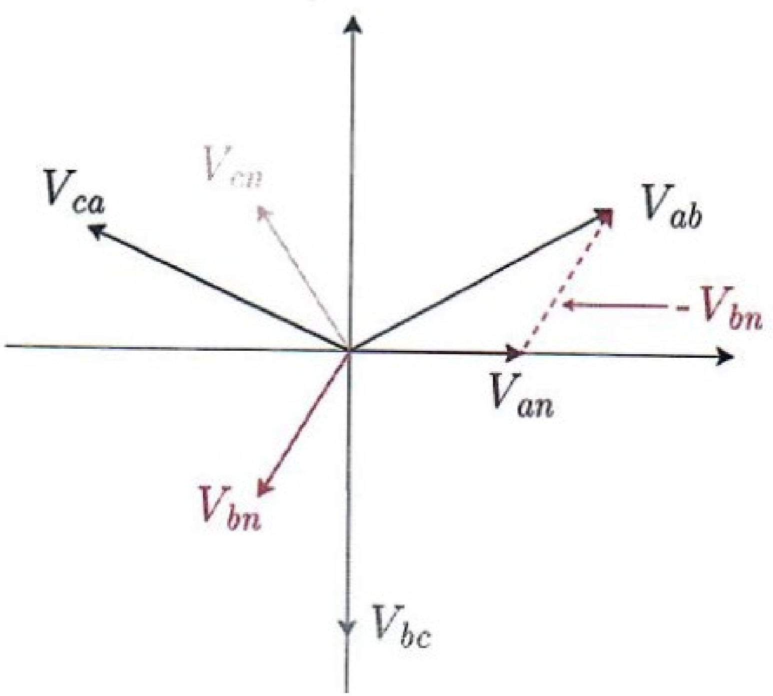
Y-Connected 3-Phase Voltage Source

$$V_{an} = V\angle\alpha$$
$$V_{bn} = V\angle\alpha - 120°$$
$$V_{cn} = V\angle\alpha - 240°$$

Or,

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an}\ \angle 30°$$
$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_{bn}\ \angle 30°$$
$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_{cn}\ \angle 30°$$

Line to Line voltages are also balanced.



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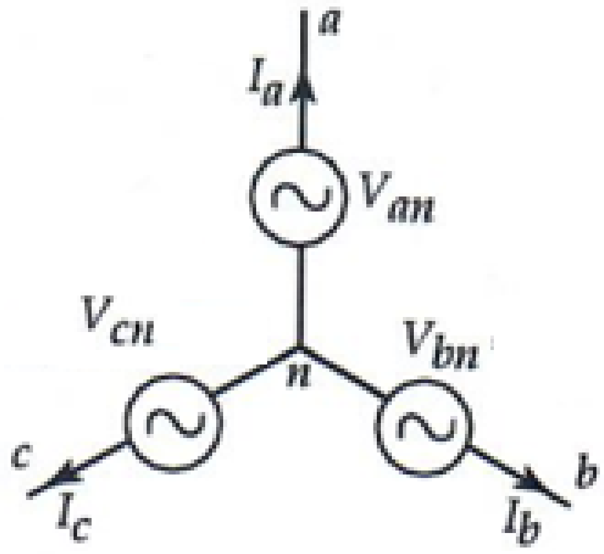
So, in a Y-connected source

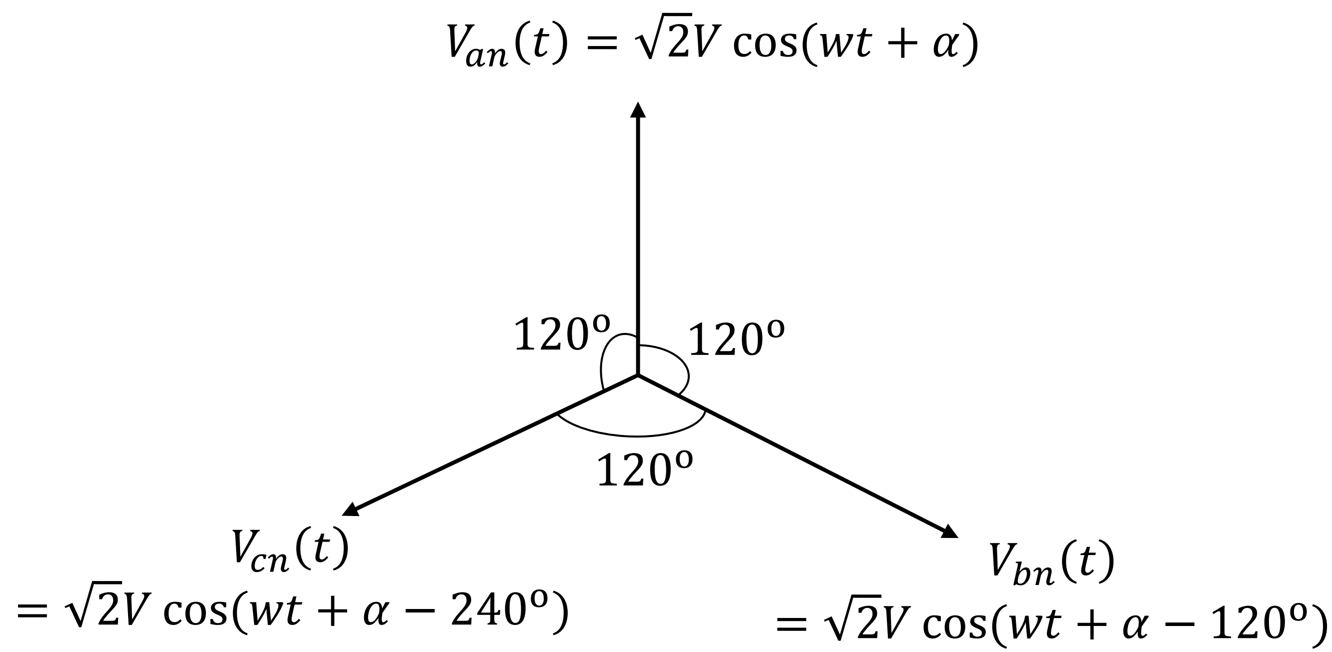
- $V_{line} = \sqrt{3}V_{phase}\,\angle 30°$
- $I_{line} = I_{phase}$
- $S_{3-\emptyset} = 3V_{phase}\,\mathrm{I}^{*}$



Y-Connected

Summary of 3-Phase, Y-Connection

Time Domain Representation



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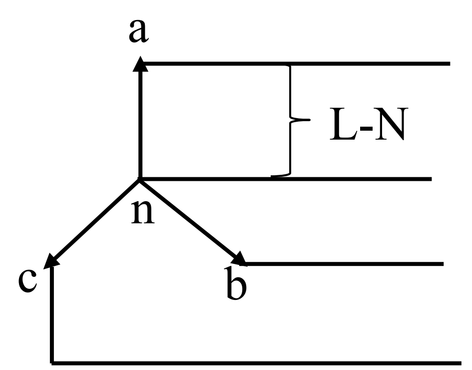
Phasor Domain Representation

$$\left.\begin{aligned} V_{an} &= V\angle\alpha \\ V_{bn} &= V\angle(\alpha - 120°) \\ V_{cn} &= V\angle(\alpha - 240°) \end{aligned}\right\}$$

Line to Neutral (L-N) Voltage, also called Phase Voltage



Relationship between Line to Line (L-L) and L-N voltage

$$V_{ab} = V_{an} - V_{b_n} \quad = V\angle\alpha - V\angle(\alpha - 120°)$$
$$= \sqrt{3}V\angle(\alpha + 30°)$$

$$V_{bc} = V_{bn} - V_{c_n} = \sqrt{3}V_{bn}\angle 30°; \text{where } V_{bn} = V\angle(\alpha + 120°)$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_{cn}\angle 30°; \text{where } V_{cn} = V\angle(\alpha - 240°)$$

Contd…

Conclusion: In Y- connection

- $$\underbrace{V_{line}}_{\substack{V_{ab} \\ V_{bc} \\ V_{ca}}} = \underbrace{\sqrt{3}V_{phase}}_{\substack{V_{an} \\ V_{bn} \\ V_{cn}}} . e^{j30^{o}}$$

- $$I_{line} = I_{phase}$$

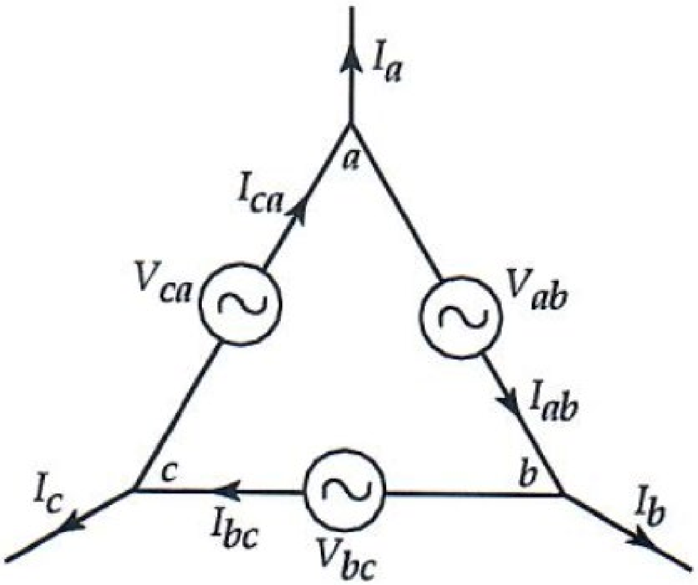
- $$S_{3-\emptyset} = 3V_{phase}\, \mathrm{I}^{*}_{\mathrm{phase}}$$

3-Phase Voltage Source – (Δ-Connection)

(Δ-Connection) 3-Phase Voltage Source

For the **Δ** connection ,

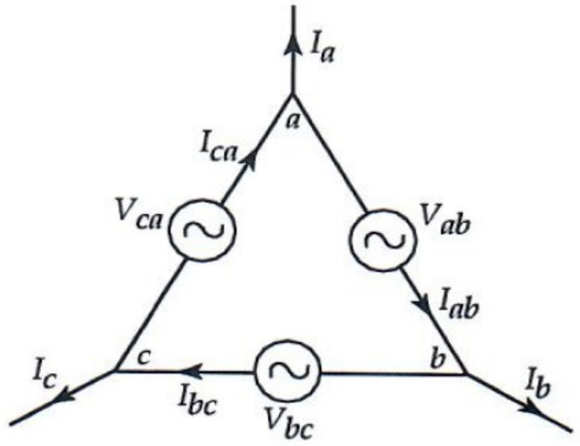
Phase voltages equal Line voltages.



Δ-Connected

Contd…

(Δ-Connection) 3-Phase Voltage Source

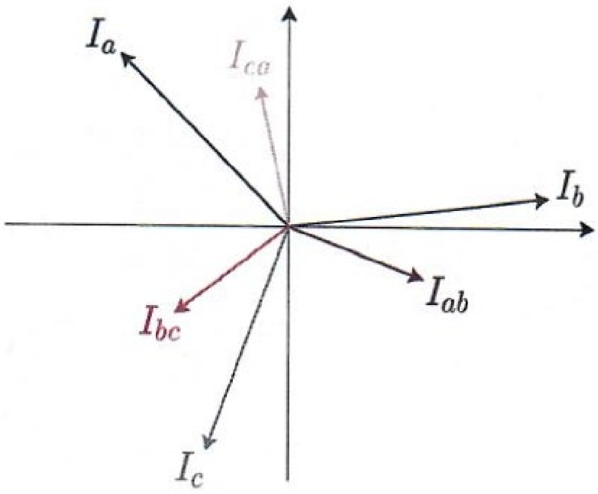


Δ-Connected

For the **Δ** connection , phase voltages equal line voltages.

Moreover,

$$I_a = I_{ca} - I_{ab} = \sqrt{3} I_{ca} \angle 30^\circ$$
$$I_b = I_{ab} - I_{bc} = \sqrt{3} I_{ab} \angle 30^\circ$$
$$I_c = I_{bc} - I_{ca} = \sqrt{3} I_{bc} \angle 30^\circ$$



Line Currents are also balanced.

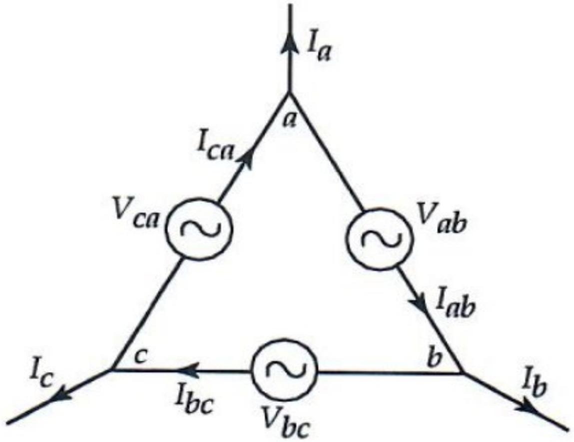
Contd…

So, in a Δ-Connection source:

$$V_{line} = V_{phase}$$

$$I_{line} = \sqrt{3} I_{phase} \angle 30°$$

$$S_{3\phi} = 3V_{phase}\, I^*_{phase}$$

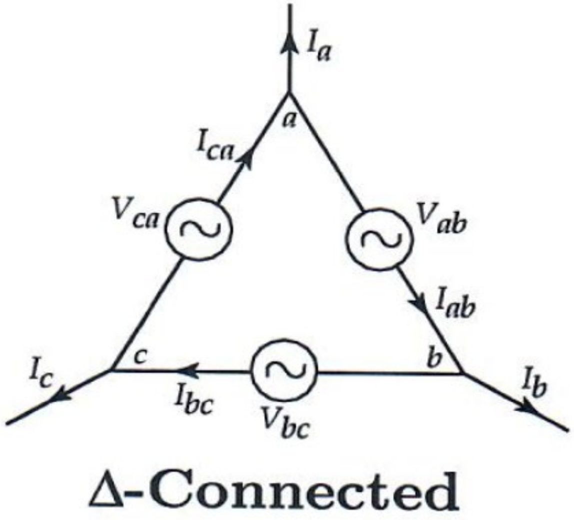


Δ-Connected

Summary of 3-Phase, Δ -Connection Source

$$V_{line} = V_{phase}$$

- Measurement from outside, the voltages are; $V_{ab}/\ V_{bc}/\ V_{ca}$
- (**<u>Note:</u>** Line voltage=Phase Voltage in Delta Connection)
- Measurement from outside, the currents are; $I_a/\ I_b/\ I_c$

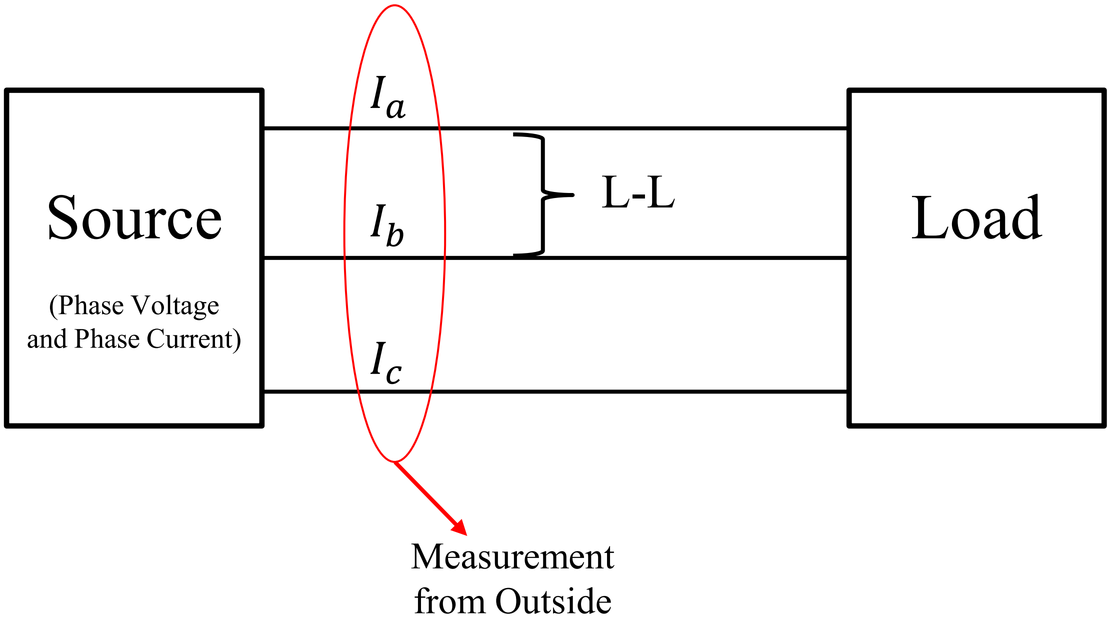


$$I_a = I_{ca} - I_{ab} = \sqrt{3}\ I_{ca}\ e^{j30^o}$$
$$I_b = I_{ab} - I_{bc} = \sqrt{3}\ I_{ab}\ e^{j30^o}$$
$$I_c = I_{bc} - I_{ca} = \sqrt{3}\ I_{bc}\ e^{j30^o}$$

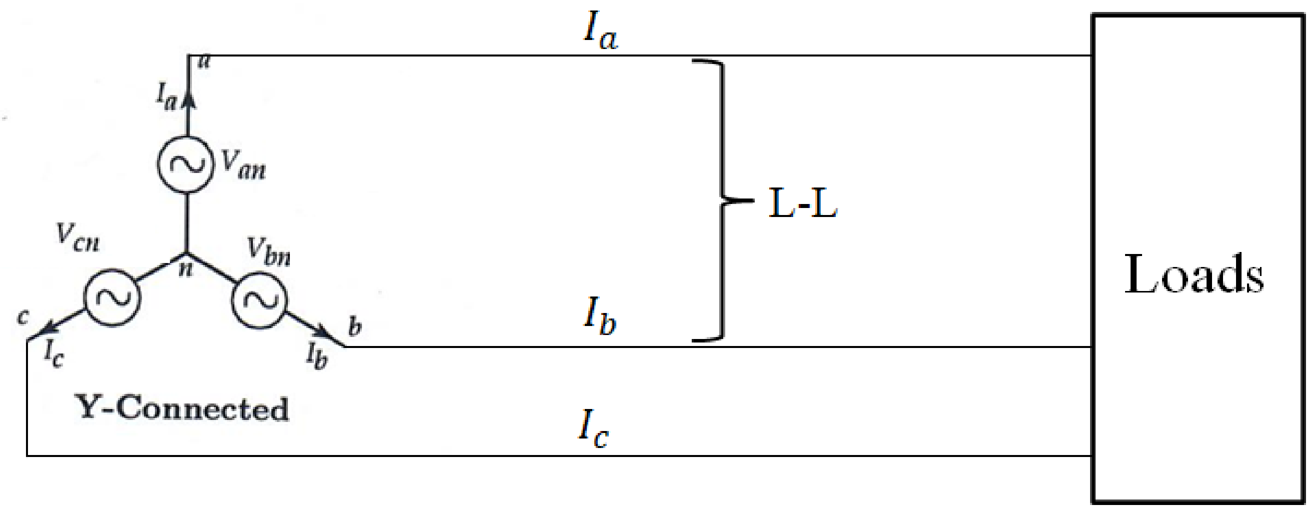
Contd…

Conclusion: In Δ - connection

- $V_{line} = V_{phase}$
- $I_{line} = \sqrt{3} I_{phase}\, e^{j30^{o}}$
- $S_{3-\emptyset} = 3V_{phase}\, \mathrm{I}^{*}_{phase}$

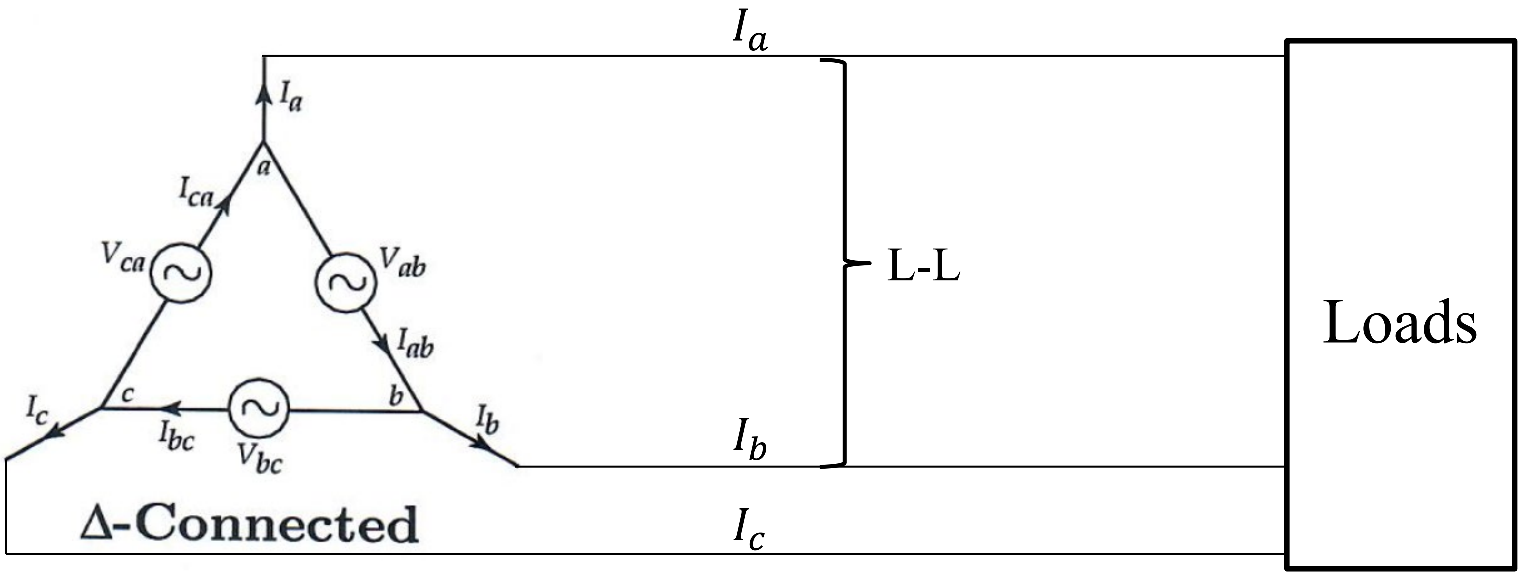


3-Phase Voltage Source: (Y-Connected Source)



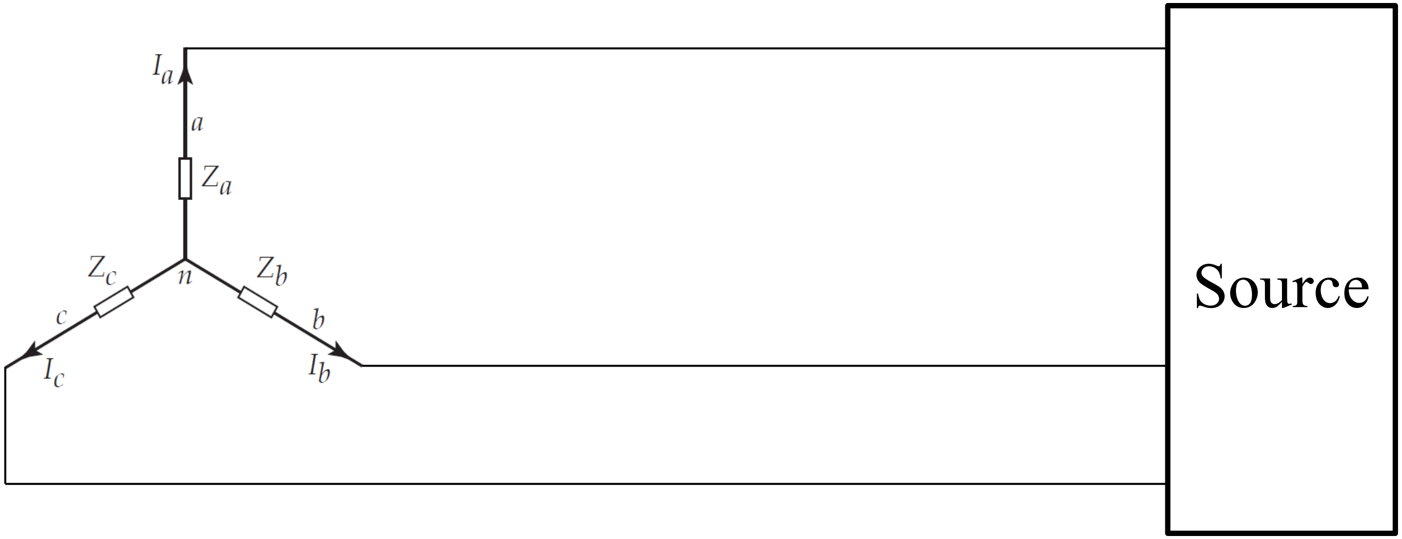
- $V_{line} = V_{phase}$
- $I_{line} = \sqrt{3} I_{phase}\, e^{j30^o}$
- $S_{3-\emptyset} = 3V_{phase}\, I_{phase}^{*} = V_{an} \,.\, I_a^{*} + V_{bn} \,.\, I_b^{*} + V_{cn} \,.\, I_c^{*}$

3-Phase Voltage Source: (Δ -Connected Source)



- $V_{line} = V_{phase}$
- $I_{line} = \sqrt{3} I_{phase}\, e^{j30^{o}}$
- $S_{3-\emptyset} = 3 V_{phase}\, \mathrm{I}_{phase}^{*}$

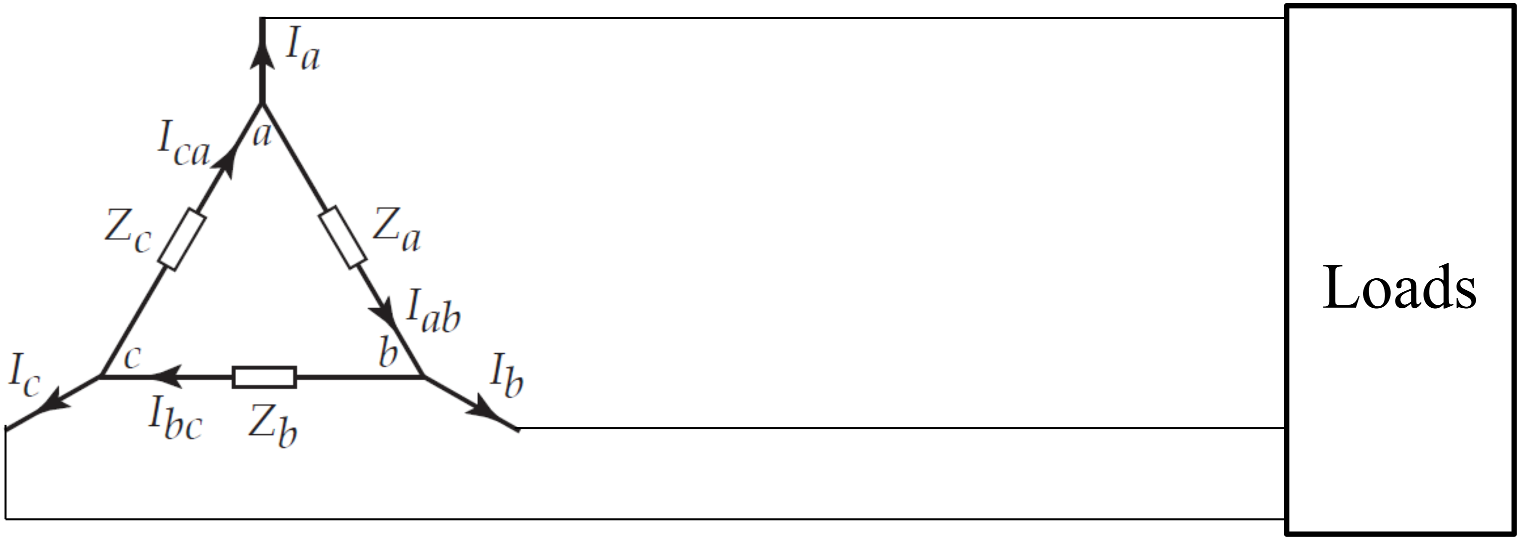
3-Phase Load: (Y-Connected Load)



Phase & Line Currents are the same.

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an} \cdot e^{j30^\circ}$$
$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_{bn} \cdot e^{j30^\circ}$$
$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_{cn} \cdot e^{j30^\circ}$$

3-Phase Load: (Δ -Connected Load)

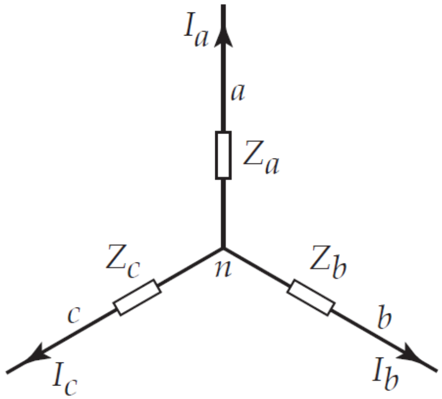


Phase & Line Voltages are the same.

$$I_a = I_{ca} - I_{ab} = \sqrt{3}\, I_{ca}\, e^{j30^o}$$
$$I_b = I_{ab} - I_{bc} = \sqrt{3}\, I_{ab}\, e^{j30^o}$$
$$I_c = I_{bc} - I_{ca} = \sqrt{3}\, I_{bc}\, e^{j30^o}$$

3-Phase Load: Summary

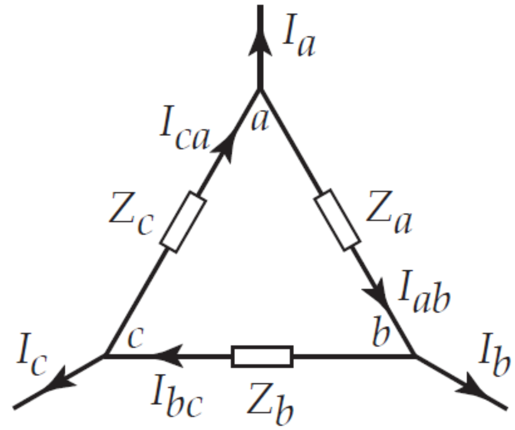
Y -Connected Load



Phase & Line Currents are the same.

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an} \cdot 1\angle 30^\circ$$
$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_{bn} \cdot 1\angle 30^\circ$$
$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_{cn} \cdot 1\angle 30^\circ$$

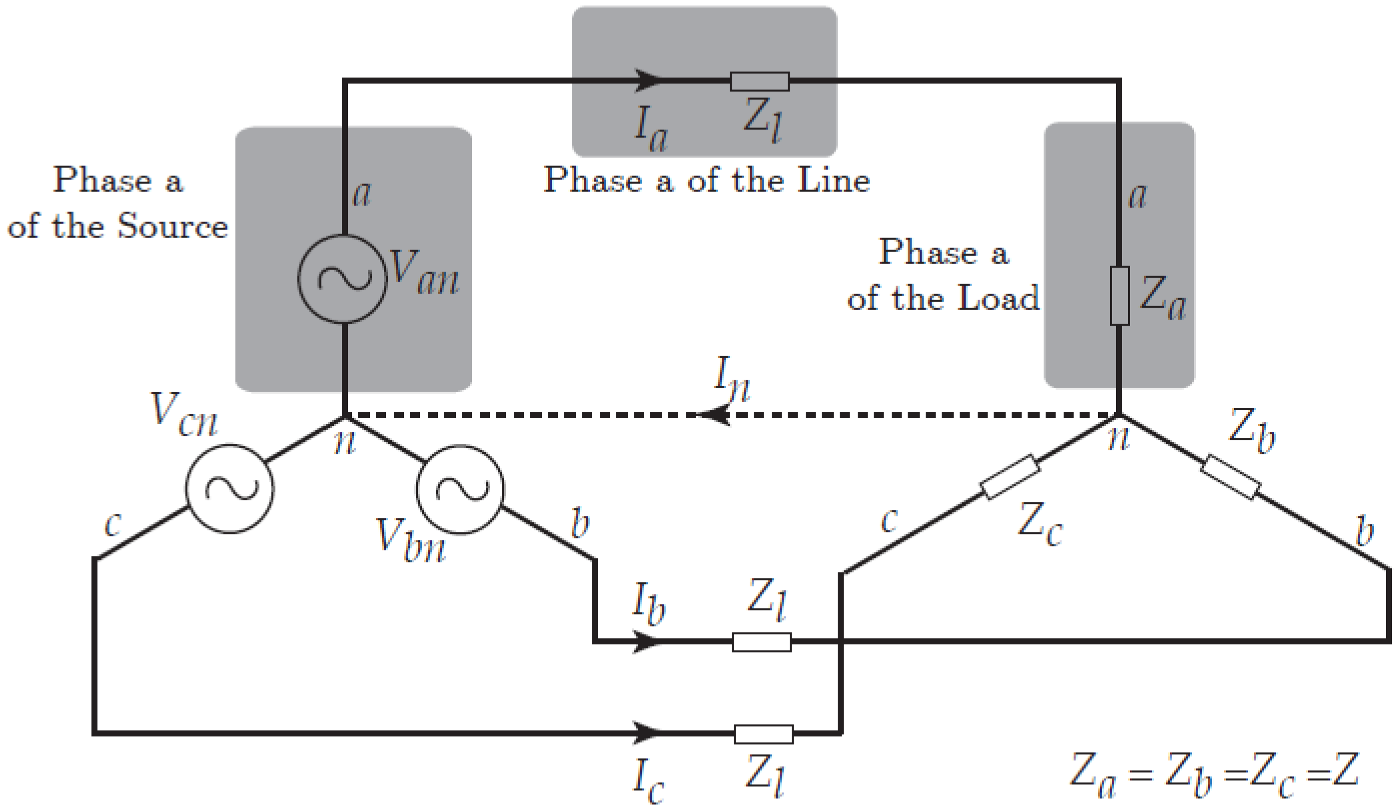
Δ -Connected Load



Phase & Line Voltages are the same.

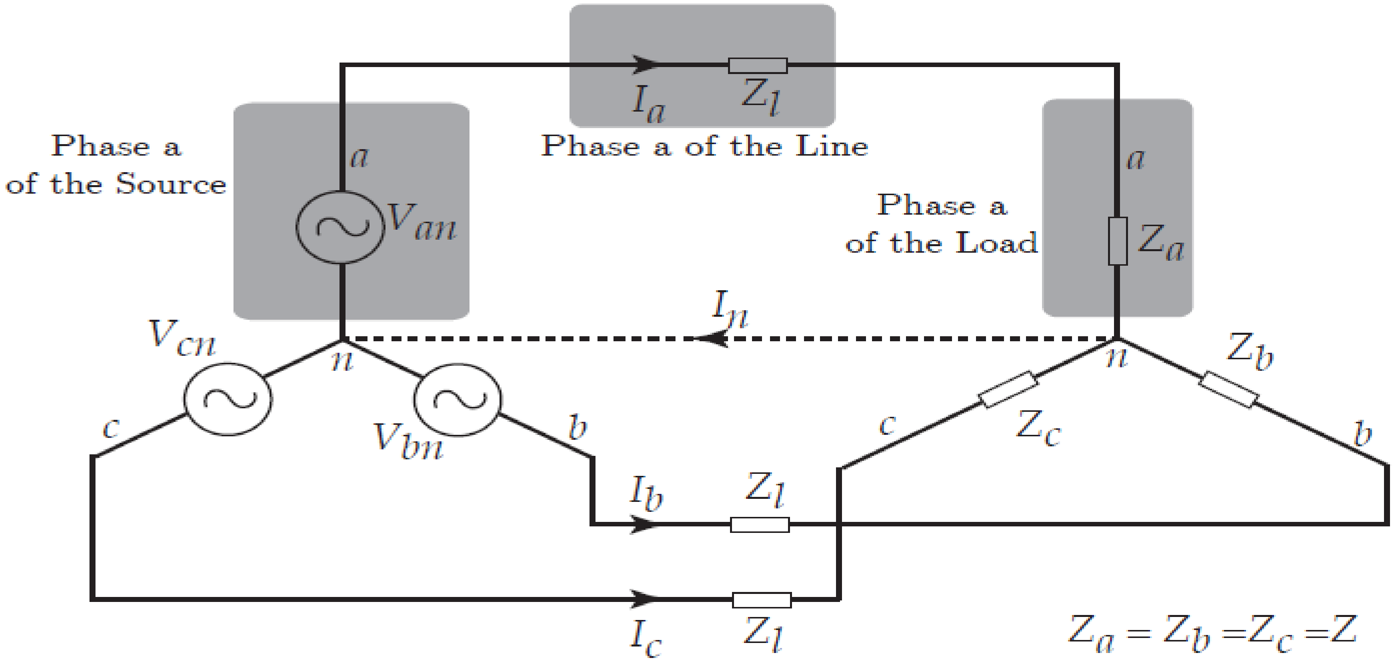
$$I_a = I_{ca} - I_{ab} = \sqrt{3}\ I_{ca}\ 1\angle 30^\circ$$
$$I_b = I_{ab} - I_{bc} = \sqrt{3}\ I_{ab}\ 1\angle 30^\circ$$
$$I_c = I_{bc} - I_{ca} = \sqrt{3}\ I_{bc}\ 1\angle 30^\circ$$

Neutral Point



$$I_n = I_a + I_b + I_c$$

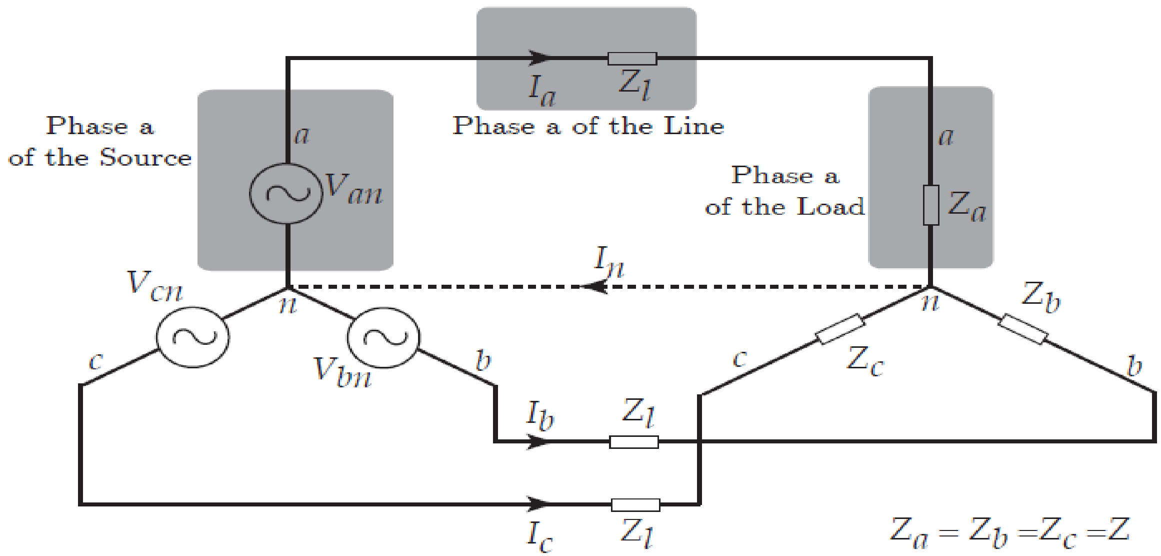
Contd…



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V_{an}}{Z_l + Z_a} + \frac{V_{b_n}}{Z_l + Z_b} + \frac{V_{cn}}{Z_l + Z_c} = \frac{1}{Z_l + Z_a}(V_{an} + V_{bn} + V_{cn})$$

Contd…



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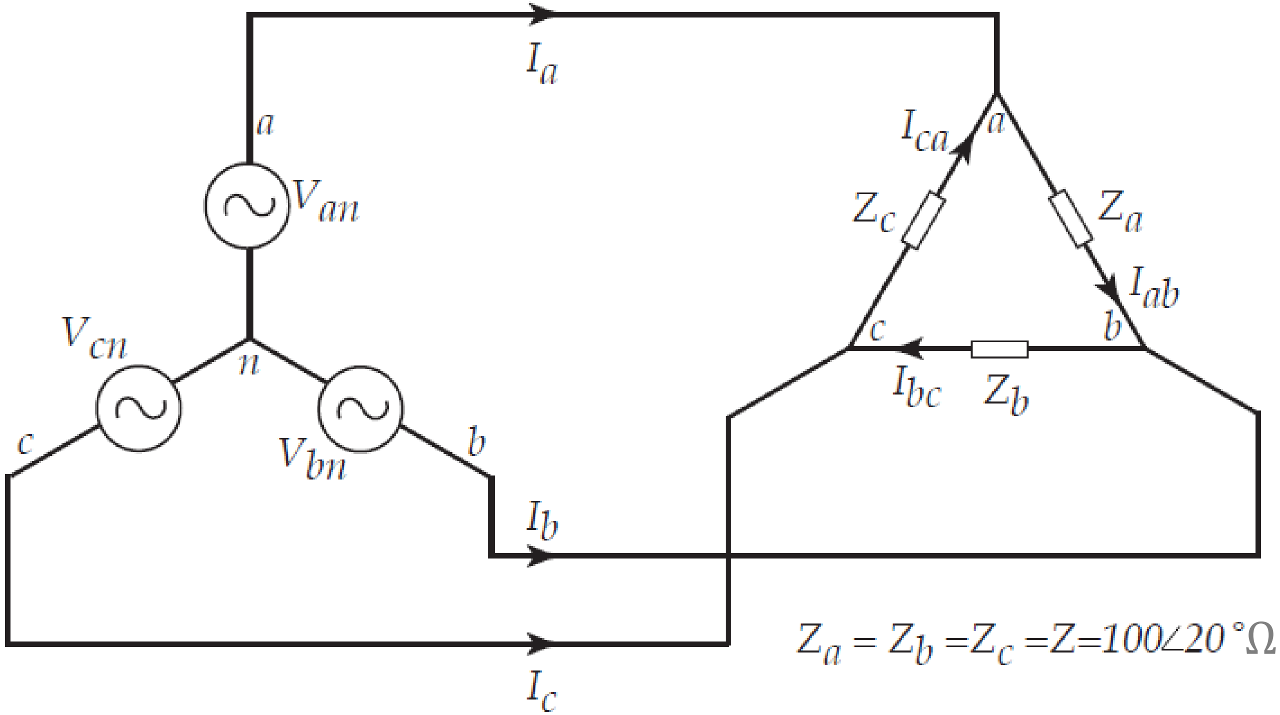
$$I_n = \frac{1}{Z_l + Z_a}(0) = 0$$

Example

Q. Assume a Δ-connected load is supplied from a 3-phase 13.8kV (L-L) source with $Z = 100\angle 20°\Omega$. Find I_a, I_b, and I_c. Moreover, determine the load apparent power of the source.



Contd…

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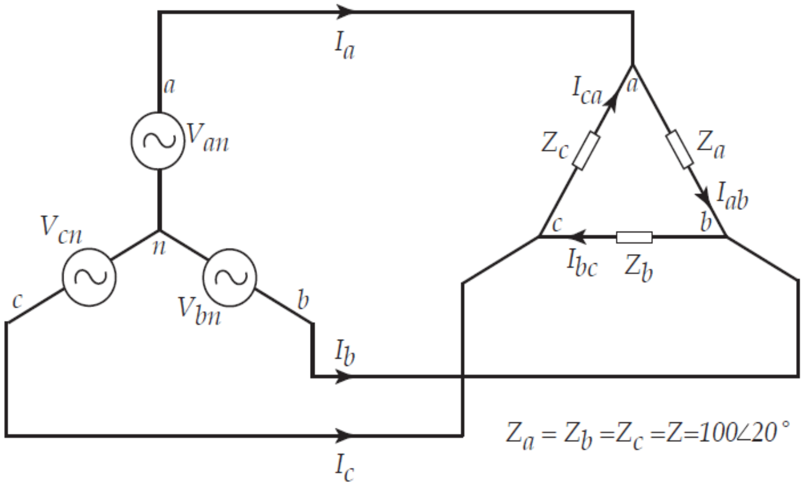
Solution,

$V_{ab} = 13.8\angle 0°\text{kV}$
$V_{bc} = 13.8\angle -120°\text{kV}$
$V_{ca} = 13.8\angle -240°\text{kV}$



Contd…

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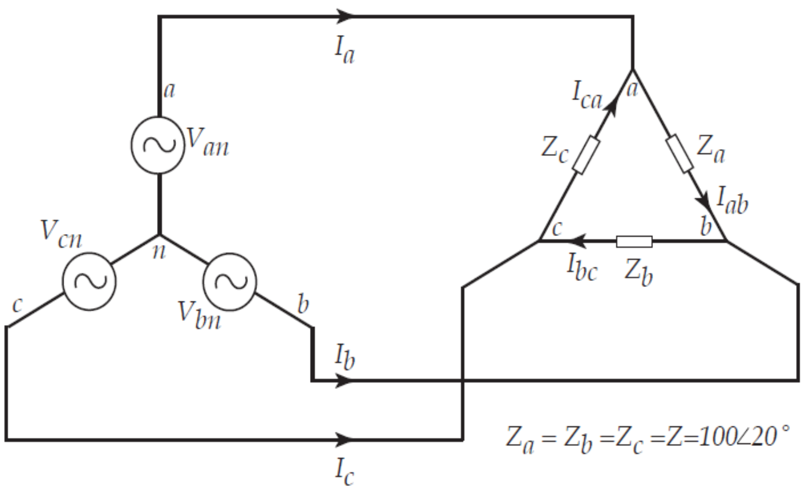

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$$I_{ab} = \frac{13.8k\angle 0°}{100\angle 20°} = 138\angle -20°\text{ A}$$

$$I_{bc} = \frac{13.8k\angle -120°}{100\angle 20°} = 138\angle -140°\text{ A}$$

$$I_{ca} = \frac{13.8\text{k}\angle -240°}{100\angle 20°} = 138\angle -260°\text{A}$$

Contd...

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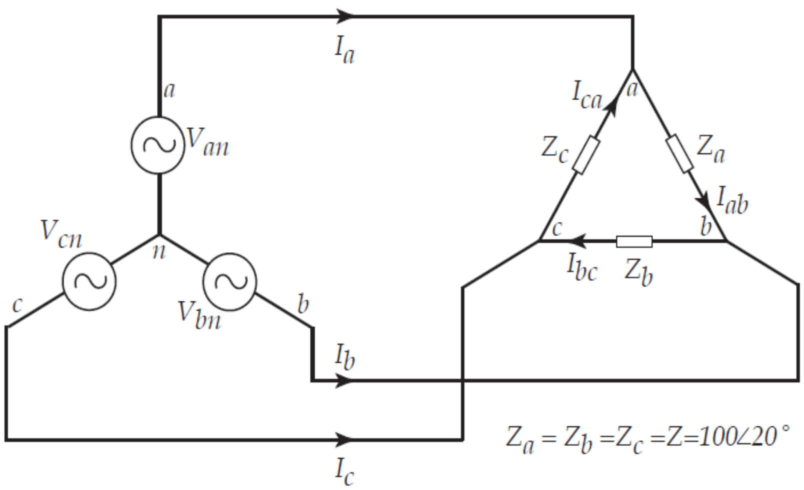

$$I_a = I_{ab} - I_{ca} = 138\angle -20° - 138\angle -260°$$
$$I_a = 239\angle -50°\text{A}$$
$$I_b = 239\angle -170°\text{A}$$
$$I_c = 239\angle -290°\text{A}$$

Contd…

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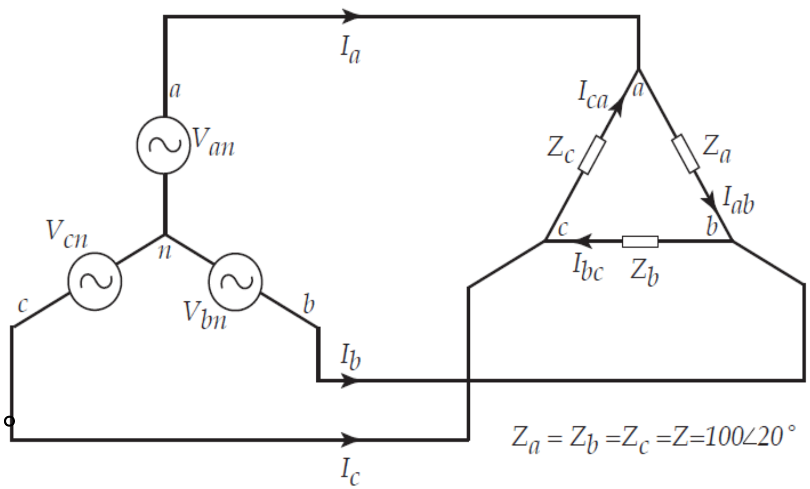
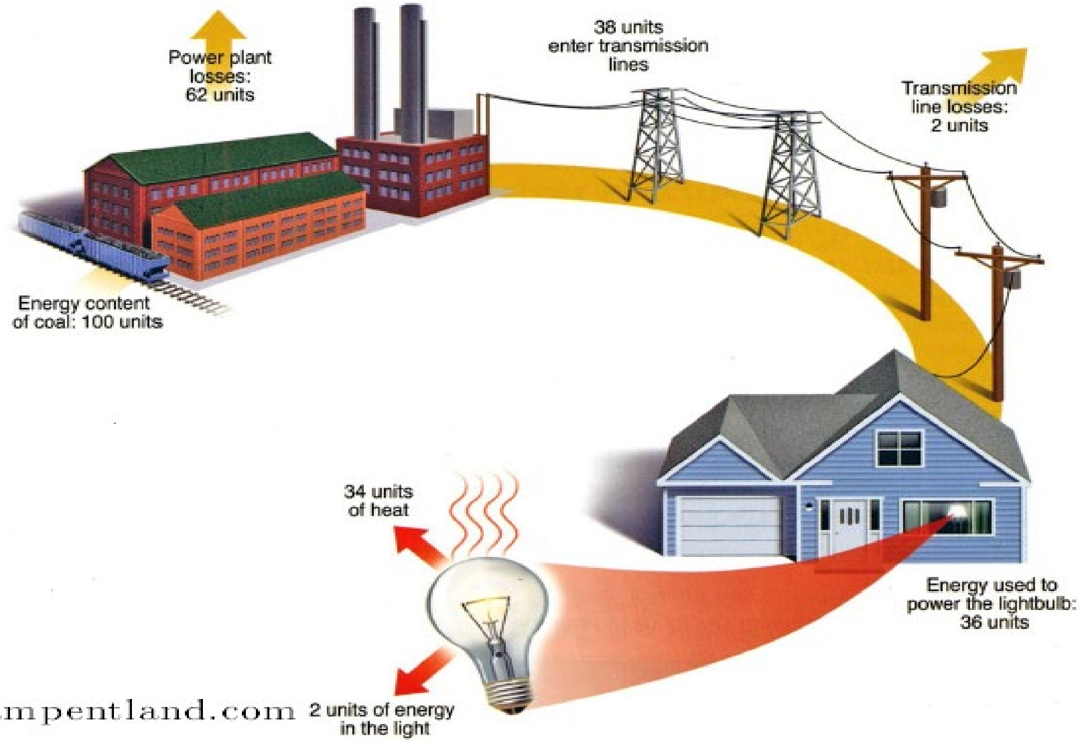


$$S = 3 \times V_{ab} I_{ab}^* = 3 \times 13.8\angle 0°\text{kV} \times 138\angle 20° = 5.7\angle 20°\text{MVA} = 5.37 + j1.95\text{MVA}$$

Fun Fact

Imagine you have a power plant that generates electricity from coal and transmits the generated electrical energy to your home via transmission lines.

If you use this energy to power a conventional lightbulb, how much do you think is the overall efficiency of the system (coal to light)?



Source: www.williampentland.com

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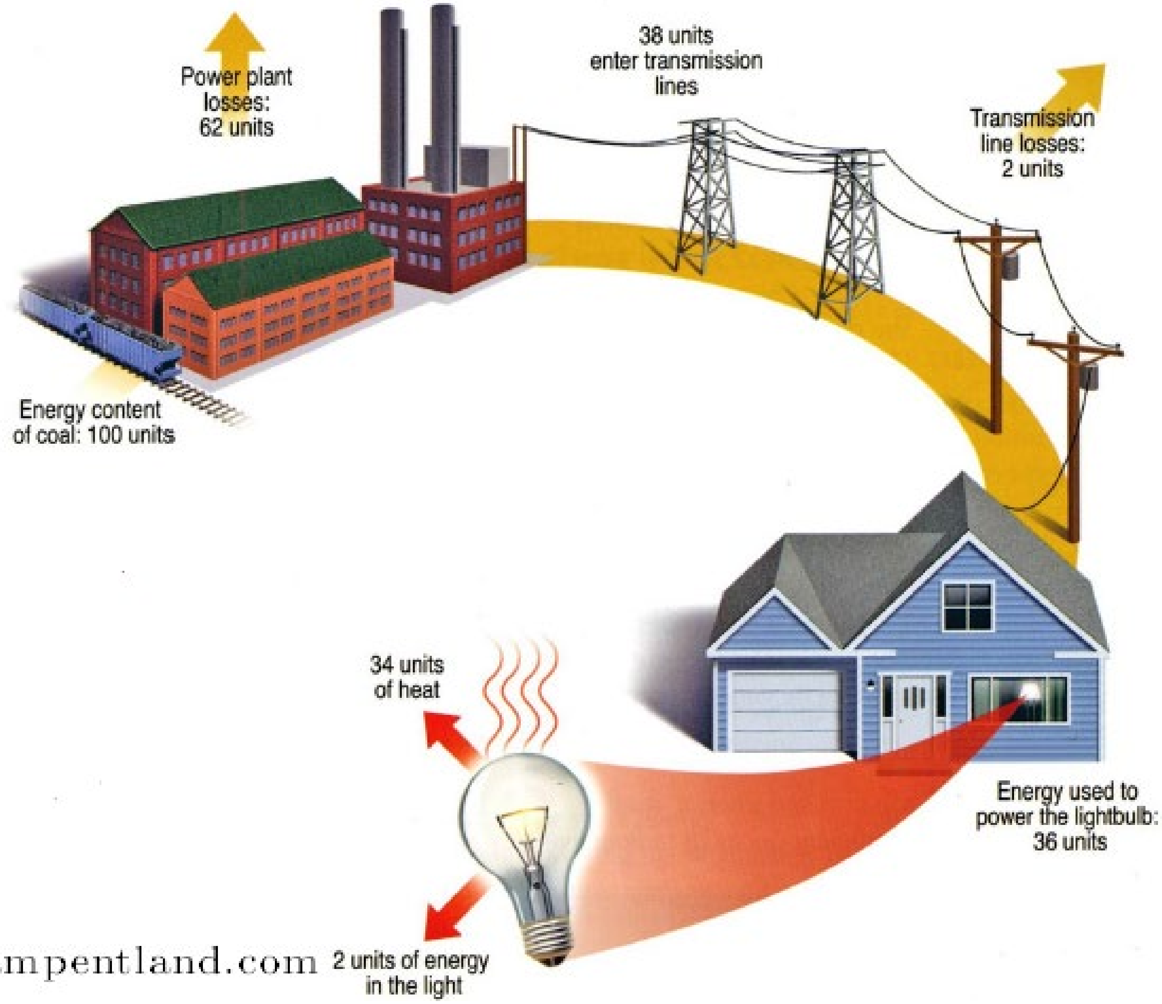


Only 2%

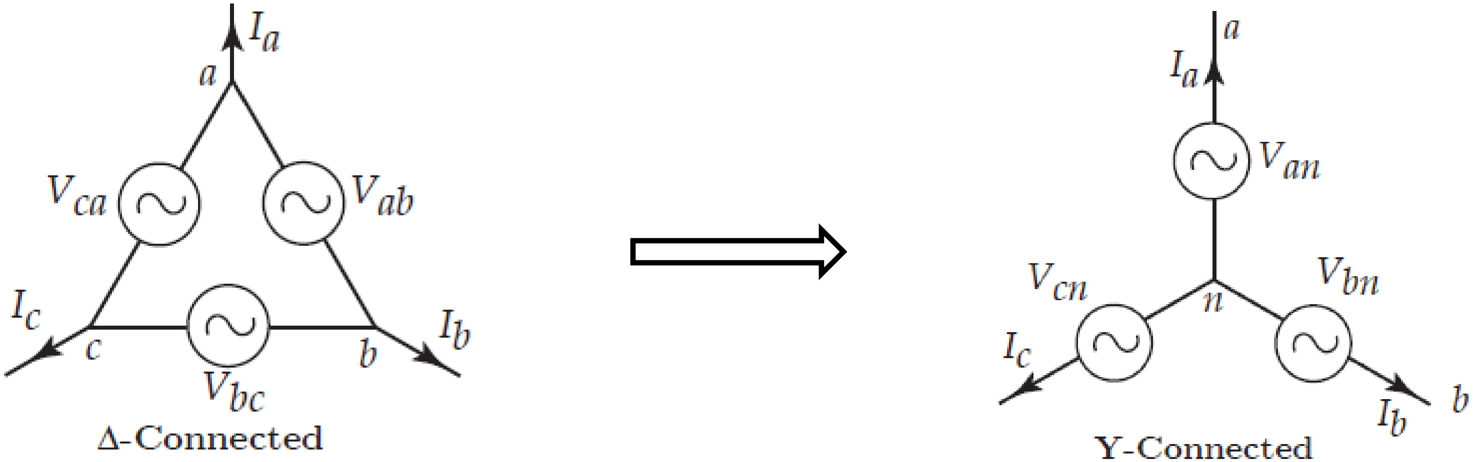
Source: www.williampentland.com

Δ- Y Transformation

To simplify analysis of balanced 3 Phase systems:

- **Δ**-connected sources can be replaced by an equivalent Y-connected sources with $V_{phase\,,Y} = \frac{V_{line,\Delta}}{\sqrt{3}\angle 30°}$
- **Δ**-connected loads can be replaced by an equivalent Y-connected loads with $Z_Y = \frac{Z_\Delta}{3}$.

Δ- Y Transformation - Source



We would like to have:

$$V_{ab,\Delta} = V_{ab,Y}$$
$$V_{bc,\Delta} = V_{bc,Y}$$
$$V_{ca,\Delta} = V_{ca,Y}$$

$$V_{ab,Y} = V_{an,Y} - V_{bn,Y} = \sqrt{3}V_{an,Y} \cdot e^{j30^\circ}$$
$$V_{ab,\Delta} = \sqrt{3}V_{an,Y} \cdot e^{j30^\circ}$$
$$V_{an,Y} = \frac{V_{ab,\Delta}}{\sqrt{3} \cdot e^{j30^\circ}}$$

Three Phase Transformers Combinations

Three phase transformers are of **Δ - Δ**, Y - Y, **Δ** - Y, and Y - **Δ** combinations.

Δ - Δ

- Common in Transmission Systems
- Reliability
- 3rd harmonics propagation
- Cannot be grounded
- Cannot supply very unbalanced system
- cannot handle a large amount of single-phase load.

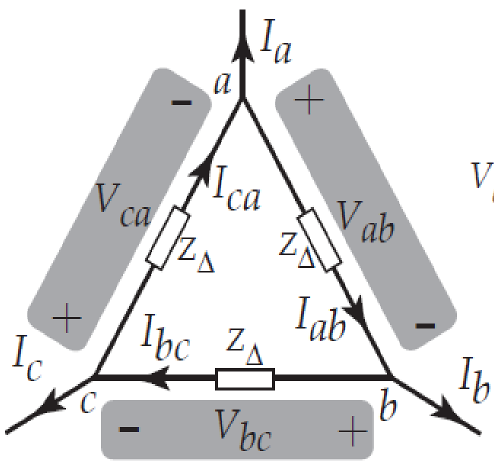
Y - Y

- Can be grounded on each side.
- No phase shift
- No 3rd harmonics propagation.

Δ - Y

- Most commonly used connection scheme (Step-down transformer between Transmission and Distribution.
- Presence of Neutral/ Can be grounded.
- 3rd harmonics propagation.
- Reliable
- Phase shift

Δ- Y Transformation - Loads

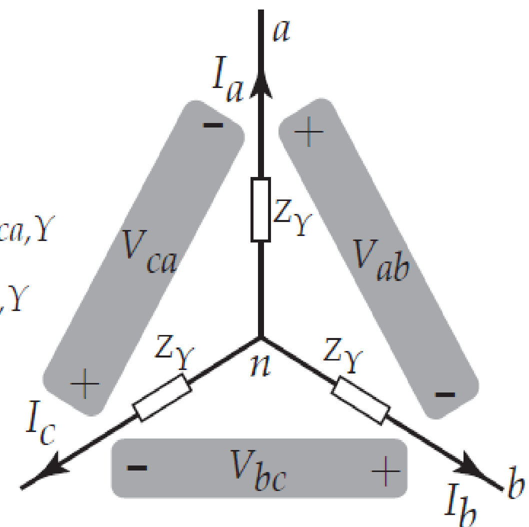


Δ-Connected

Equivalent if:

$V_{ab,\Delta}=V_{ab,Y}$ $\quad V_{bc,\Delta}=V_{bc,Y}$ $\quad V_{ca,\Delta}=V_{ca,Y}$

$I_{a,\Delta}=I_{a,Y}$ $\quad I_{b,\Delta}=I_{b,Y}$ $\quad I_{c,\Delta}=I_{c,Y}$



Y-Connected

Contd…

Δ- connected loads can be replaced by Y-connected loads with $Z_Y = \frac{z_\Delta}{3}$.

From Δ-side: we have,

$$I_a = I_{ca} - I_{ab} = \frac{V_{ca}}{z_\Delta} - \frac{V_{ab}}{z_\Delta} = \frac{V_{ca} - V_{ab}}{z_\Delta}$$

From Y-side: we have,

$$V_{ab} = V_{an} - V_{bn} = I_b \cdot z_Y - I_a \cdot z_Y.$$

$$V_{ca} = V_{cn} - V_{an} = I_a \cdot z_Y - I_c \cdot z_Y$$

$$V_{ca} - V_{ab} = Z_Y(2I_a - I_b - I_c)$$

$$I_a + I_b + I_c = 0 \Rightarrow I_a = -I_b - I_c \cdot \frac{Z_0}{3}$$

$$z_Y = \frac{z_\Delta}{3}$$

$$V_{ca} - V_{ab} = z_Y \cdot 3I_a$$

Contd…

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$$V_{ca} - V_{ab} = z_Y \cdot 3I_a$$

$$\boxed{z_Y = \frac{z_\Delta}{3}}$$

Per Phase Analysis

Per phase analysis allows analysis of balanced 3-phase systems with the same effort as for a single-phase system. Per phase analysis is applicable to balanced 3-phase systems with all loads and sources Y-connected and no mutual inductance between phases.

In such systems:

- All neutrals are at the same potential.
- All phases are COMPLETELY decoupled.
- All system values are the same sequence as sources. The sequence order we have been using (phase b lags phase a and phase c lags phase b) is known as positive sequence.
- Convert all Δ load/sources to equivalent Y's.
- Keep in mind that all naturals are at the same potential.

Contd…

- Solve phase "a" independent of the other phases.
- Total system power is then $S = 3V_a I_a^*$.
- If desired, phase "b" and "c" values can be determined by inspection (i.e., $\pm 120°$ degree phase shifts).
- If necessary, go back to original circuit to determine line-line values or internal Δ values.

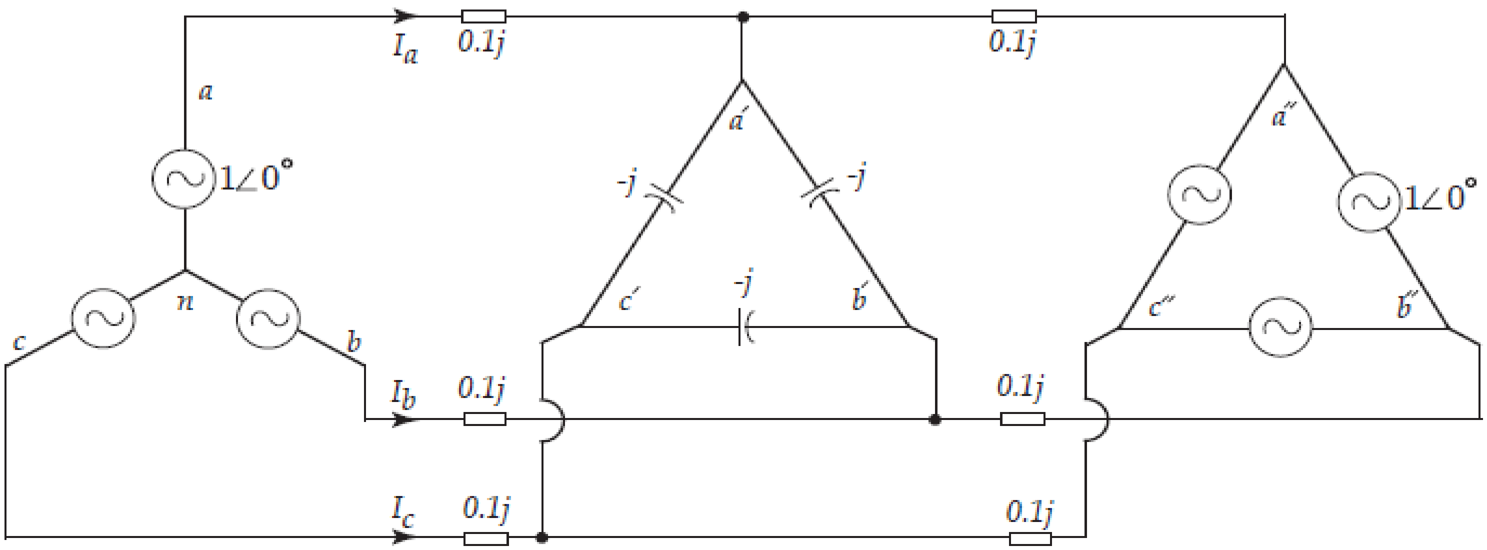
Per Phase Analysis Example

Assume a 3phase, Y-connected generator with $V_{an} = 1\angle 0°$ volts supplies a Δ-connected load with $Z_\Delta = -j$ through a transmission line with impedance of $0.1j$ per phase. The load is also connected to a Δ connected generator with $V_{ajbj} = 1\angle 0°$ through a second transmission line which also has an impedance of $j0.1$ per phase.

Find:

1. The load voltage $V_{a'b'}$
2. The total power supplied by each generator, S_Y and S_Δ.

Contd…



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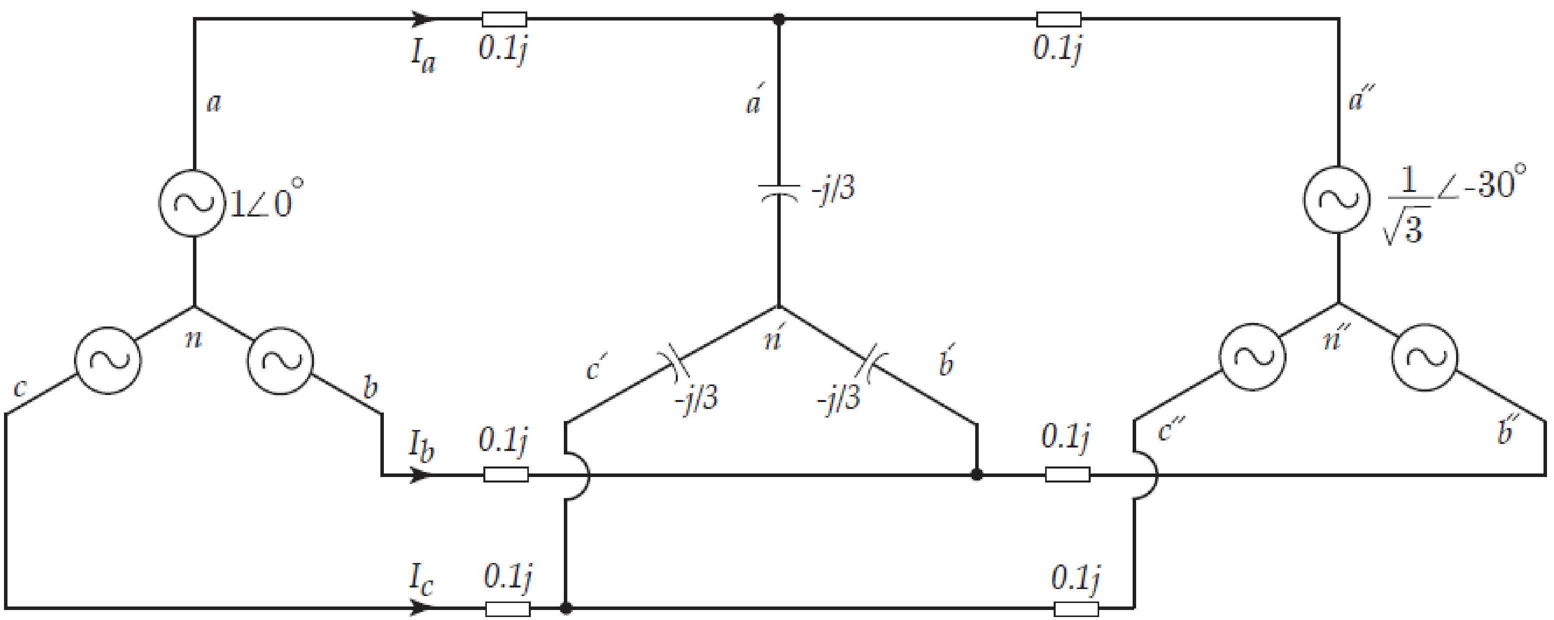
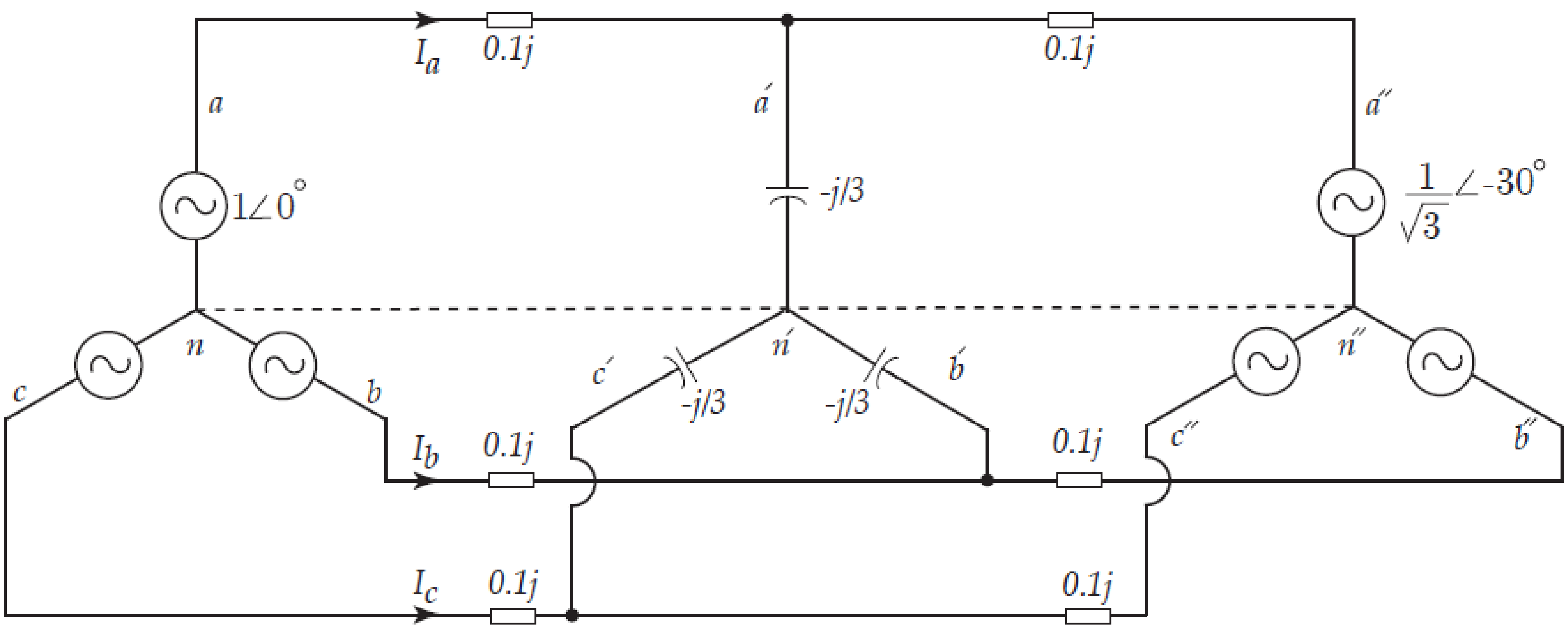
Contd…



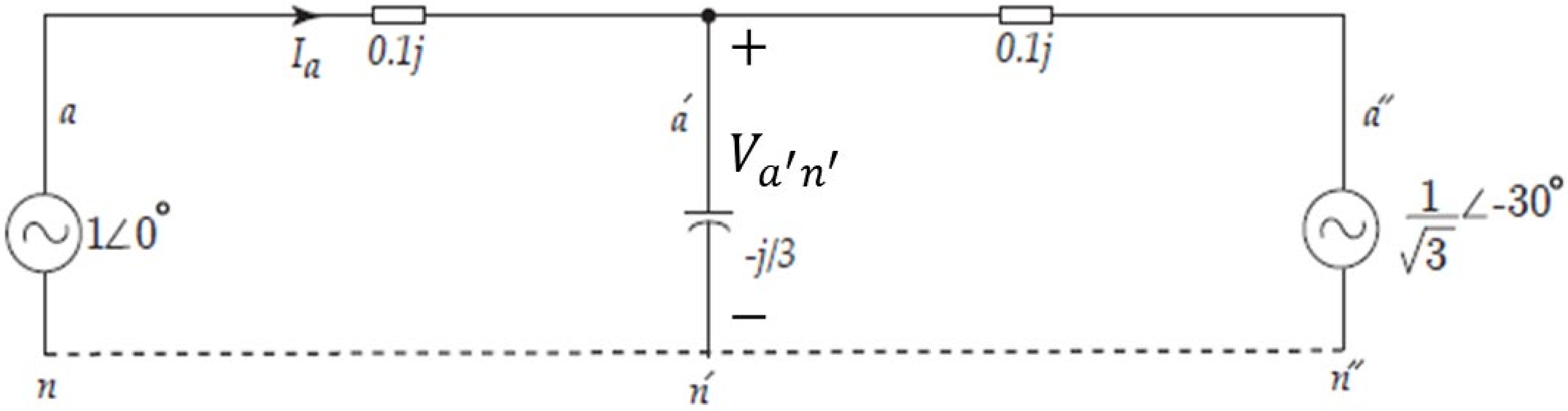
First, convert the Δ load and sources to their equivalent Y load and sources.

Contd…



Since all neutral points are at the same potential, we may assume a line between them to better illustrate phase a.

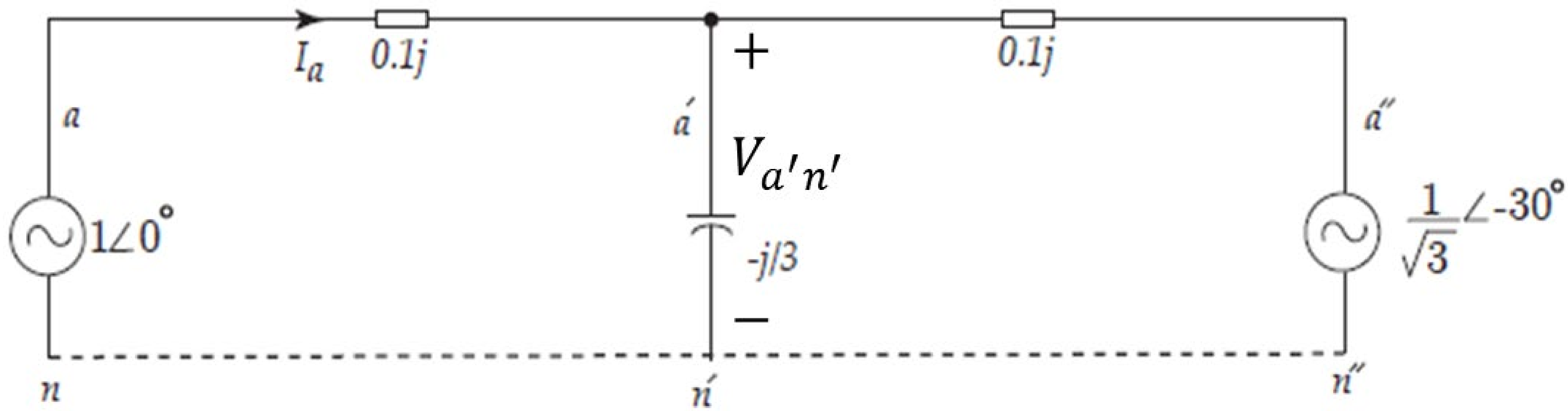
Contd…



We draw phase a and solve the circuit by writing KCL at a’:

$$(V_{a'n'} - 1\angle 0)(-10j) + V_{a'n'}(3j) + \left(V_{a'n'} - \frac{1}{\sqrt{3}}\angle -30^\circ\right)(-10j) = 0$$

Contd…



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Therefore,

$$\left(10j + \frac{10}{\sqrt{3}}\angle 60^\circ\right) = V_{a'n'}(10j - 3j + 10j) \rightarrow V_{a'n'} = 0.9\angle -10.9^\circ\mathrm{V}$$

Contd…

Therefore,

$V_{a'n'} = 0.9\angle -10.9°$ Volts

$V_{b'n'} = 0.9\angle -130.9°$ Volts

$V_{c'n'} = 0.9\angle -250.9°$ Volts

Contd…

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And,

$V_{a'b'} = V_{a'n'} - V_{b'n'} = 1.56\angle 19.1°$ volts

Contd...

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And,

$V_{a'b'} = V_{a'n'} - V_{b'n'} = 1.56\angle 19.1°$ volts

Thus,

$$S_{Y,Gen} = 3V_{an}I_a^* = 3V_{an}\left(\frac{V_{an} - V_{a'n'}}{j0.1}\right)^* = 5.1 + j3.5\text{VA}$$

$$S_{\Delta,Gen} = 3V_{a''n''}I_{a''}^* = 3V_{a''n''}\left(\frac{V_{a''n''} - V_{a'n'}}{j0.1}\right)^* = -5.1 - j4.7\text{VA}$$

Contd…

Thus,

$$S_{Y,Gen} = 3V_{an}I_a^* = 3V_{an}\left(\frac{V_{an} - V_{a'n'}}{j0.1}\right)^* = 5.1 + j3.5\text{VA}$$

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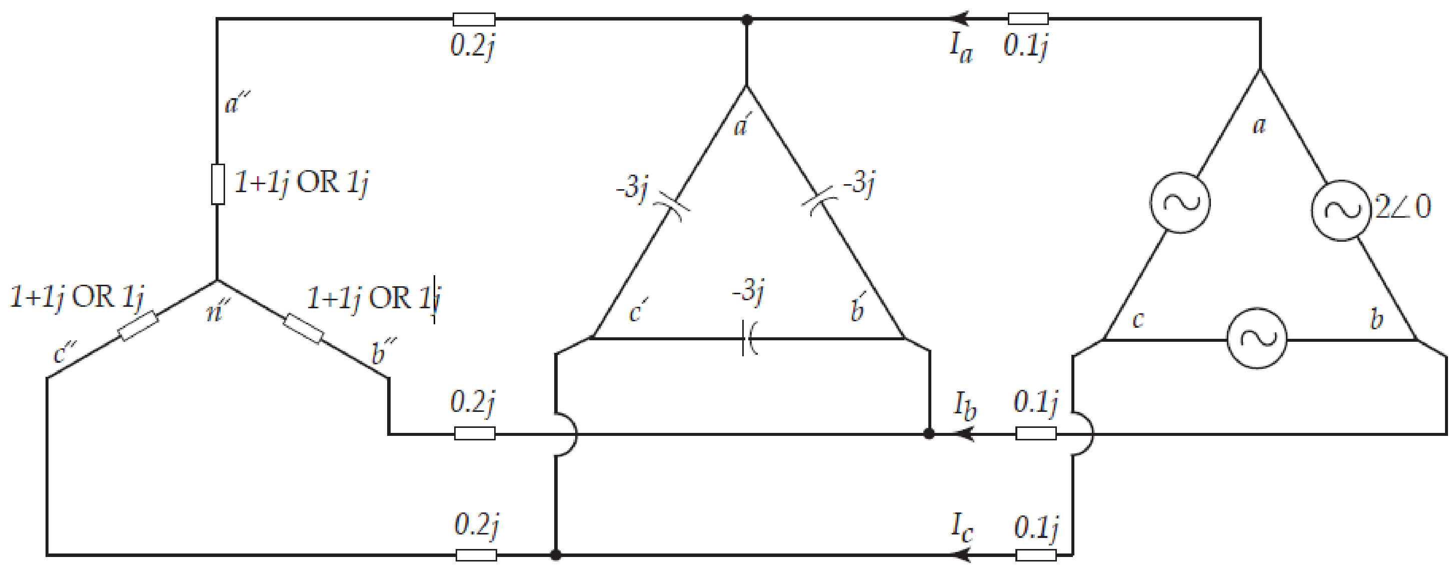
Note that we calculate the complex power in the transformed circuit. Since Δ source and its transformed Y source are equivalent, their complex power is the same.

Per Phase Analysis Example 2

Assume a 3-phase, Δ-connected generator with $V_{ab} = 2\angle 0°$ volts supplies a Δ-connected load with $Z_\Delta = -3j\Omega$ through a transmission line with impedance of $0.1j\Omega$ per phase. The load is also connected to another Y-connected load with impedance of Z_{load} through a second transmission line which has an impedance of $0.2j\ \Omega$ per phase.

Calculate the complex power of the second load for two scenarios:
i) $Z_{load} = 1j\Omega$ and
ii) $Z_{load} = 1 + 1j\Omega$

Per Phase Analysis Example 2



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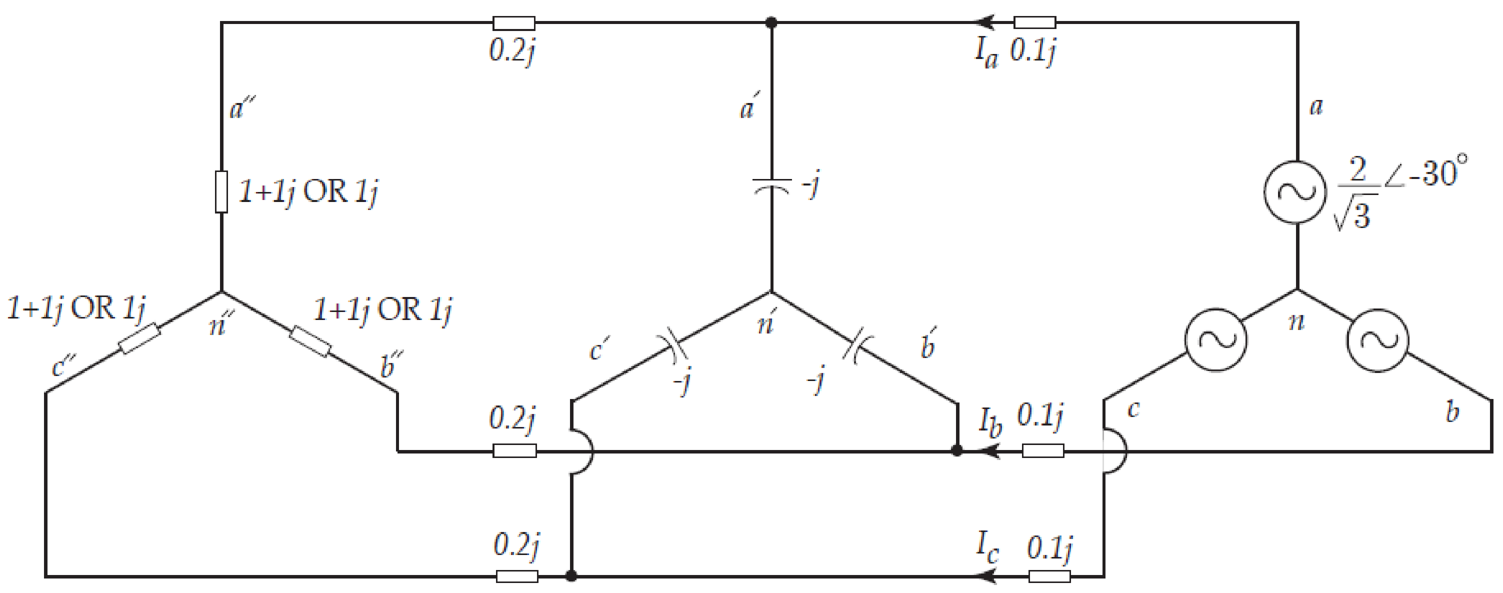
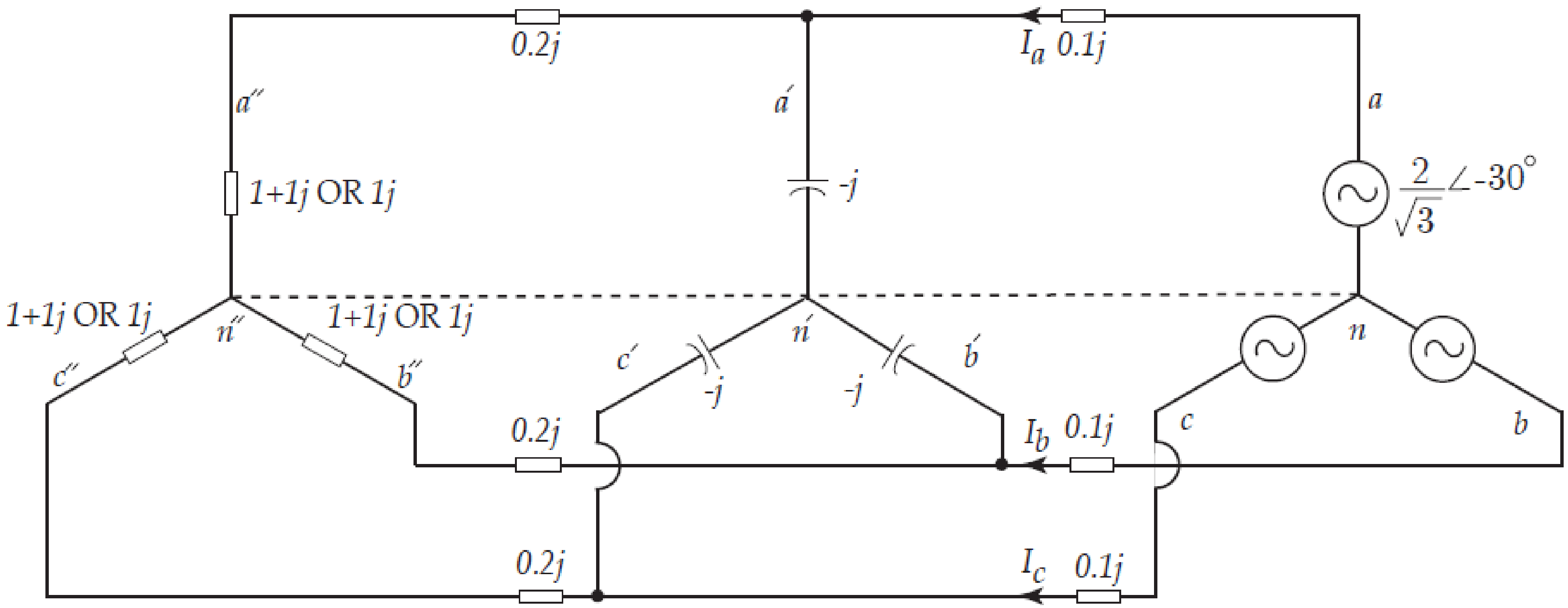


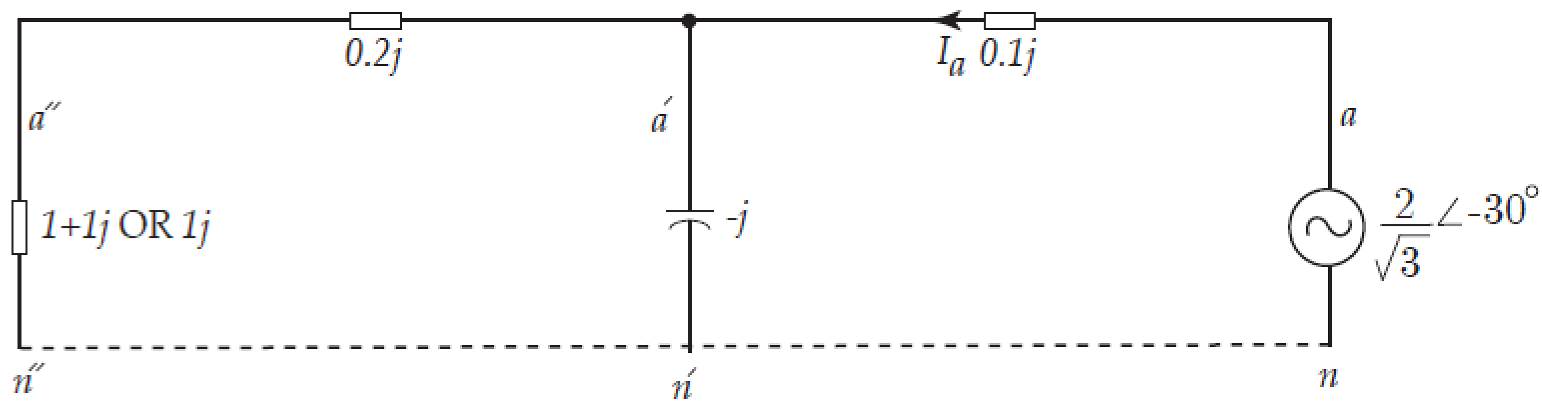
First, convert the Δ load and sources to their equivalent *Y* load and sources.

Per Phase Analysis Example 2



Since all neutral points are at the same potential, we may assume a line between them to better illustrate phase a.

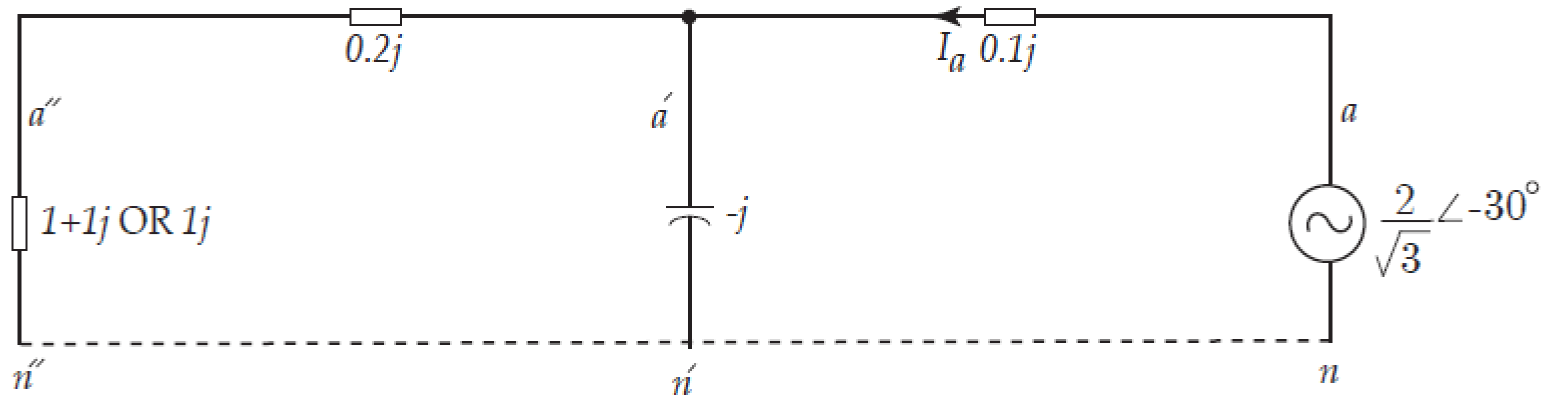
Per Phase Analysis Example 2



We draw phase a and solve the circuit by writing KCL at a':

$$\frac{V_{a'n'}}{0.2j+1+1j}+\frac{V_{a'n'}}{-j}+\frac{V_{a'n'}-\frac{2}{\sqrt{3}}\angle-30^\circ}{0.1j}=0$$

Per Phase Analysis Example 2



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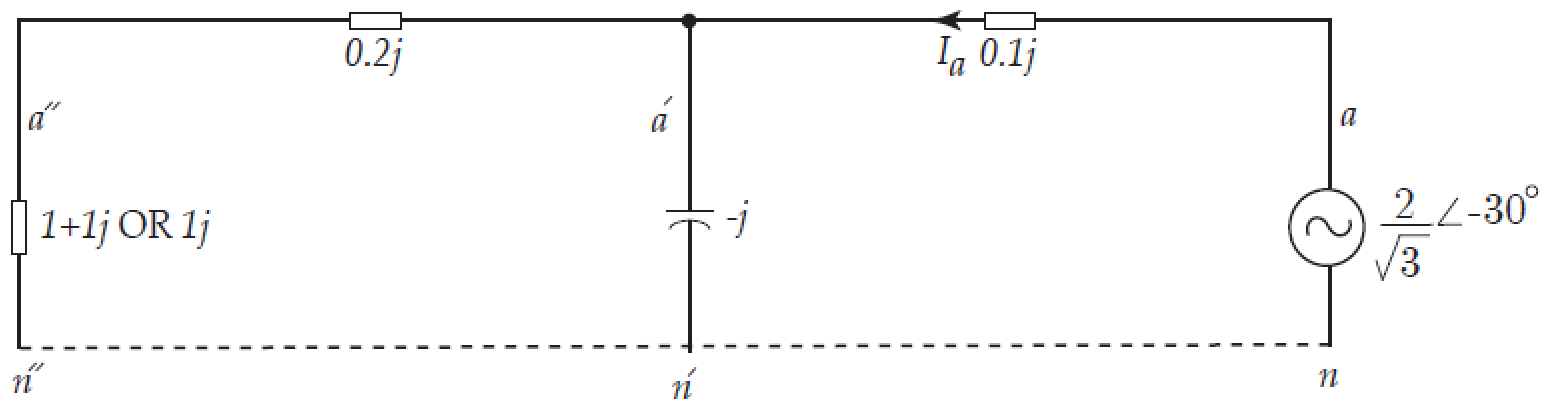
$$\frac{V_{a'n'}}{0.2j+1+1j}+\frac{V_{a'n'}}{-j}+\frac{V_{a'n'}-\frac{2}{\sqrt{3}}\angle-30^\circ}{0.1j}=0$$

Therefore,

$$\frac{-20}{\sqrt{3}}\angle 60^\circ = V_{a'n'}\left(\frac{1}{1+1.2j}+j-10j\right)\rightarrow V_{a'n'}=1.21\angle-32.47^\circ\mathrm{V}$$

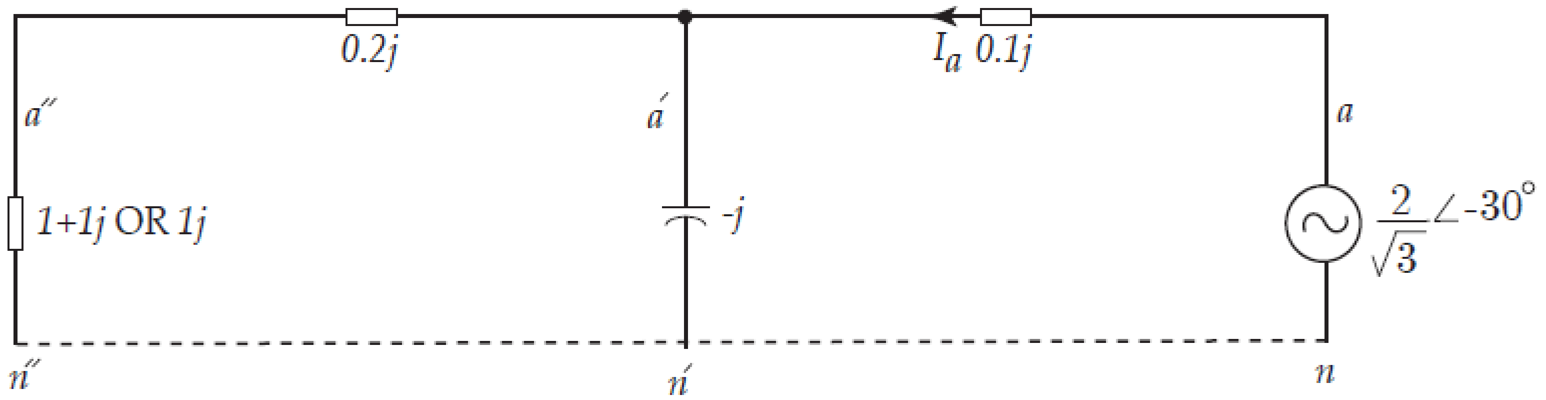
Per Phase Analysis Example 2



Therefore,

$$I_{load\ 1,a} = \frac{V_{a'n'}}{1 + 1j + 0.2j} = 0.78\angle - 82.66°\text{amps}$$

Per Phase Analysis Example 2



Therefore,

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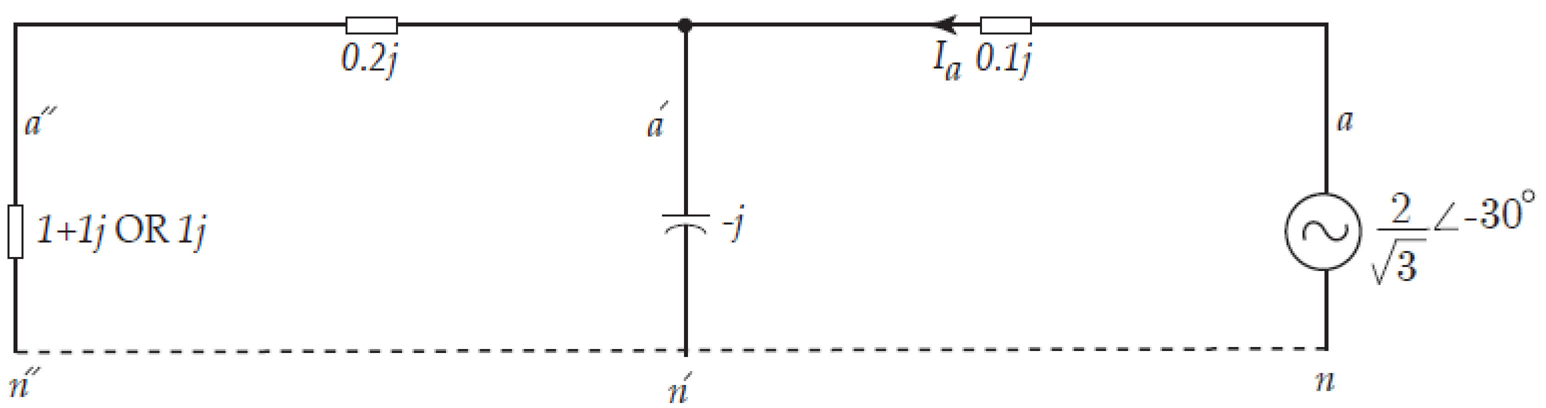
Thus,

$$V_{a''n''} = I_{load1,a}\,(1+1j) = 1.1\angle-37.68°\text{ volts}$$

And,

$$S_{load\ 1} = 3V_{a''n''}I^{*}_{load1,a} = 1.81 + j1.81\text{VA}$$

Per Phase Analysis Example 2



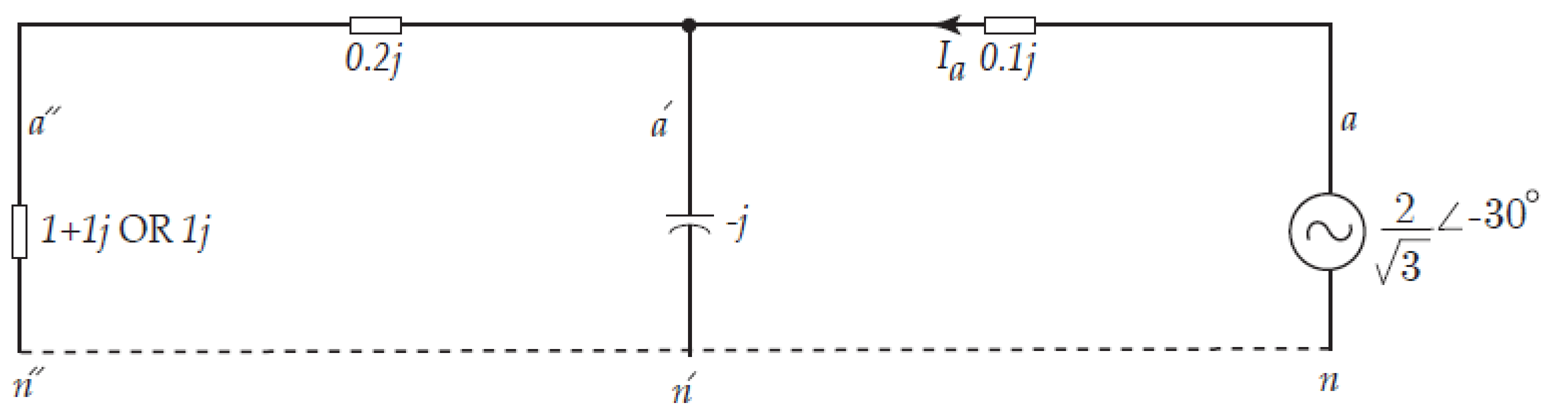
For the other load, we write the same KCL at a':

$$\frac{V_{a'n'}}{0.2j+1j}+\frac{V_{a'n'}}{-j}+\frac{V_{a'n'}-\frac{2}{\sqrt{3}}\angle-30^\circ}{0.1j}=0$$

Therefore,

$$\frac{-20}{\sqrt{3}}\angle 60^\circ = V_{a'n'}\left(\frac{1}{1.2j}+j-10j\right)\rightarrow V_{a'n'}=1.17\angle-30^\circ \text{ volts}$$

Per Phase Analysis Example 2



Therefore,

$$I_{load\ 2,a} = \frac{V_{a'n'}}{1j + 0.2j} = 0.98\angle - 120°\text{amps}$$

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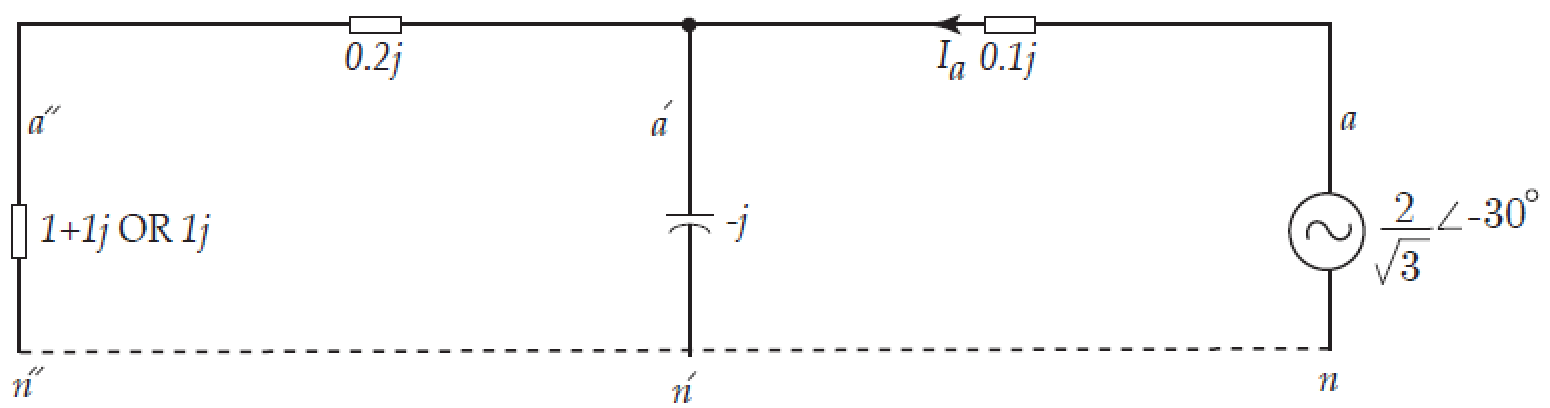

Therefore,

$$I_{load\ 2,a} = \frac{V_{a'n'}}{1j + 0.2j} = 0.98\angle - 120°\text{amps}$$

Thus,

$$V_{a''n''} = I_{\text{load } 2,a}(1j) = 0.98\angle - 30°\text{ volts} \quad \text{and,}$$

$$S_{load\ 2} = 3V_{a''n''}I^{*}_{load\ 2,a} = 0 + j2.87\text{VA}$$

Key Messages

- Three-phase systems are usually used at high power/voltage levels.
- Three-phase systems are more economical for transferring energy as they need less amount of wire to transmit the same amount of energy compared to single or two-phase systems.
- To analyze complicated three-phase BALANCED systems, $\Delta - \mathrm{Y}$ transformation is the key element.
- In three-phase BALANCED systems, when all Δ loads and sources are transformed to Y connections, neutral points are at the same potential.

Power in $1-\emptyset$ Circuits

- The instantaneous power delivered to a load is,

$$
\begin{aligned}
p(t) &= \mathrm{v(t)i(t)} \\
&= \sqrt{2}\,\mathrm{V}\sin\omega t\sqrt{2}I\sin(\omega t-\phi) \\
&= \mathrm{VI}\cos\phi - \mathrm{V}I\cos(2\omega t-\phi)
\end{aligned}
$$

- This power clearly has an average value, $P = VI\cos\phi$ (W). This is referred to as the "real power." The second, double frequency, term has no time averaged value, and therefore does no work.
- $\cos\phi$ is the "power factor"
- "Reactive Power," $Q = VI\sin\phi$ (VAR)
- $\mathrm{Q} > 0 \Rightarrow \phi > 0 \Rightarrow R-L$ load. $\mathrm{Q} < 0 \Rightarrow \phi < 0 \Rightarrow R-C$ load.
- Complex Power, $\mathrm{S} = \mathrm{P} + j\mathrm{Q} = \mathrm{VI}^*$ (* is complex conjugate)

Power in $3-\emptyset$ Circuits

- The instantaneous power is:

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

$$p_a(t) = \sqrt{2}V_{an}\cos(\omega t)\sqrt{2}I_a\cos(\omega t - \phi)$$

$$p_b(t) = \sqrt{2}V_{an}\cos(\omega t - 2\pi/3)\sqrt{2}I_a\cos(\omega t - 2\pi/3 - \phi)$$

$$p_c(t) = \sqrt{2}V_{an}\cos(\omega t + 2\pi/3)\sqrt{2}I_a\cos(\omega t + 2\pi/3 - \phi)$$

$$\begin{aligned} p(t) &= 2V_{an}I_a[\cos(\omega t)\sqrt{2}\cos(\omega t - \phi) + \cos(\omega t - 2\pi/3)\cos(\omega t - 2\pi/3 - \phi) + \cos(\omega t + 2\pi/3)c \\ &= V_{an}I_a[\cos(2\omega t - \phi)\cos(\phi) + \cos(2\omega t - 4\pi/3 - \phi)\cos(\phi) + \cos(2\omega t + 4\pi/3 - \phi)\cos(\phi)] \\ &= 3V_{an}I_a\cos(\phi) \\ &= P \end{aligned}$$

3x power with one more wire compared to single phase

Thank You!