IOWA STATE UNIVERSITY

ECpE Department

EE 303 Energy Systems and Power Electronics

Magnetics, Transformer and Harmonics

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Ampere’s Law

Ampere's circuital law, discovered by Andre-Marie Ampere in 1826, relates the integrated magnetic field around a closed loop to the electric current passing through the loop:

$$I_e = \oint \mathbf{H}.d\mathbf{l} = F$$

In which,

$\oint$ = Line integral along a closed path ($d\mathbf{l}$ is tangent to the path)
I_e = Algebraic sum of current linked by the path
$\mathbf{H}$ = Magnetic Field Intensity (amp-turns/meter)
$d\mathbf{l}$ = Vector differential path length (meters)
F = mmf = magnetomotive force (amp-turns)

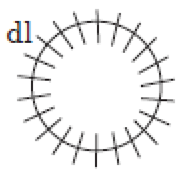
What is Line Integral?

Line integrals are a generalization of traditional integration.

$$\int_a^b f(x)dx$$

Integration along the x-axis

$$\int_\Gamma H.dl$$



Integration along a general path which could be closed

Ampere's law is most useful in cases of symmetry, such as with an infinitely long line.

Magnetic Flux Density

Magnetic fields are usually measured in terms of flux density, **B**, whose unit is Tesla [T] or Gauss [G], and 1 T = 10,000 G.

For a linear magnetic material:

$\mathbf{B} = \mu\mathbf{H}$ where μ is the called the permeability.

$\mu = \mu_0\mu_r$

μ_0 = permeability of free space $= 4\pi \times 10^{-7}\,\mathrm{H/m}$

μ_r = relative permeability $= 1$ for air

Contd…

Total flux passing through a surface A is:

$$\phi = \int_A B \cdot da$$

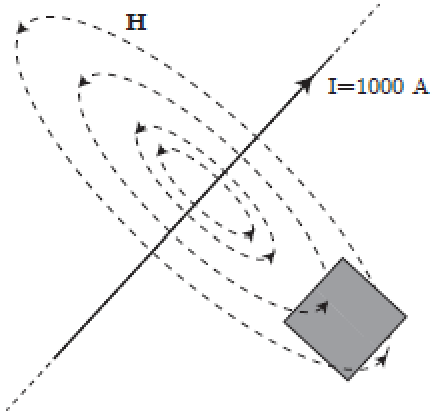
Where, da is a vector with direction normal to the surface.

If flux density B is uniform and perpendicular to an area A, then:

$$\phi = BA$$

Magnetic Fields from Single Wire

Q. Assume we have an infinitely long wire with current of 1000 A. How much magnetic flux passes through a 1-meter square, located between 4 and 5 meters from the wire?



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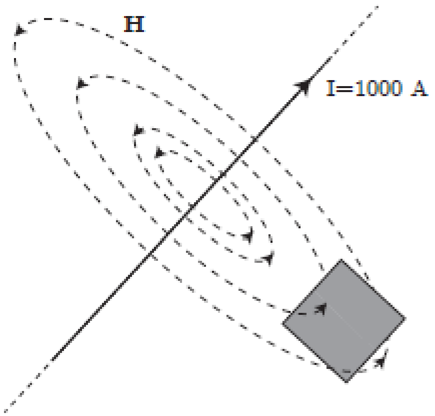


Solution:

The easiest way to solve the problem is to take advantage of symmetry. For an integration path we will choose a circle with a radius of x.

Contd…

$$I_e = \oint \mathbf{H.dl}$$

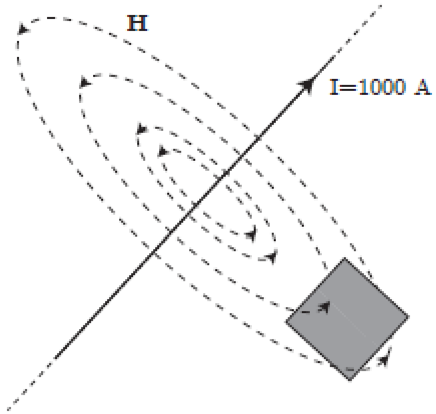
$$I = 2\pi x H \rightarrow H = \frac{I}{2\pi x}$$

$$B = \mu_0 H$$

$$B = \frac{2\times 10^{-4}}{x}\ \mathrm{T} = \frac{2}{x}\ \mathrm{Gauss}$$

$$\phi = \int_A \mu_0 H.dA = \int_4^5 \frac{\mu_0 I}{2\pi x} dx$$

$$\phi = \mu_0 \frac{I}{2\pi} \ln\frac{5}{4} = 4.46\times 10^{-5}\ \mathrm{Wb}$$



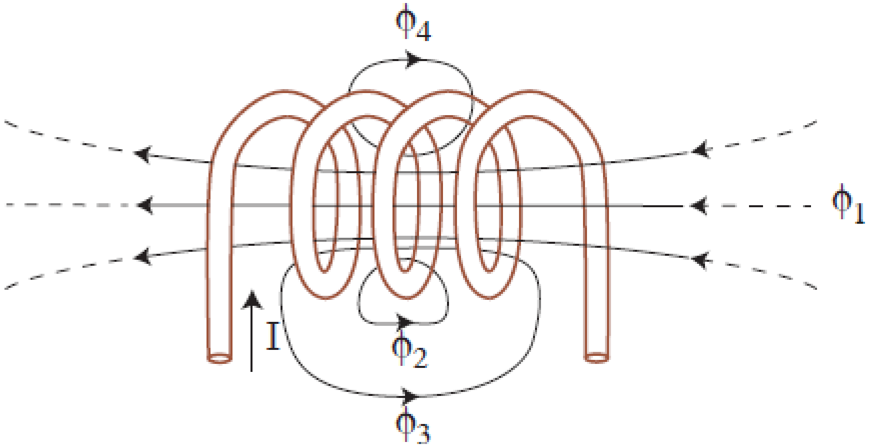
For reference, the earth's magnetic field is about 0.6 Gauss (Central US).

Flux Linkages and Faraday's Law

Flux linkage, λ, is defined from Faraday's law:

$V = \frac{d\lambda}{dt}$ where V = voltage and λ = flux linkage.

The flux linkage is the sum of the flux cut (linked) by each turn of wire. If each turn cuts (or links) flux ϕ, the total flux linkage for N turns must be $N\phi$.



Inductance

For a linear magnetic system in which

$$B = \mu H$$

we can define the inductance, L, to be the constant relating the current and the flux linkage

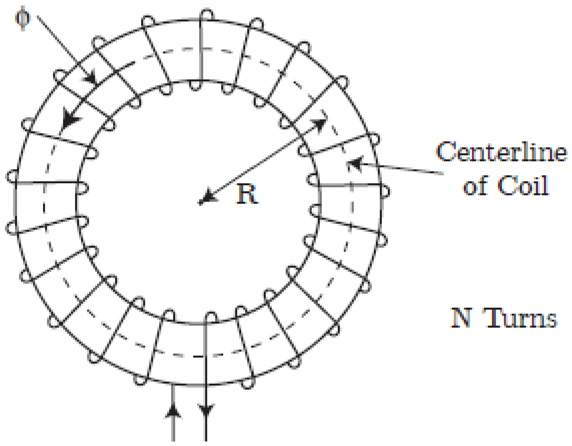
$$\lambda = Li$$

where, the unit of L is Henry (H).

Inductance Example

Calculate the inductance of an N turn coil wound tightly on a toroidal iron core that has a radius of R and a cross-sectional area of A. Assume:

1) all flux is within the coil.
2) all flux links each turn.



Inductance Example: Solution

$I_e = \int_\Gamma \mathbf{H} \cdot d\mathbf{l}$

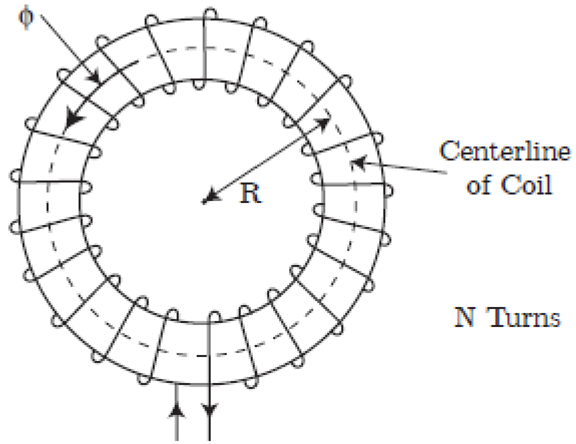
$NI = H2\pi R$ (path length is $2\pi R$)

$H = \dfrac{NI}{2\pi R}$ $\quad$ $B = \mu H = \mu_r \mu_0 H$

$\phi = AB$ $\quad$ $\lambda = N\phi = LI$

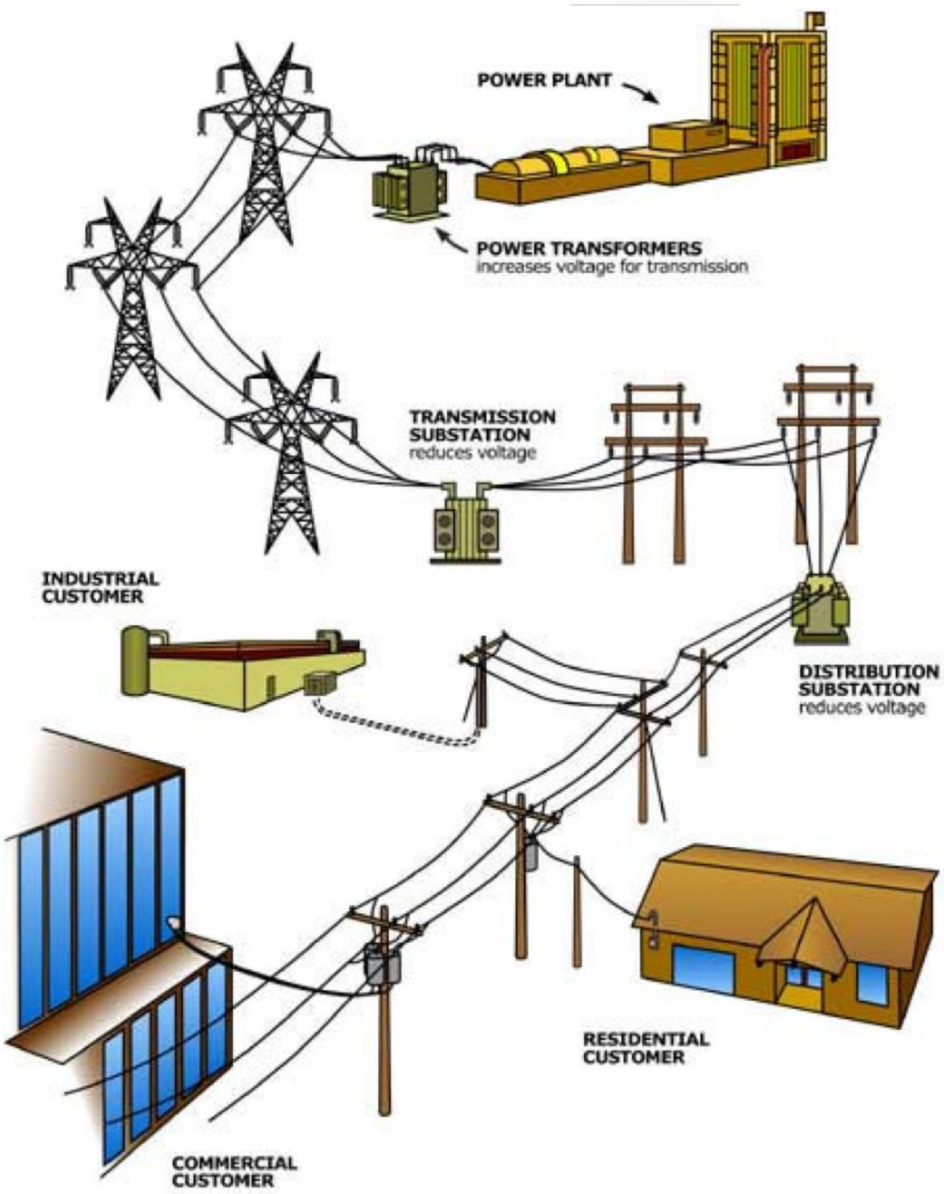
$\lambda = NAB = NA\mu_r\mu_0 \dfrac{NI}{2\pi R}$

$L = \dfrac{N^2 A \mu_r \mu_0}{2\pi R} H$



Transformers

- Power systems are characterized by many different voltage levels, ranging from 765kV down to 240/120 volts.
- Transformers are used to transfer power between different voltage levels.
- The ability to inexpensively change voltage levels is a key advantage of ac systems over dc systems.



How do they look like?

Low Power Transformers

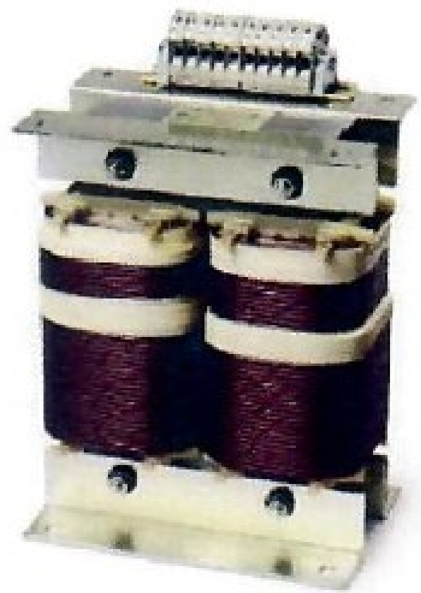


Image Source: Google Images

Transmission Transformers

Distribution Transformers

100kVA, 11KV to 240kV

Ideal Transformers

First, we review the voltage/current relationships for an ideal transformer assuming

- no real power losses,
- magnetic core has infinite permeability,
- no leakage flux.

We define the "primary" side of the transformer as the side that 17 usually takes power, and the secondary as the side that usually delivers power.

Primary is usually the side with the higher voltage, but may be the low voltage side on a generator step-up transformer.

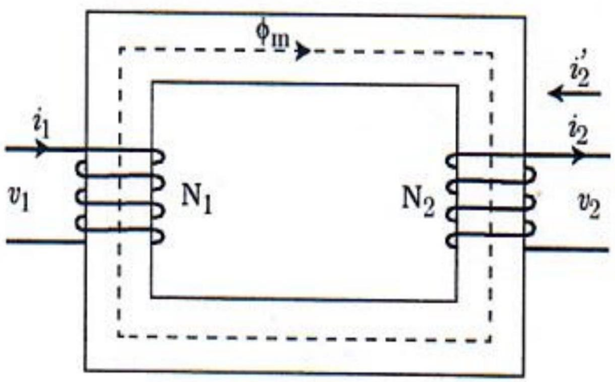
Ideal Transformer Relationships

Assume we have flux ϕ_m in magnetic material. Then:

$$\lambda_1 = N_1\phi_m \text{ and } \lambda_2 = N_2\phi_m$$

$$v_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi_m}{dt} \text{ and } v_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\phi_m}{dt}$$

$$\frac{d\phi_m}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2} \rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2} = a = \text{turn ratio}$$



Note:

- $V_1 I_1 = V_2 I_2$
- $\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$

Current/Voltage Relationships

To get the current relationships, use ampere's law:

$$\oint H.dl = N_1 i_1 + N_2 i_2'$$

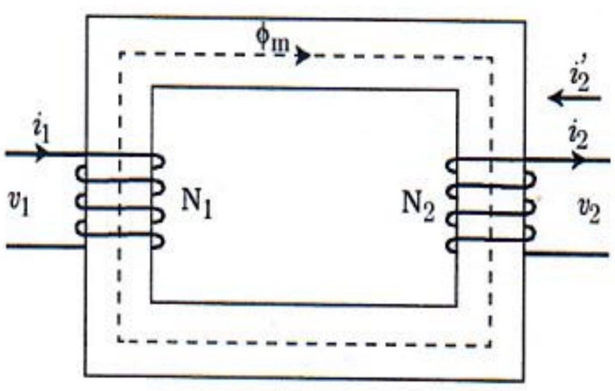
where, $i_2' = -i_2$

So, the equation can also be written as,

$$\oint H.dl = N_1 i_1 - N_2 i_2$$

or, $H * length = N_1\ i_1 + N_2 i_2'$

or, $\frac{B * length}{\mu} = N_1\ i_1 + N_2 i_2'$



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or, $\frac{B*length}{\mu} = N_1\ i_1 + N_2 i_2'$

Assuming uniform flux density in the core,

$$\frac{\phi_m \times \text{ length}}{\mu \times \text{ area}} = N_1 i_1 + N_2 i_2'$$

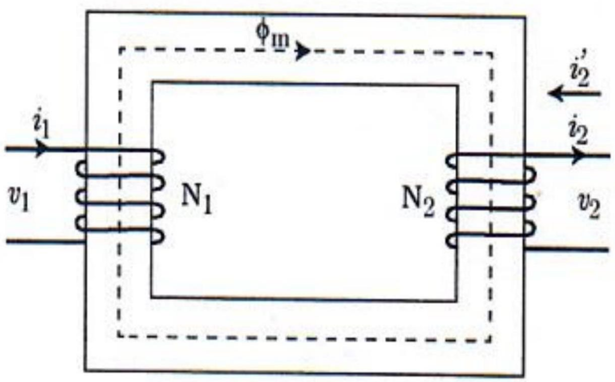
If μ is infinite, then

$$0\ = N_1 i_1 + N_2 i_2'$$

Hence,

$$\frac{i_1}{i_2'} = -\frac{N_2}{N_1}$$

$$\therefore \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$



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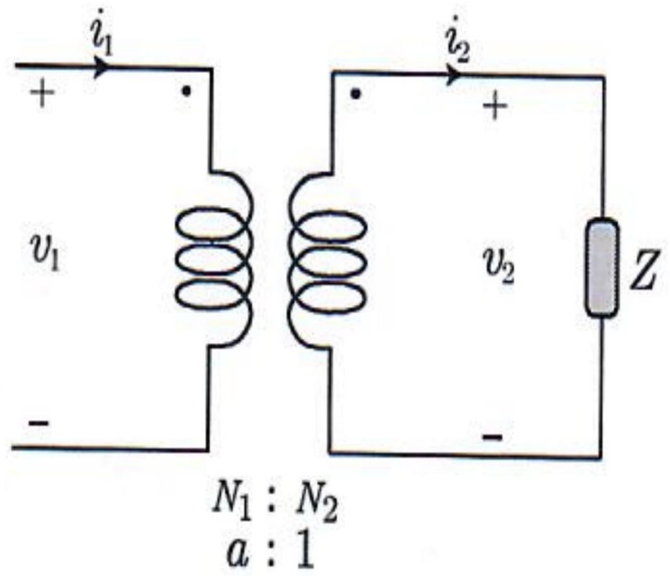
Note:

Polarity:

- Current owing into a dot on the primary winding will induce a current owing out of the dot on the corresponding secondary winding.

- Depending on how the windings are connected to the bushings, the polarities can be additive or subtractive.



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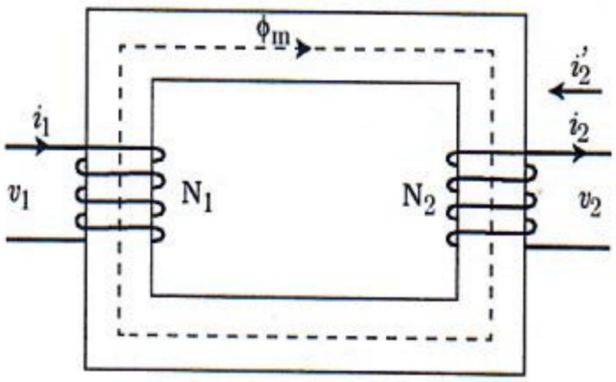
As we have derived the expression,

$$\therefore \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$



We can write in the matrix form as,

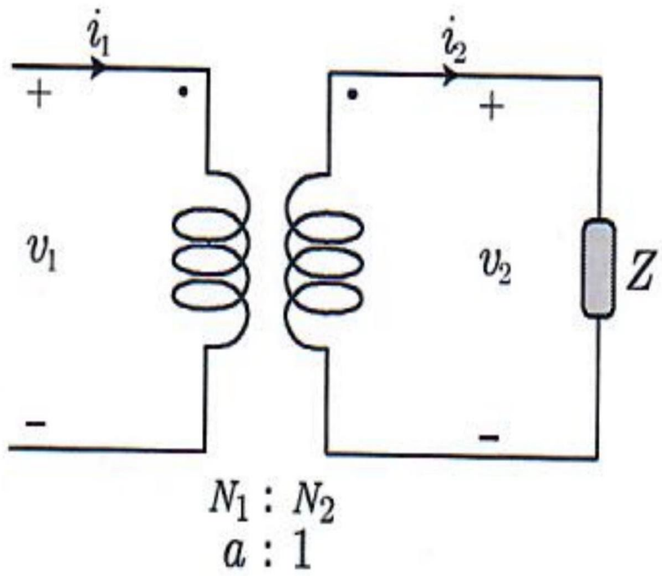
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

Impedance Transformation Example

Calculate the primary voltage and current for an impedance load on the secondary.

Solution:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ \frac{v_2}{Z} \end{bmatrix}$$



Then,

$$v_1 = a v_2 \text{ and } i_1 = \frac{v_2}{aZ}$$

Thus,

$$\frac{v_1}{i_1} = a^2 Z$$

This is the equivalent impedance in primary side.

Real Transformers

Real Transformers:

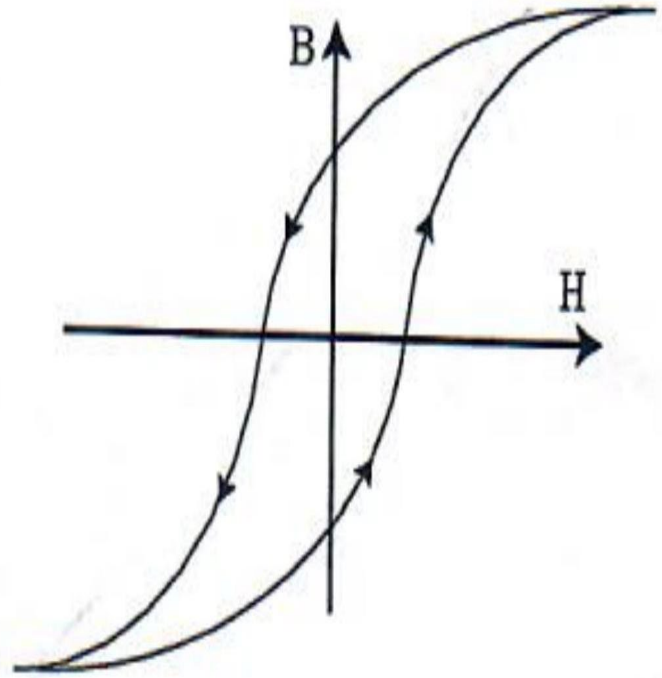
- have losses,
- have leakage flux,
- have finite permeability of magnetic core.

Transformer Losses

Real power losses are due to resistance in windings (i^2R) and core losses due to eddy currents and hysteresis.

Eddy currents arise because of changing flux in core. Eddy currents are reduced by laminating the core.

Hysteresis losses are proportional to area of BH curve and the frequency. These losses are reduced by using material with a thin BH curve.

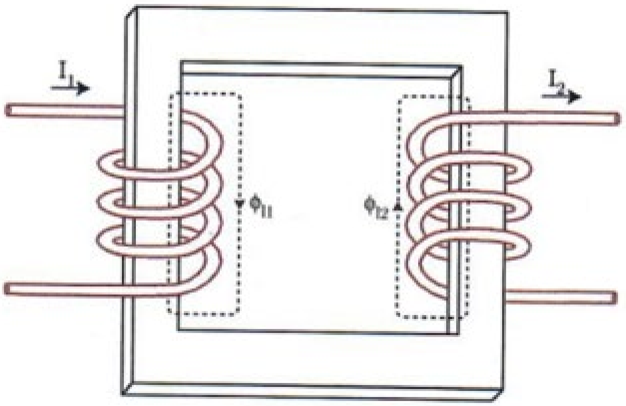


Effect of Leakage Flux

Not all flux is within the transformer core.

$$\lambda_1 = \lambda_{l1} + N_1\phi_m$$

$$\lambda_2 = \lambda_{l2} + N_2\phi_m$$



Assuming a linear magnetic medium, we get:

$$\lambda_{l1} \triangleq L_{l1}i_1 \text{ and } \lambda_{l2} \triangleq L_{l2}i_2'$$

Thus,

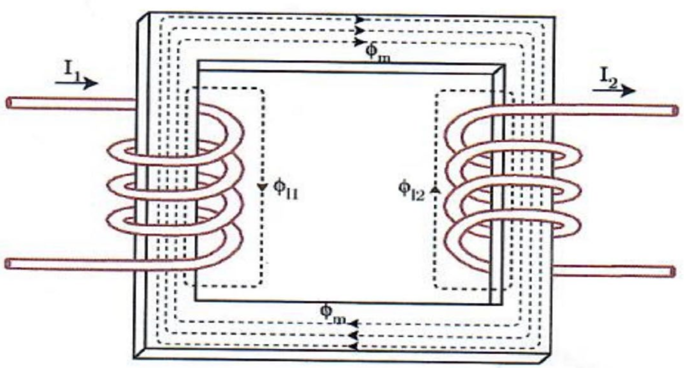
$$v_1 = r_1 i_1 + L_{l1}\frac{di_1}{dt} + N_1\frac{d\phi_m}{dt}$$

$$v_2 = r_2 i_2' + L_{l2}\frac{di_2'}{dt} + N_2\frac{d\phi_m}{dt}$$

Voltage induced by ϕ_m

Resistance of the wire

Voltage induced by flux leakage



Effect of Finite Core Permeability

Finite core permeability means a non-zero mmf is required to maintain ϕ_m in the core:

$$N_1 i_1 - N_2 i_2 = \frac{\text{length}}{\mu \times \text{area}} \phi_m = \mathcal{R}\phi_m$$

This value is usually modelled as magnetizing current.

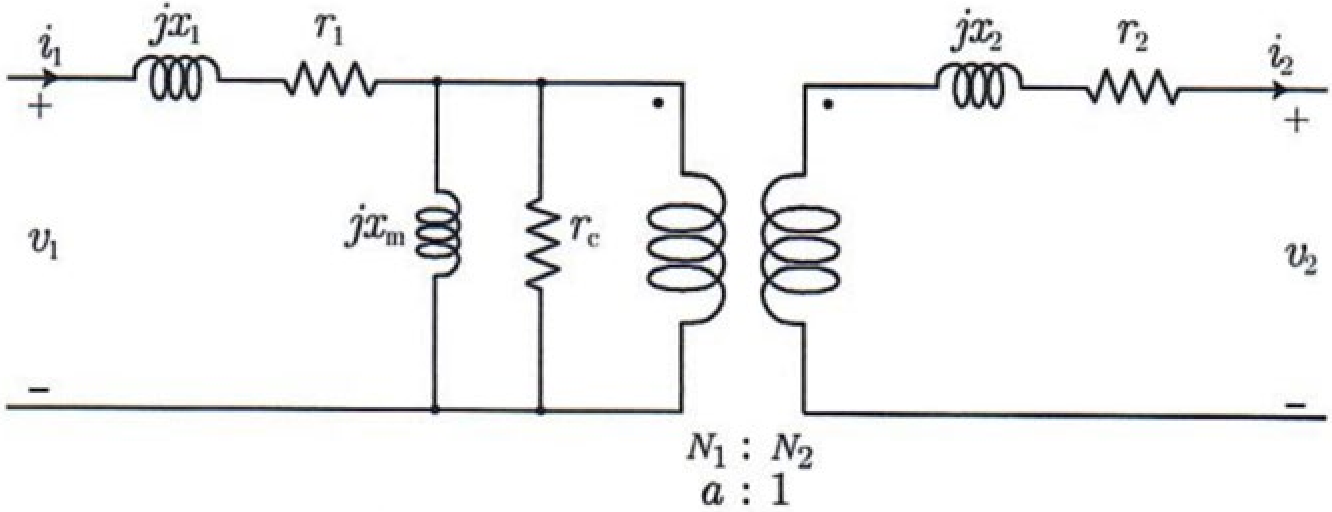
$$i_1 = \frac{\mathcal{R}\phi_m}{N_1} + \frac{N_2}{N_1} i_2$$

$$i_1 = i_m + \frac{N_2}{N_1} i_2 \ ;$$

where $i_m = \frac{\mathcal{R}\phi_m}{N_1}$ is the magnetizing current.

Transformer Equivalent Circuit

Using the previous relationships, we can derive an equivalent circuit model for the real transformer.



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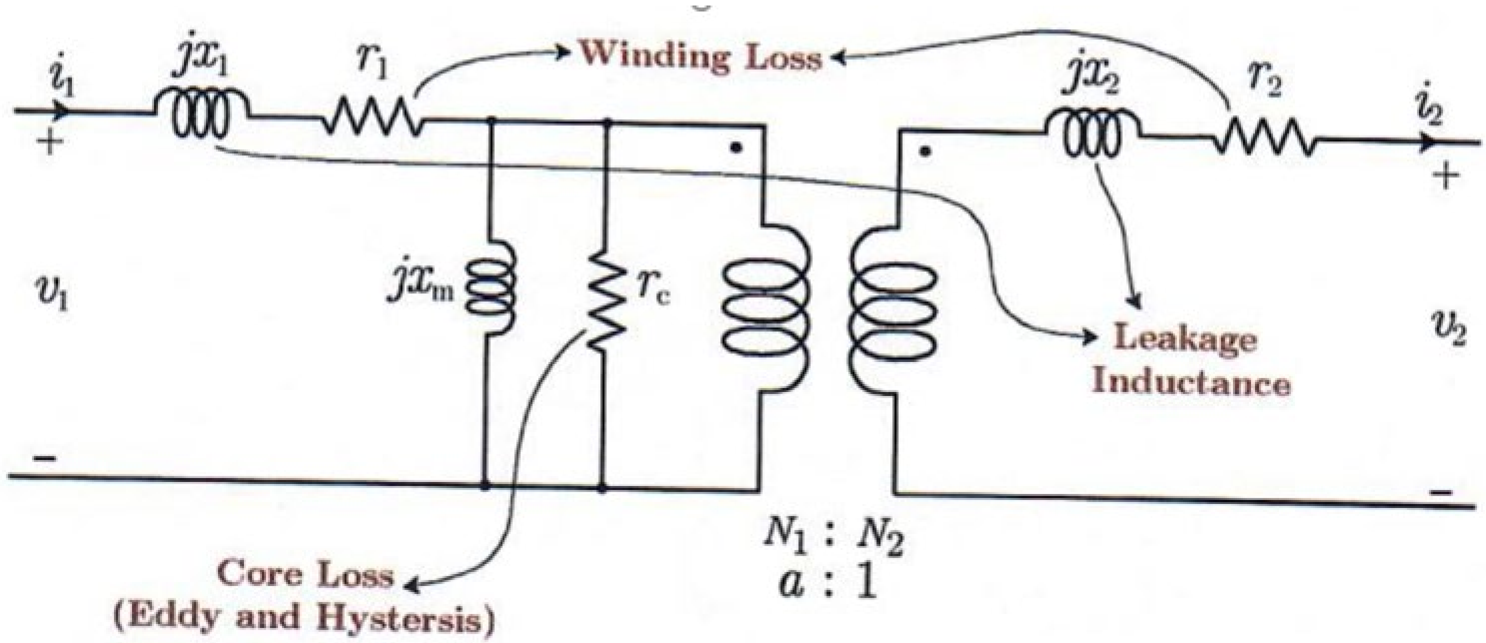
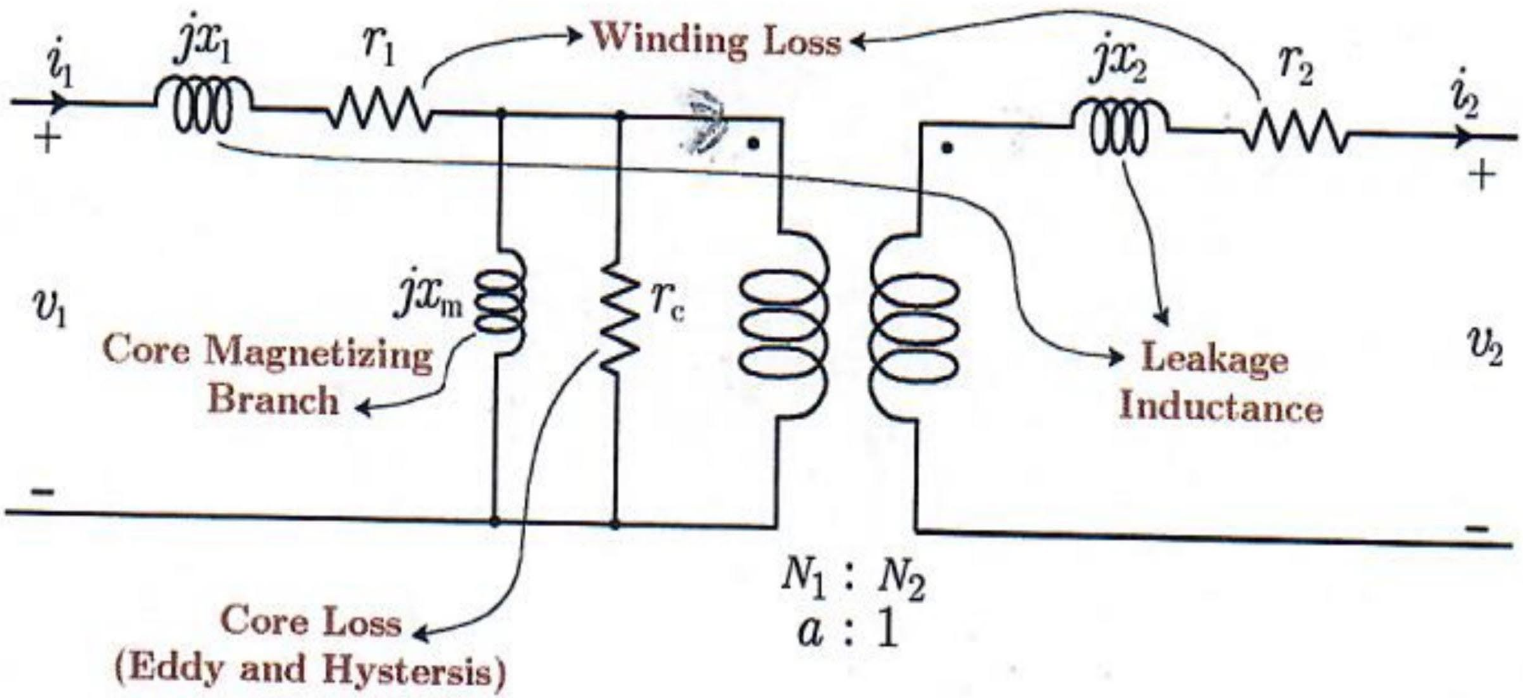


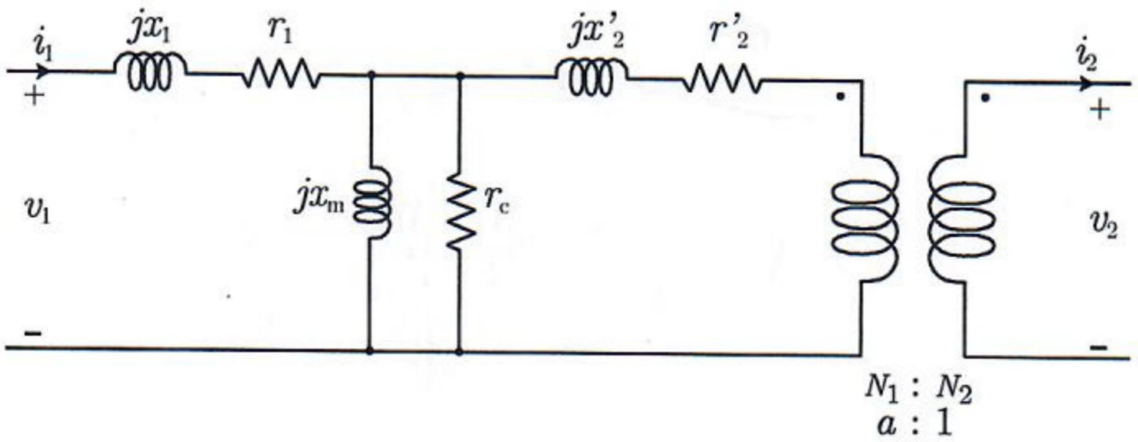
i_1
jx_1
r_1
Winding Loss
jx_2
r_2
i_2
+
+
v_1
jx_m
r_c
Leakage
Inductance
v_2
−
−
$N_1 : N_2$
$a : 1$
Core Loss
(Eddy and Hysteresis)

Contd…



i_1
jx_1
r_1
Winding Loss
jx_2
r_2
i_2
+
+
v_1
jx_m
r_c
v_2
Core Magnetizing Branch
Leakage Inductance
-
-
$N_1 : N_2$
$a : 1$
Core Loss (Eddy and Hysteresis)

Contd…

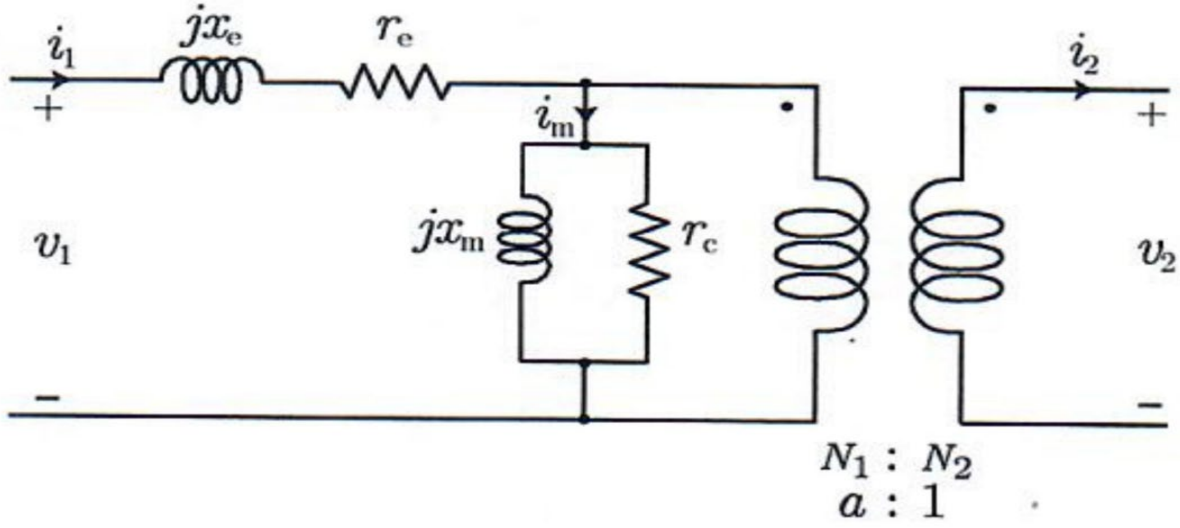


This model is further simplified by referring all impedances to the primary side:

$$r_2' = a^2 r_2$$

$$x_2' = a^2 x_2$$

Contd…



Moreover, since the impedance of the parallel branch is so high, we can approximate the equivalent circuit:

$$r_e = r_1 + r_2'$$
$$x_e = x_1 + x_2'$$

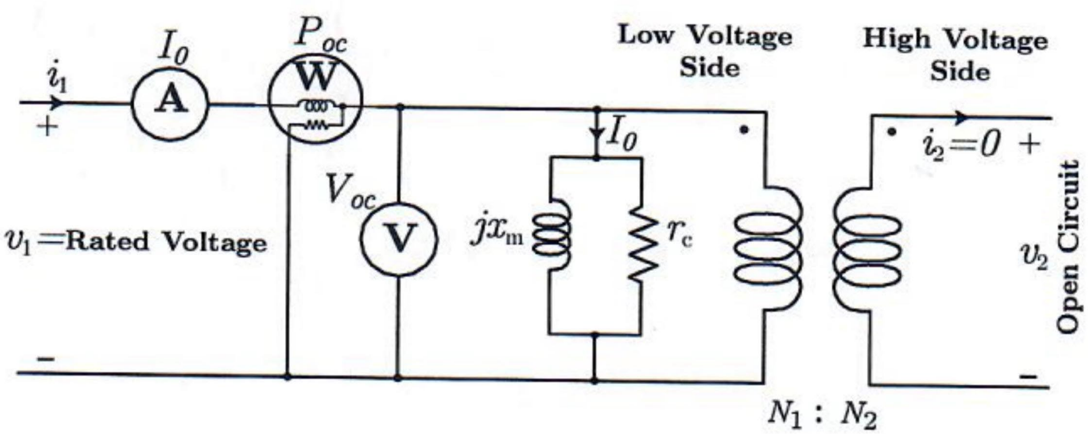
Calculation of Model Parameters

The parameters of the model are determined based upon:

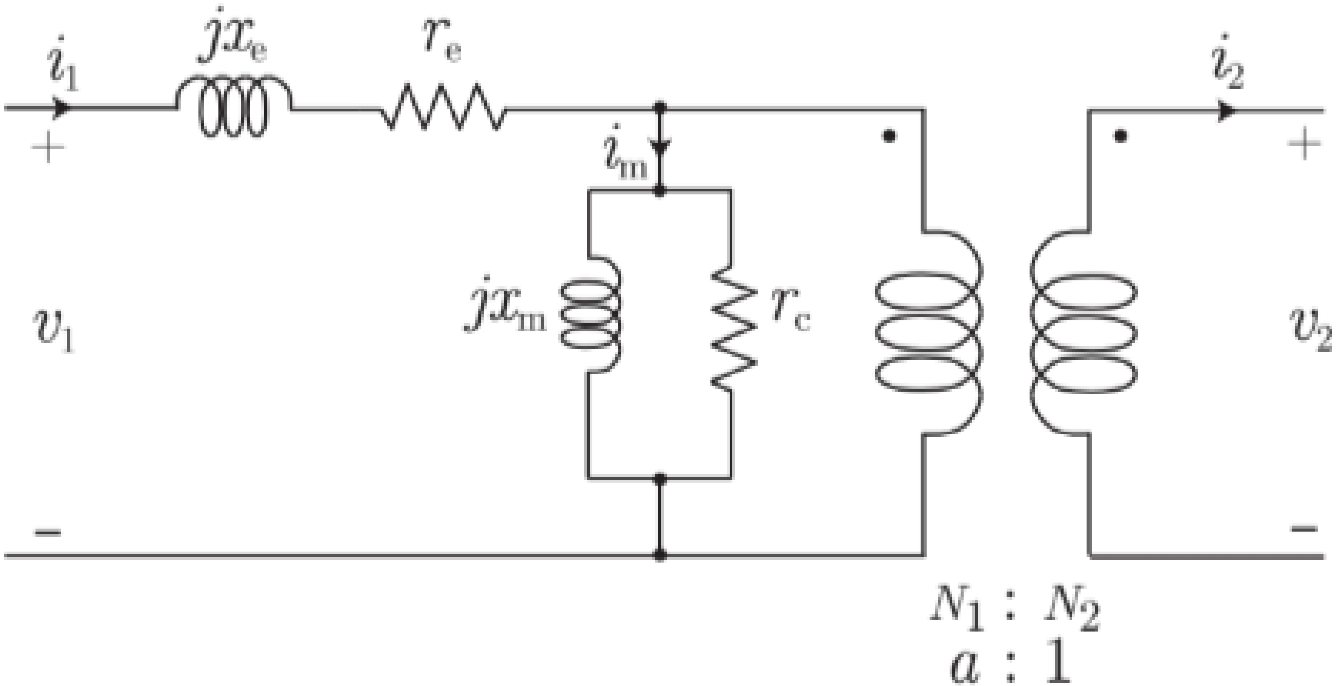
- **Nameplate data:** Gives the rated voltages and power.
- **Open circuit test:** Rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
- **Short circuit test:** With secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.

Open Circuit Test

The open circuit test is conducted to determine the parameters of the magnetizing branch: x_m and r_c. In this test, usually the high-voltage side is open-circuited, and the rated voltage is applied to the low-voltage side. Then, the current, voltage, and power at the low-voltage side are measured by using a voltmeter, an ammeter, and a watt-meter. In such a case, the voltage drop in the leakage inductance and winding resistance is negligible due to very low primary current. Therefore, the transformer equivalent circuit is as shown below.



Simplified Equivalent Circuit



Calculation of Model Parameters

The parameters of the model are determined based on:

- **Nameplate data:** Gives the rated voltages and power

- **Open Circuit Test:**

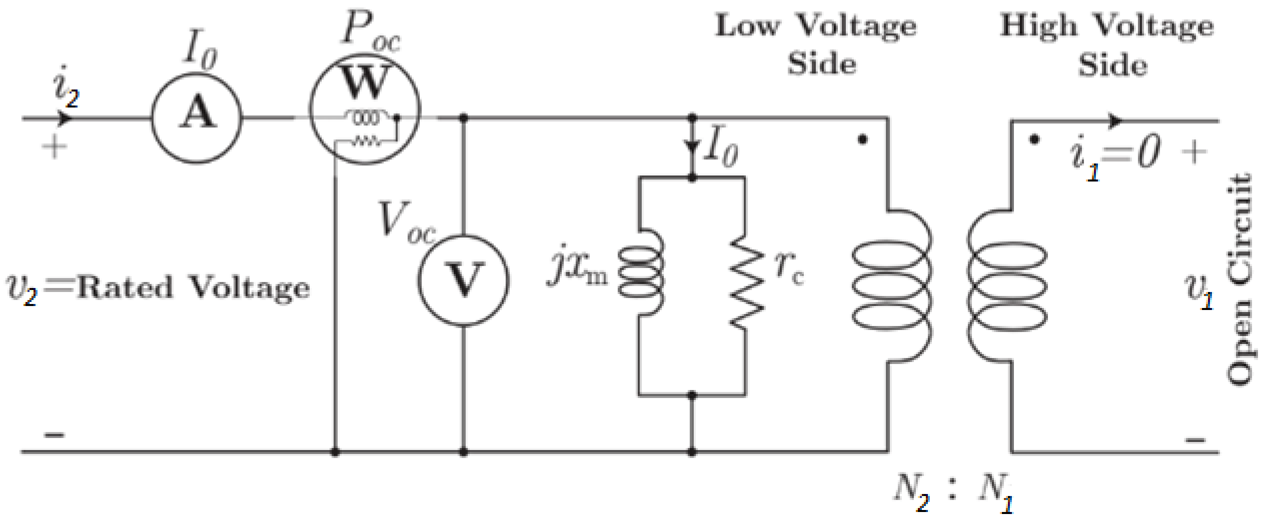
Rated voltage is applied to primary with secondary open;

Measure the primary current and losses (the test may also be done applying the rated voltage to the secondary, calculating the values, then referring the values back to the primary side).

- **Short Circuit Test:**

With secondary shorted, apply voltage to primary to get rated current to flow; Measure voltage and losses.

Open Circuit Test



The watt-meter shows the core loss. Therefore,

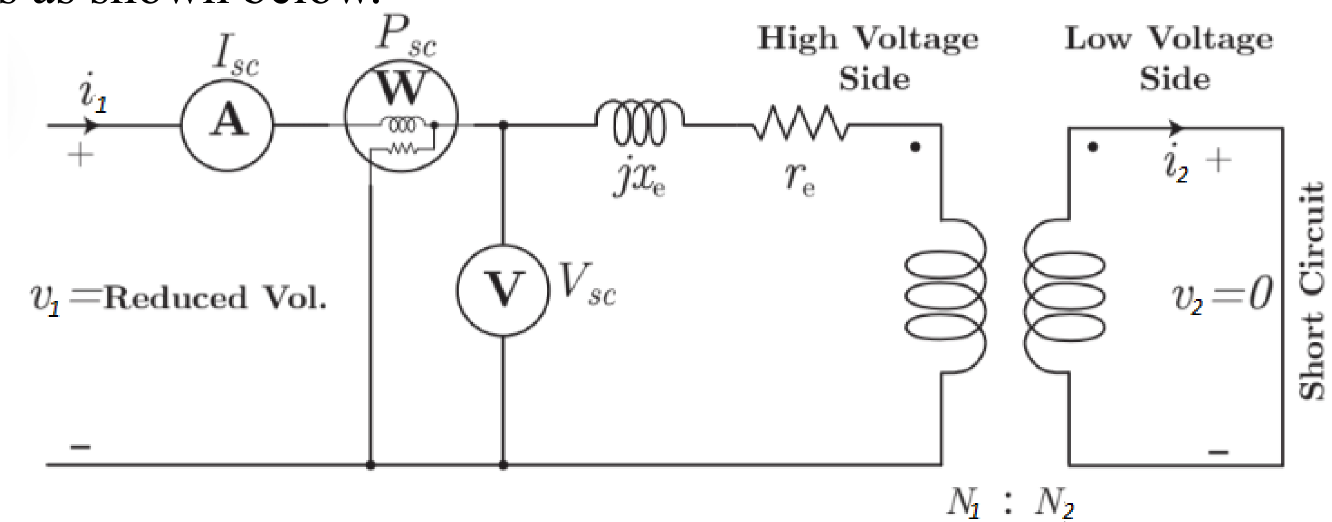
$$r_c = \frac{V_{oc}^2}{P_{oc}}$$

Moreover, the magnitude of the admittance of the branch is:

$$\left\| \frac{1}{r_c} + \frac{1}{jx_m} \right\| = \sqrt{\frac{1}{r_c^2} + \frac{1}{x_m^2}} = \frac{I_0}{V_{oc}} \rightarrow x_m = \frac{1}{\sqrt{\left(\frac{I_0}{V_{oc}}\right)^2 - \frac{1}{r_c^2}}}$$

Short Circuit Test

The short circuit test is conducted to determine the parameters of the windings: x_e and r_e. In this test, usually the low-voltage side is short-circuited, and a reduced voltage is applied to the high-voltage side such that the rated current flows. Then, the current, voltage, and power at the high-voltage side are measured by using a voltmeter, an ammeter, and a watt-meter. In such a case, the magnetizing branch can be neglected since the magnetizing current is negligible compared to the rated current. Therefore, the transformer equivalent circuit is as shown below.



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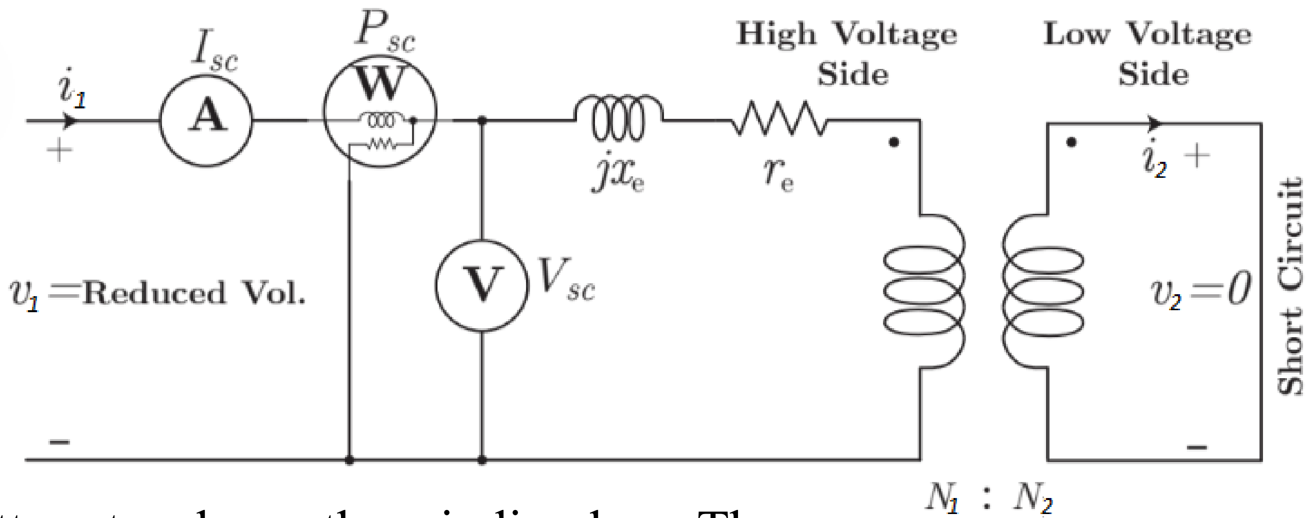


The watt-meter shows the winding loss. Thus,

$$r_e = \frac{P_{sc}}{I_{sc}^2}$$

The magnitude of the winding impedance is:

$$\|r_e + jx_e\| = \frac{V_{sc}}{I_{sc}}$$

Therefore,

$$x_e = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - r_e^2}$$

Contd…

It must be noted that the values obtained in the short circuit test are at the high-voltage side, while the parameters obtained in the open circuit test are at the low-voltage side. Therefore, to obtain the final equivalent model, all parameters must be transformed to either the low-voltage or the high-voltage side.

Transformer Parameters Calculation Example

These data were obtained when open-circuit and short circuit tests were conducted on a single-phase transformer with the power rating of 50kVA with the rated frequency of 60 Hz. The rated voltages of the high-voltage and low-voltage sides are 2400 V and 240 V, respectively.

	Voltage (V)	Current (A)	Power (W)
HV Winding Open Circuit	240	4.85	180
LV Winding Short Circuit	50	20.8	600

Calculate the equivalent circuit of the transformer referred to the low voltage side.

Solution:

	Voltage (V)	Current (A)	Power (W)
HV Winding Open Circuit	240	4.85	180
LV Winding Short Circuit	50	20.8	600

Using the open-circuit test data, r_c and x_m can be calculated:

$$r_c = \frac{V_{oc}^2}{P_{oc}} = \frac{240^2}{180} = 320\Omega.$$

Then, $x_m = \dfrac{1}{\sqrt{\left(\frac{I_0}{V_{oc}}\right)^2 - \frac{1}{r_c^2}}} = \dfrac{1}{\sqrt{\frac{4.85}{240} - \frac{1}{320^2}}} = 50.08\Omega.$

Contd…

	Voltage (V)	Current (A)	Power (W)
HV Winding Open Circuit	240	4.85	180
LV Winding Short Circuit	50	20.8	600

Using the short-circuit test data, r_e and x_m can be calculated:

$$r_e = \frac{P_{sc}}{I_{sc}^2} = \frac{600}{20.8^2} = 1.38\Omega.$$

Then, $x_e = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - r_e^2} = \sqrt{\left(\frac{50}{20.8}\right)^2 - 1.38^2} = 1.96\Omega.$

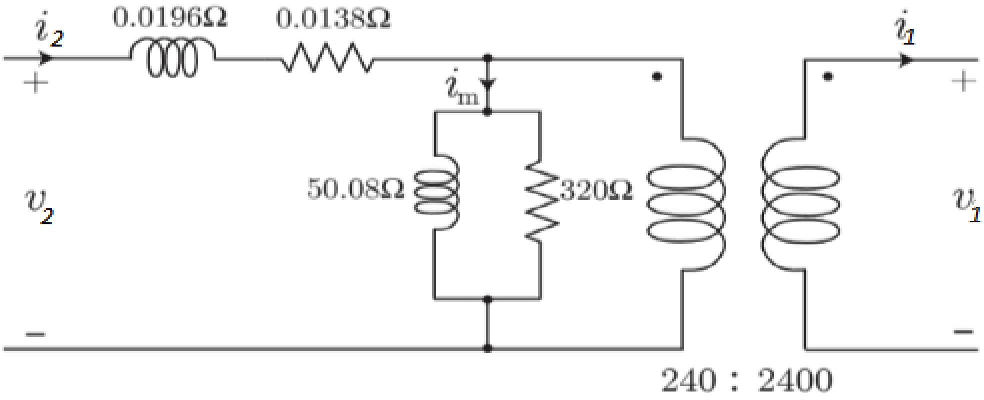
Contd…

It must be noted that x_e and r_e are at the high-voltage side. Since the equivalent model at the low-voltage side is asked, x_e and r_e must be referred to the low voltage side.

$$d = \frac{N_2}{N_1} = \frac{V_{\text{low}}}{V_{\text{high}}} = \frac{240}{2400} = 0.1$$

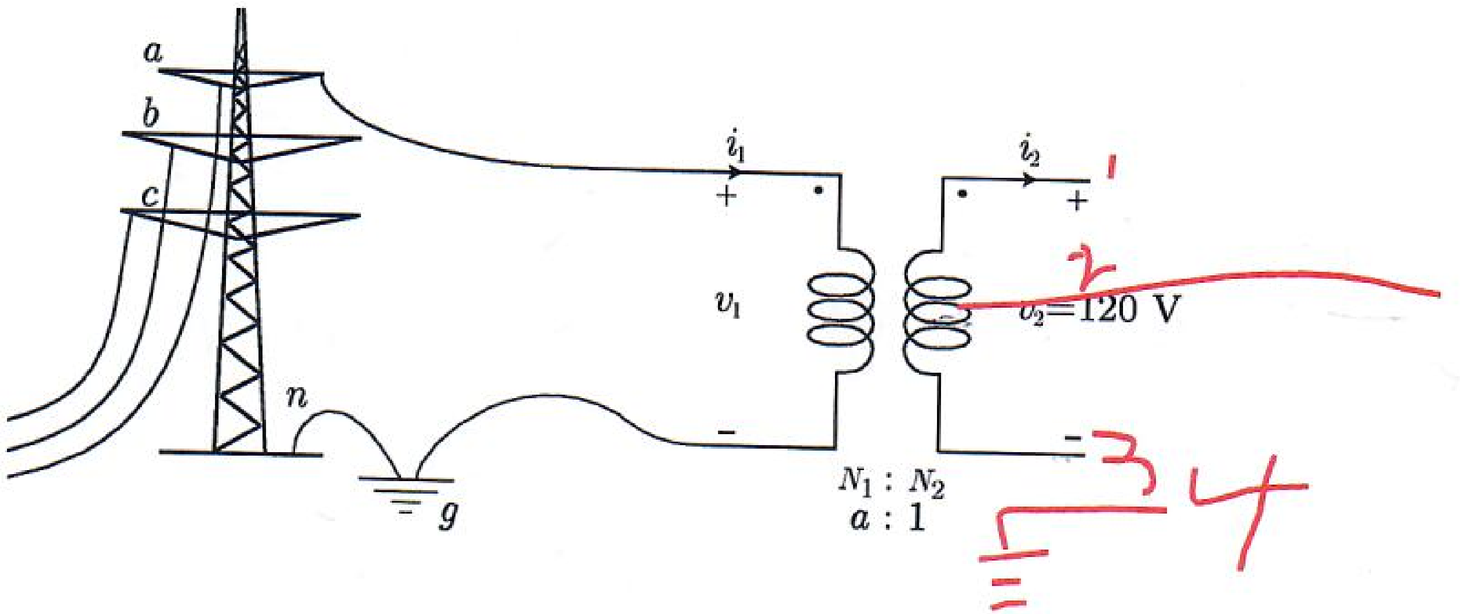
Then,

$$r_e' = d^2 \times r_e = 0.1^2 \times r_e = 0.0138\Omega$$
$$x_e' = d^2 \times x_e = 0.1^2 \times x_e = 0.0196\Omega$$



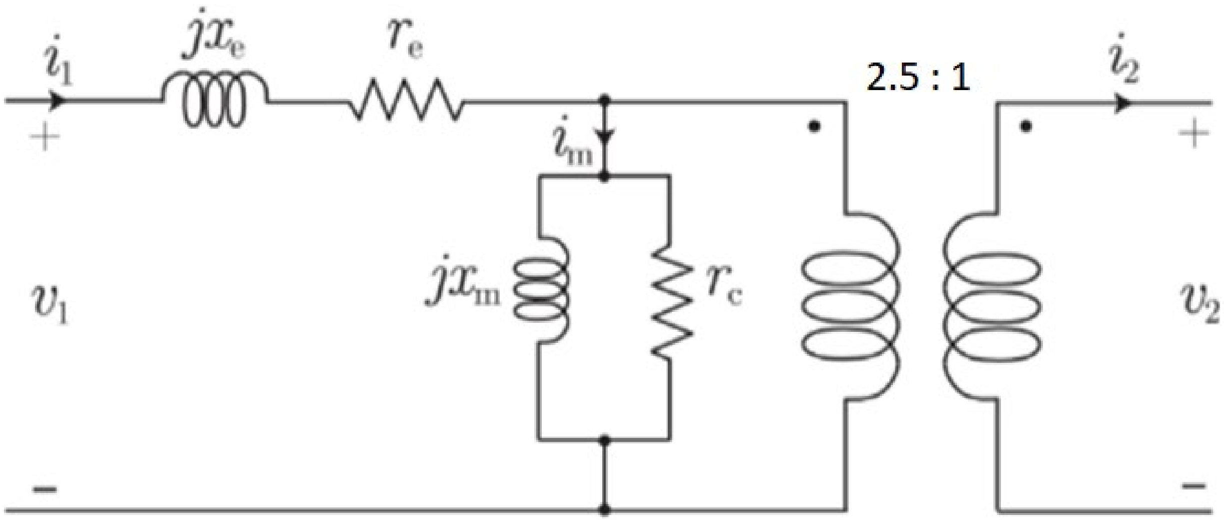
Single Phase Transformer Applications

Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a grounded, common neutral.



Transformer Test Example

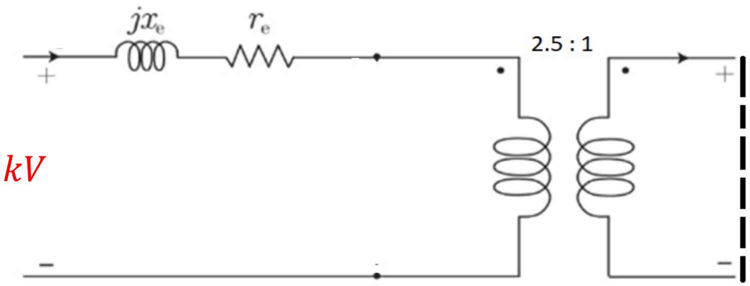
1. Determine the model parameters for a single phase, 100MVA, 200/80kV transformer with the following test data:
   - Short circuit: 30 kV, with 500 kW losses
   - Open circuit: 20 amps, with 10 kW losses
2. Determine the model parameters.



Contd…

$P_{sc} = 500 \text{ kW}$
$I_{sc} = 500\text{A} = I_{I,Rated}$

$V_{sc} = 30kV$

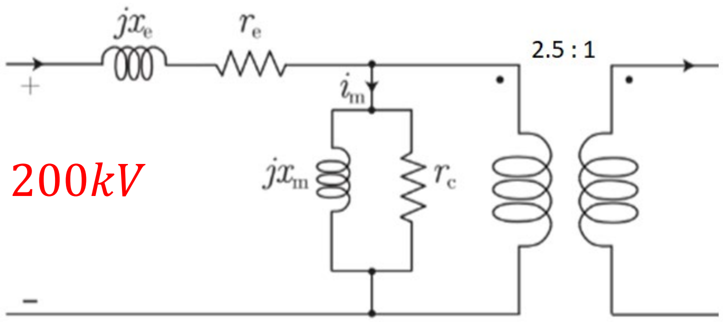


From the short circuit test

- $I_{sc} = \frac{100\text{MVA}}{200\text{kV}} = 500 \text{ A}, \ \ |R_e + jX_e| = \frac{30\text{kV}}{500 \text{ A}} = 60\Omega$
- $P_{sc} = R_e I_{sc}^2 = 500 \text{ kW} \rightarrow R_e = 2\Omega,$
- Hence $X_e = \sqrt{60^2 - 2^2} = 60\Omega$

Contd…

$P_{sc} = 10\ kW$
$I_{oc} = 20A$
$V_{oc} = 200kV$



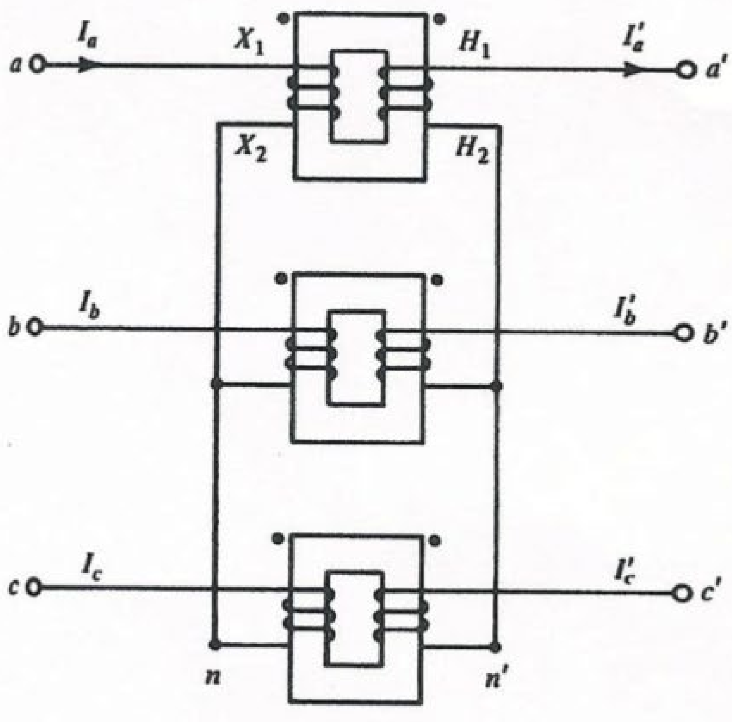
From the open circuit test

- $R_c = \frac{200\text{kV}^2}{10\text{ kW}} = 4M\Omega$
- $\left\| \frac{1}{r_c} + \frac{1}{jx_m} \right\| = \frac{200\text{kV}}{20\text{ A}} = X_m = 10{,}000\Omega$

$3 - \phi$ Phase Transformers Interconnections

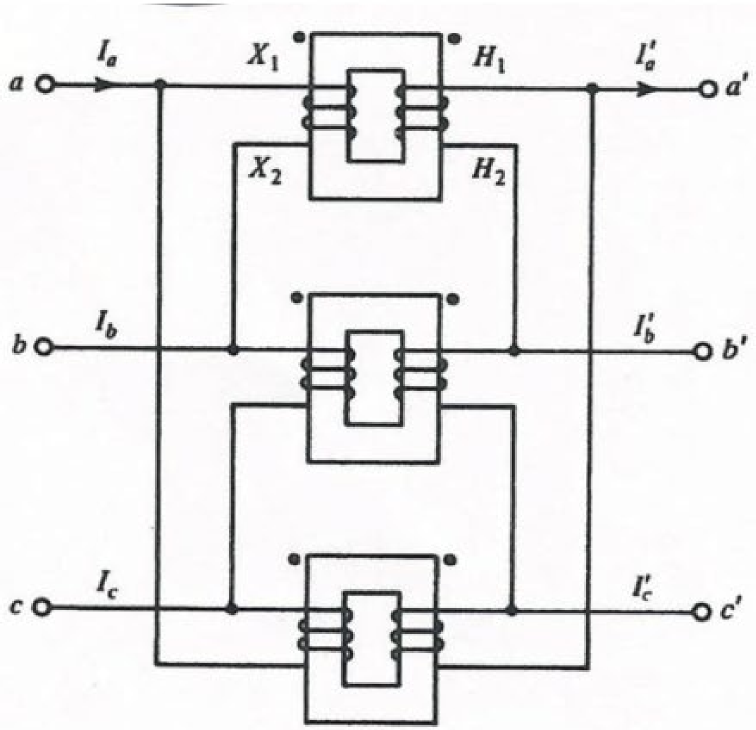
There are 4 different ways to connect 3ϕ transformers.

Y-Y



Y-Y Connection

Δ-Δ

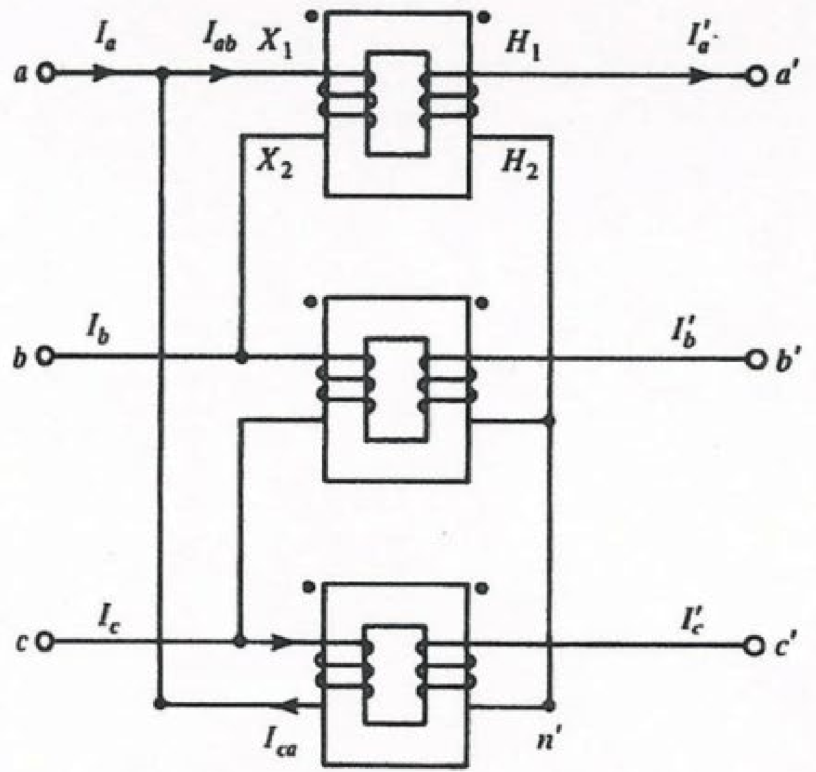


Δ-Δ Connection

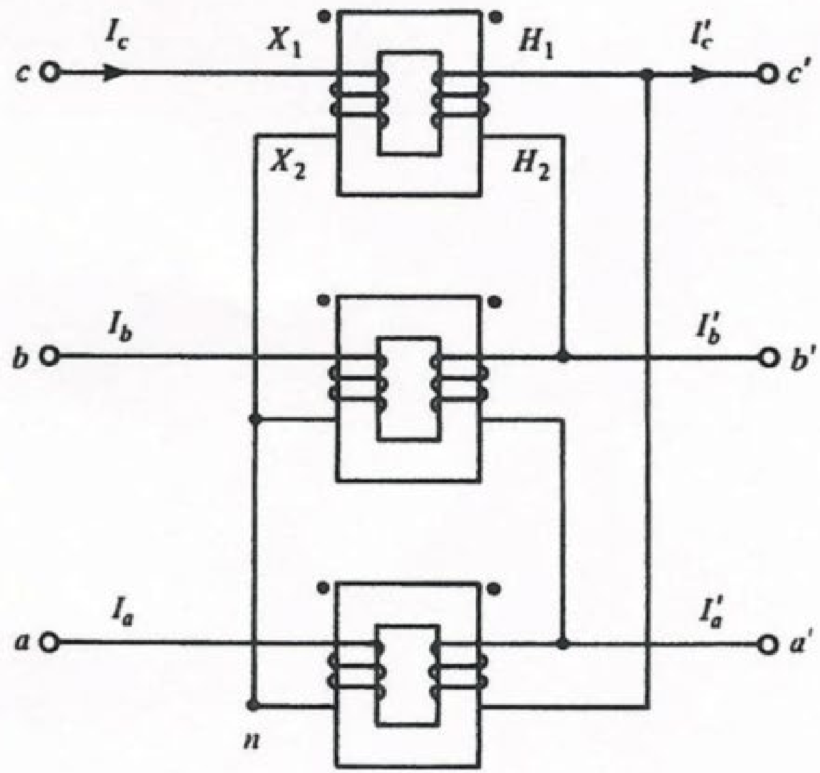
Contd…

Δ-Y

Y-Δ



Δ-Y Connection



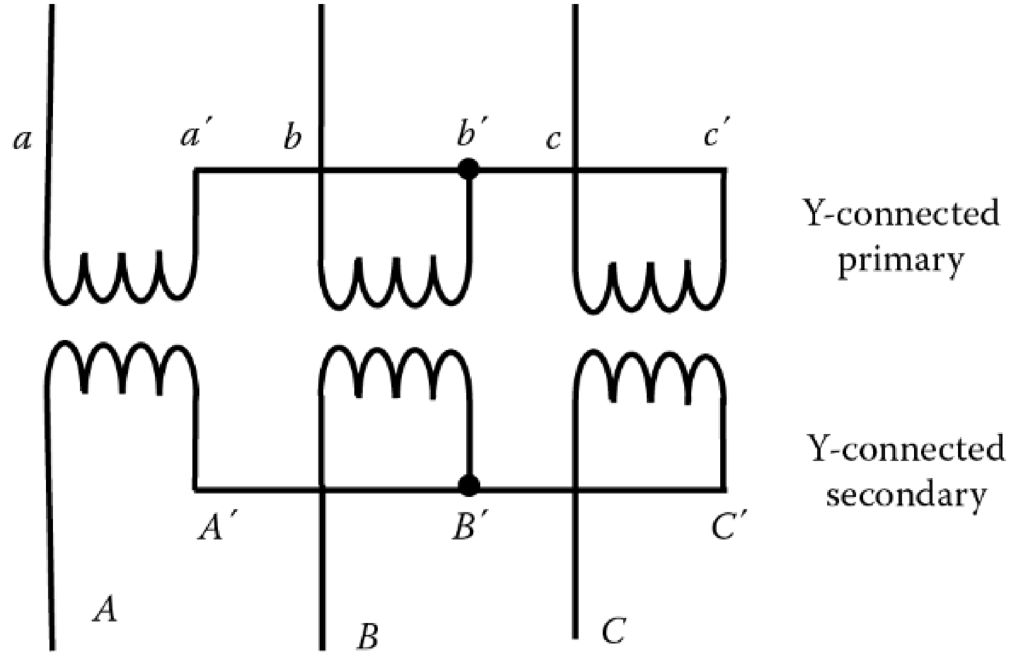
Y-Δ Connection

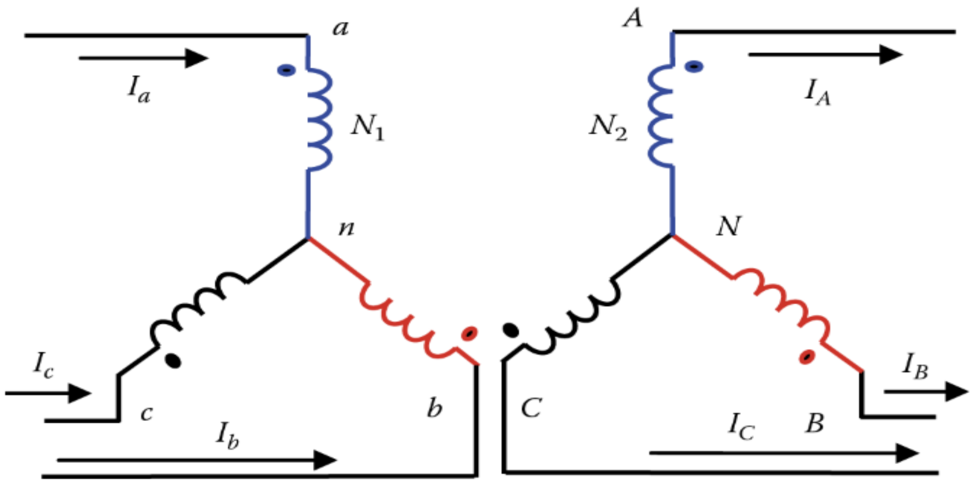
Y-Y Connection

- The top figure shows the wiring of the transformer where terminals a', b', and c' of the primary windings are connected to a common point called neutral n.

- The secondary windings are similarly connected.

- A more convenient schematic is the one at the bottom of the figure.





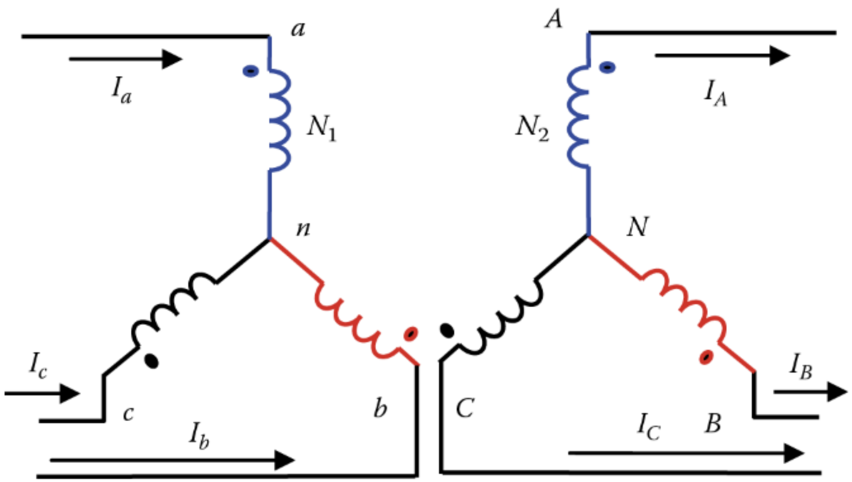
Y-Y Connection

- The turns ratio of the transformer is N_{I}/N_2, while the line-to-line voltage ratio of the transformer is V_{ab}/V_{AB}.

$$\frac{N_1}{N_2} = \frac{V_{an}}{V_{AN}} = \frac{\sqrt{3}V_{an}}{\sqrt{3}V_{AN}} = \frac{V_{ab}}{V_{AB}}$$

- The current ratio is the inverse of the turns ratio.

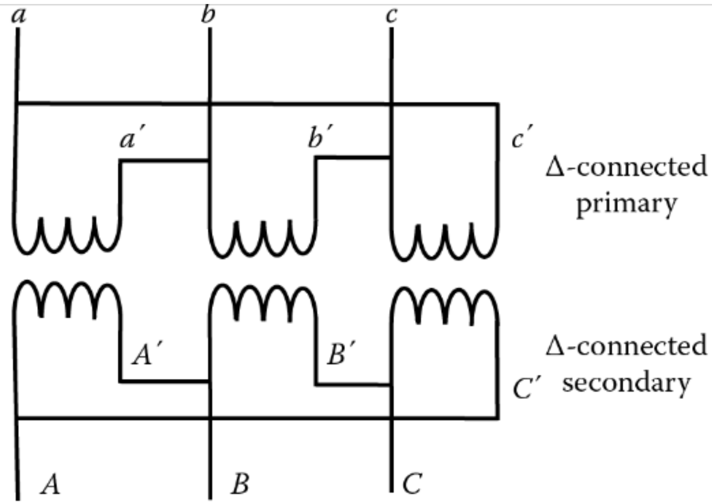
$$\frac{N_1}{N_2} = \frac{I_A}{I_a}$$

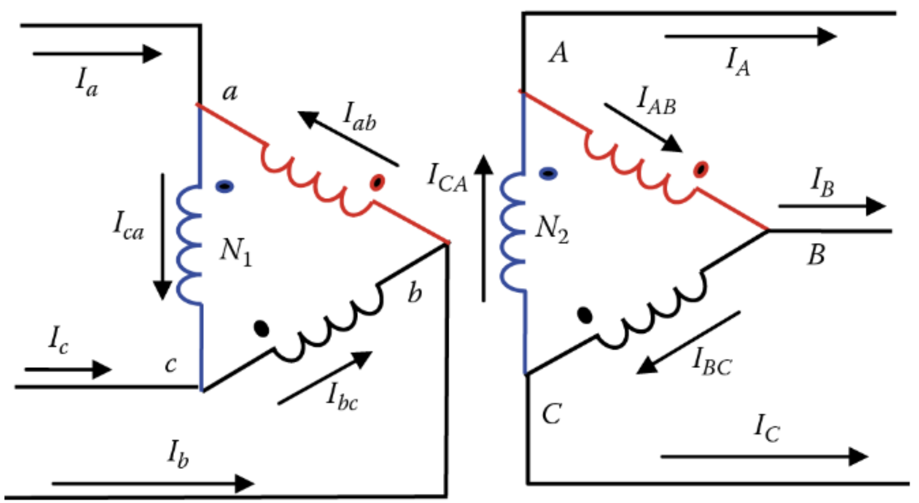


- Y-Y connections are common in transmission systems.
- Key advantages are the ability to ground each side and there is no phase shift introduced.

Δ - Δ Connection

- NOTE that the currents in the transformer windings are phase currents, and the currents in the circuit feeding the transformers are the line currents.



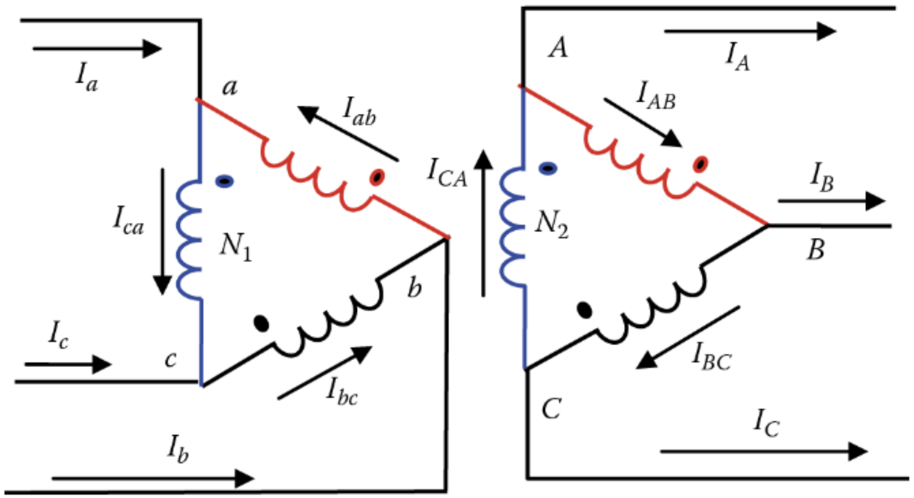


Δ - Δ Connection

- The turns ratio of the transformer is N_I/N_2. The voltage ratio of the transformer is the ratio of the voltages V_{ab}/V_{AB}, which are the same as the ratio of the line-to-line voltages in the delta configuration.
- Keep in mind that the voltage per turn is constant in any winding. Hence,

$$V_T = \frac{V_{ab}}{N_1} = \frac{V_{AB}}{N_2}$$

$$\frac{N_1}{N_2} = \frac{V_{ab}}{V_{AB}}$$



Using the magnetomotive force equation, we can compute the current ratio

$$\mathfrak{I} = I_{ab}N_1 = I_{AB}N_2$$

$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_{ab}}$$

- Key disadvantage is D-D connections can not be grounded; not commonly used.

Δ - Y Connection

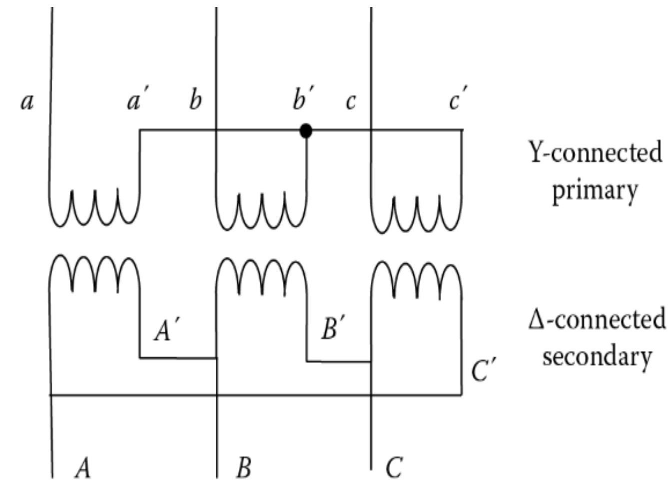
- The analysis of the wye–delta transformer requires some attention when computing the various ratios.

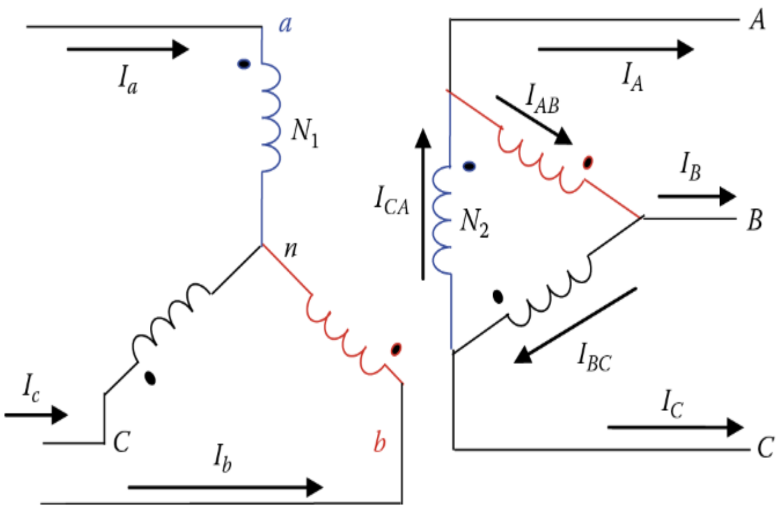
- The voltage across N_1 is the phase voltage of the primary circuit V_{an} and the voltage across N_2 is the line-to-line voltage of the secondary circuit V_{AB}. Since the voltage turns ratio as

$$V_T = \frac{V_{an}}{N_1} = \frac{V_{AB}}{N_2}$$

$$\frac{N_1}{N_2} = \frac{V_{an}}{V_{AB}}$$

$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_a}$$



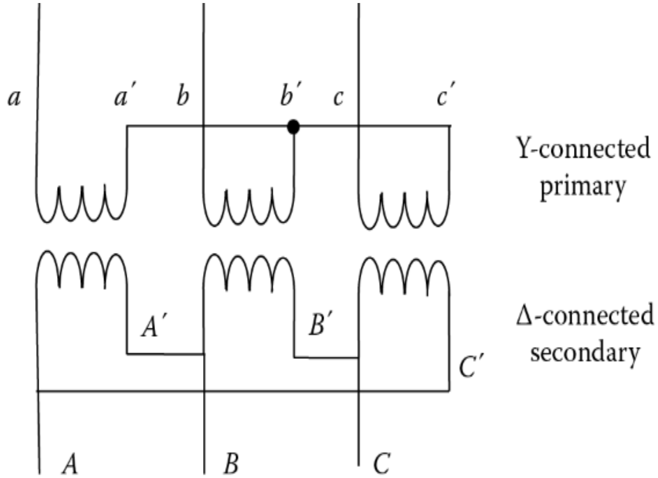


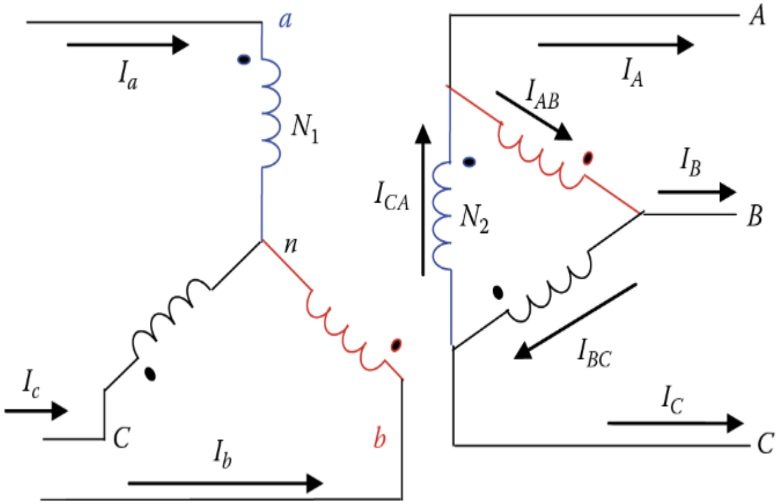
Δ - Y Connection

- Similarly, the current through N_1 is the line current of the primary circuit I_a and the current through N_2 is the phase current of the secondary winding I_{AB}.
- Hence, we can compute the current ratio using the ampere-turn equation.

$$\Im = I_a N_1 = I_{AB} N_2$$

$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_a}$$





Δ - Y Connection: V/I Relationships

For Voltage, we get,

- $V_{AB} = \sqrt{3} V_{An} e^{j30^\circ}$
- $V_{AB} = a \cdot V_{an}$
- $V_{an} = \dfrac{\sqrt{3} V_{An} \cdot e^{j30^\circ}}{a}$

For Current, we get,

- $\dfrac{I_{AB}}{I_a} = \dfrac{1}{a} \rightarrow I_a = a I_{AB}$
- $I_A = \sqrt{3} I_{AB} \angle -30^\circ \rightarrow I_{AB} = \frac{1}{\sqrt{3}} I_A \angle 30^\circ$
- $I_a = a \frac{1}{\sqrt{3}} I_A \angle 30^\circ$

Example 1:

A 25kVA transformer has a voltage ratio of $20\text{kV}(\mathbf{Y})/10\text{kV}(\boldsymbol{\Delta})$. Compute the following:

1. Winding voltages
2. Turns ratio
3. Line currents in the primary and secondary circuits
4. Phase currents of the transformer

Example 1: Solution

1.

a) The primary windings of the transformer are connected in wye. Hence, the voltage across the winding is

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} = \frac{20}{\sqrt{3}} = 11.55\text{kV}$$

b) The secondary windings are connected in delta. Hence, the winding voltage is equal to the line-to-line voltage

$$V_{AB} = 10\text{kV}$$

2. The turns ratio is the ratio of the voltages of the windings

$$\frac{N_1}{N_2} = \frac{11.55}{10} = 1.155$$

Example 1: Solution

3.

a) The line current in the primary circuit is

$$I_a = \frac{S}{\sqrt{3}V_{ab}} = \frac{25}{\sqrt{3} \times 20} = 0.7217 \text{ A}$$

b) The line current in the secondary circuit is

$$I_A = \frac{S}{\sqrt{3}V_{AB}} = \frac{25}{\sqrt{3} \times 10} = 1.4434 \text{ A}$$

4. .

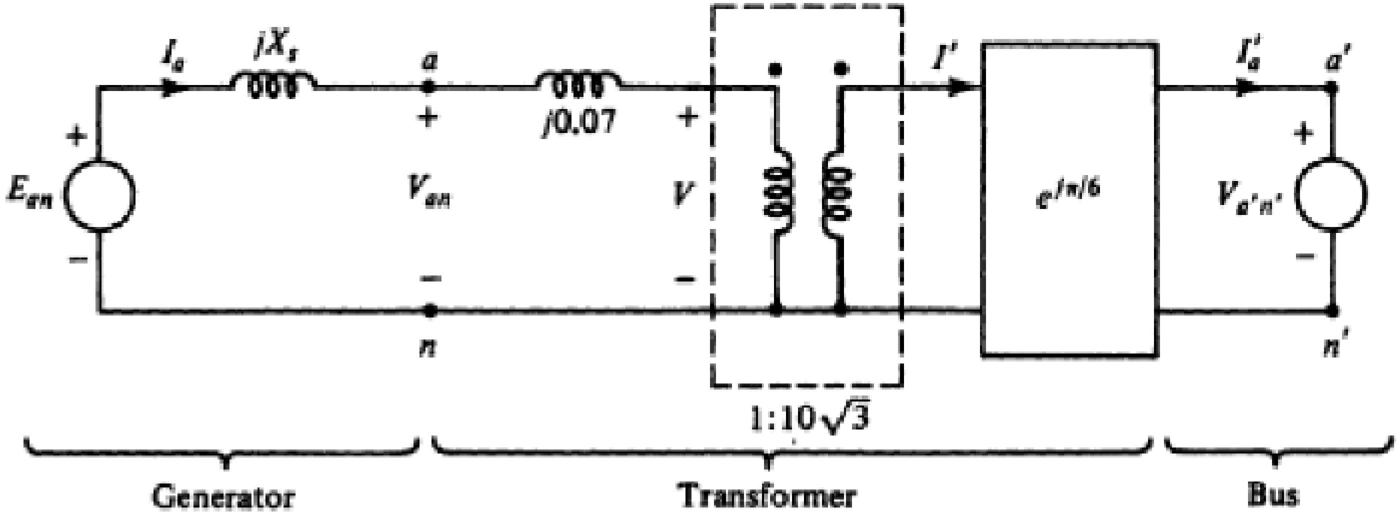
a) Since the primary windings are connected in wye, the phase current of the primary winding is the same as the line current of the primary circuit.

b) The secondary windings are connected in delta, hence the phase current of the secondary winding is

$$I_{AB} = \frac{I_A}{\sqrt{3}} = \frac{1.4434}{\sqrt{3}} = 0.8333 \text{ A}$$

Example 2

Generator with Δ-Y step up transformer. Transformer is made of identical 1ϕ transformers with $X_l = 0.21\Omega$, $n = N_1/N_2 = 10$. Each generator phase is modeled as a Thevenin equivalent circuit. The transformer delivers 100MW at 0.9 lagging power factor at 230kV to the bus.



Find the primary current, primary voltage LL, and 3ϕ complex power supplied by the generator.

Example 2: Solution

Find the primary current, primary voltage LL, and 3ϕ complex power supplied by the generator.

Solution,

$$V_{a'n'} = 230/\sqrt{3}\angle 0^\circ \text{kV} = 132.8\angle 0^\circ \text{kV}$$

$$S' = \frac{100 \times 10^6}{0.9 \times 3}\angle 25.84^\circ = 37.04\angle 25.84^\circ \text{MVA}$$

$$I'_a = \left(\frac{S'}{V_{a'n'}}\right)^* = \left(\frac{37.04\angle 25.84^\circ}{132.8\angle 0^\circ}\right)^* = 278.9\angle - 25.84^\circ$$

$$I_a = 10\sqrt{3}e^{-j\pi/6}I'_a = 4830.6\angle - 55.84^\circ$$

Contd…

$$V_{an} = V + j0.071 I_a$$

$$= \frac{1}{10\sqrt{3}} 132.8 \times 10^3 \angle -30^\circ + j0.07 \times 4830.6 \angle 55.84^o$$

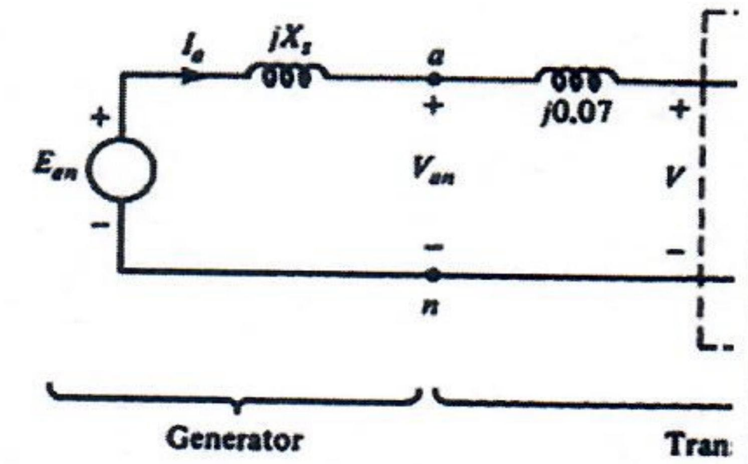
$$= 7667.2 \angle -30^o + 338.1 \angle 34.16^o$$

$$= 7820.5 \angle -27.77^\circ \text{V}$$

$$V_{ab} = \sqrt{3} e^{-j\pi/6} V_{an} = 13.55 \angle -57.77^\circ \text{kV}$$

$$S_{1\phi} = V_{an} I_a^* = 37.77 \angle 28.07^\circ \text{MVA}$$

$$S_{3\phi} = 3S = 113.33 \angle 28.07^\circ \text{MVA}$$



Transformer Efficiency

The efficiency of a transformer is given by,

$$\eta = \frac{P_O}{P_{in}} = \frac{Output\ power}{Input\ power} = \frac{Output\ power}{Output\ power + Losses}$$

$$\eta = \frac{P_O}{P_{in}} = \frac{V_2' I_2' \cos\theta_2}{V_2' I_2' \cos\theta_2 + P_{cu} + P_{iron}} \times 100\%.$$

The losses of the transformer are due to R1, R2, and Ro. The losses in R1 and R2 are the winding losses, and are named copper losses Pcu (the windings are made of copper material). The loss in Ro is known as iron loss Piron (the core is made of iron).

Voltage Regulation

The voltage regulation of a transformer indicates the voltage reduction due to various parameters of the transformer.

Mathematically,

$$\text{VR} = \frac{|V_{no\ load}| - |V_{full\ load}|}{|V_{full\ load}|}$$

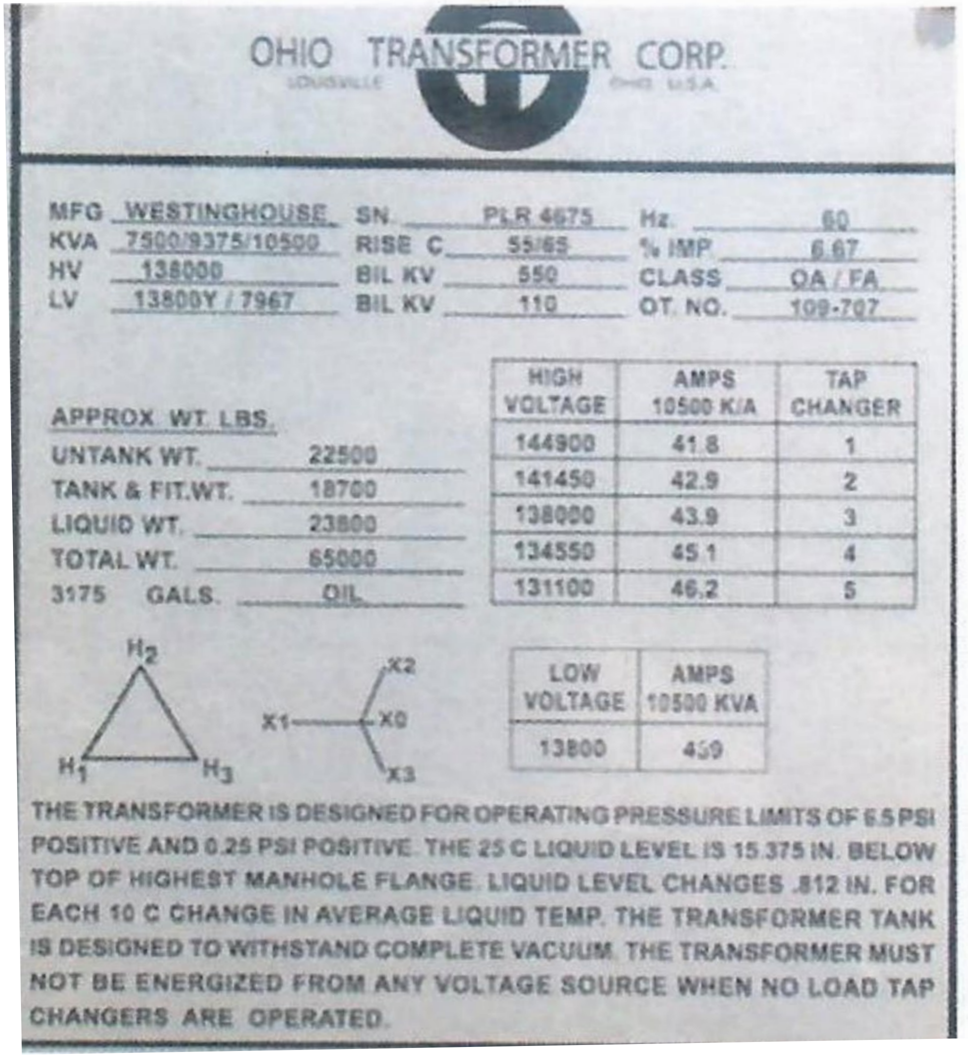
$|V_{no\ load}|$ is the magnitude of the open circuit voltage measured at the load terminals

$|V_{full\ load}|$ is the magnitude of the voltage at the load terminals when the rated current is delivered to the load

- At no load, (open circuit) $I_2 = 0, V_{no\ load} = \text{V}_1, V_{full\ load} = V_2'$

$$\text{VR} = \frac{V_1 - aV_2}{aV_2} * 100\% \text{ , } where\ a = \frac{N_1}{N_2}$$

Per Unit Change of Base Power



The base power is usually called "system base".

- Parameters for equipment are often given using power rating of equipment as the base power.
- Also called "transformer base", "generator base", etc.

Thank You!