IOWA STATE UNIVERSITY

ECpE Department

EE 303 Energy Systems and Power Electronics

Per Unit Analysis

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Per Unit Analysis

Motivation:

It would be difficult to continuously refer impedance to the different sides of transformer.

Mathematically,

$$\text{Quantity in Per Unit} = \frac{\text{Actual Quantity}}{\text{Base value of the Quantity}}$$

Normalization of all variables

S, V, Z & I   *(**Note:** Ohms law still holds)*

- Pick S_B & V_B, and calculate I_B & Z_B

$$S_B = V_B \cdot I_B^*$$

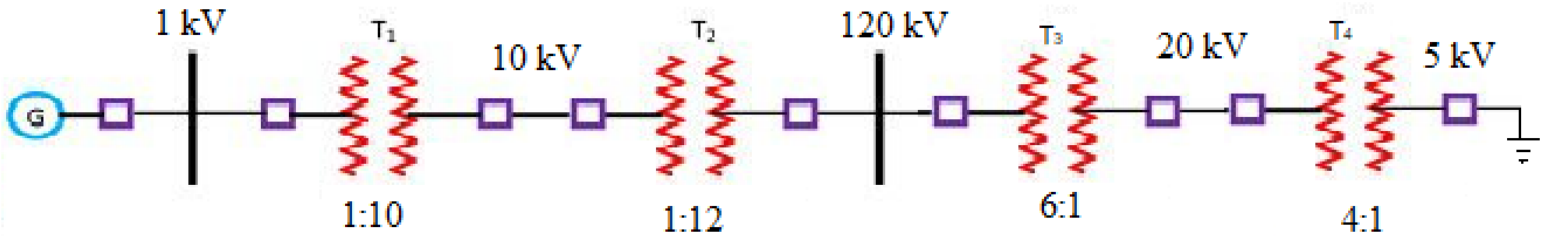
$$I_B = \left(\frac{S_B}{V_B}\right)^* = \frac{S_B}{V_B}$$

*(**Note:** Base values are real numbers)*

$$Z_B = \frac{(V_B)^2}{S_B}$$

- S_B is for the entire system.
- V_B is for each different voltage level.
- Base voltages are related by transformer turns ratio

Contd…



For a single-phase system:

- Step 1: Pick a single-phase base power for the entire system S_B.
- Step 2: Pick a base voltage for each different voltage level, V_B
  - Base voltages are related by turns ratios.
  - voltages are line-to-neutral.
- Step 3: Calculate base impedance, $Z_B = \frac{(V_B)^2}{SB}$.
- Step 4: Calculate base current, $I_B = \frac{V_B}{S_B}$

Contd…

- Step 5. Convert actual values to per unit (p.u.)

Notes:

1) Base values are real numbers. Per unit conversion only affects magnitude, not angle.
2) Per unit quantities no longer have units.

Contd…

$$V_{p \cdot u} = \frac{V}{V_B};$$

$$P_{p \cdot u} = \frac{P}{S_B};$$

$$S_{p \cdot u} = \frac{S}{S_B};$$

$$Q_{p \cdot u} = \frac{Q}{S_B};$$

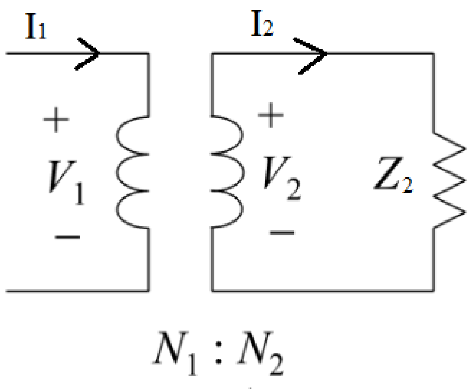
$$I_{p \cdot u} = \frac{I}{I_B};$$

$$R_{p.u} = \frac{R}{Z_B};$$

$$Z_{p \cdot u} = \frac{Z}{Z_{B.}};$$

$$X_{p \cdot u} = \frac{X}{Z_{B.}};$$

Per Unit Values of Primary and Secondary Side of a Transformer are **EQUAL**



$$a = \frac{N_1}{N_2}$$

$$n = \frac{N_2}{N_1}$$

Transformation Ratio

$$V_2 = n\,.\,V_1 = \frac{V_1}{a}$$

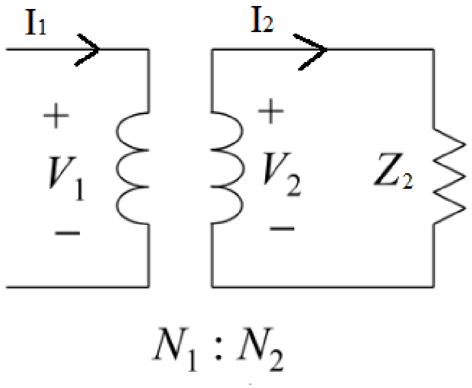
$$I_2 = \frac{I_1}{n} = a.\,I_1$$

Real Values

<u>Choose Base Values:</u>

- S_B
- V_{1B} & $V_{2B} = \frac{V_{1B}}{a} = n.\,V_{1B}$

Per Unit Values of Primary and Secondary Side of a Transformer are **EQUAL**



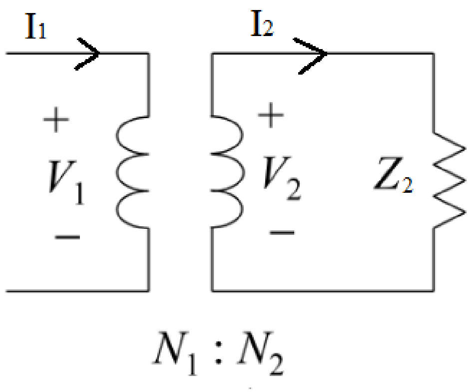
Choose Base Values:

- S_B
- V_{1B} & $V_{2B} = \frac{V_{1B}}{a} = n.V_{1B}$

$$V_{2,pu} = \frac{V_2}{V_{2B}} = \frac{n \cdot V_1}{n \cdot V_{1B}} = \frac{V_1}{V_{1B}} = V_{1,pu}$$

$$I_2, pu = \frac{I_2}{I_{2B}} = \frac{I_2}{\frac{S_B}{V_{2B}}} = \frac{V_{2B} \cdot I_2}{S_B} = \frac{n \cdot V_{1B} \cdot I_2}{S_B} = \frac{n \cdot V_{1B} \cdot a \cdot I_1}{S_B} = \frac{V_{1B} \cdot I_1}{S_B.} = \frac{I_1}{\frac{S_B}{V_{1B}}} = \frac{I_1}{I_{1B}} = I_{1,PU}.$$

Per Unit Values of Primary and Secondary Side of a Transformer are **EQUAL**



Choose Base Values:

- S_B
- V_{1B} & $V_{2B} = \frac{V_{1B}}{a} = n.V_{1B}$

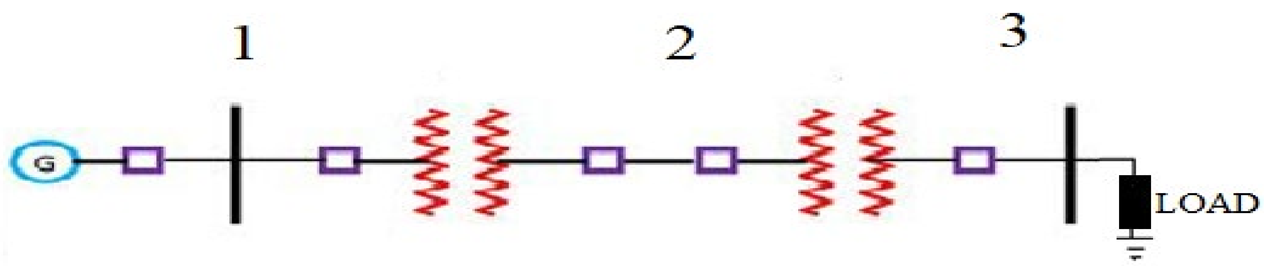
$$Z_{2B} = \frac{V_{2B}^2}{S_B} = \frac{n^2 \cdot V_{1B}^2}{S_B} = n^2 \cdot Z_{1B}$$

$$Z_{2;\,p.u} = \frac{Z_2}{Z_{2B}} = \frac{n^2 \cdot z_e}{n^2 \cdot z_{1B}} = \frac{z_e}{z_{1B}} = z_{e;\,p.u}.$$

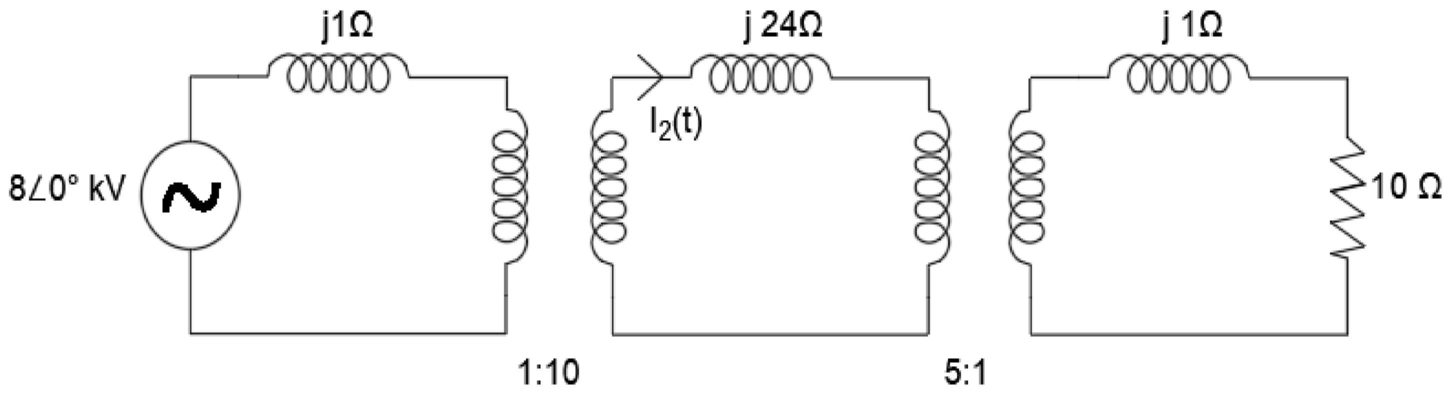
Example 1

Q. Solve for the current, load voltage & load power in the following circuit using P.U. analysis with an $S_B = 100\text{MVA}$ and base voltage of $V_{B1} = 8\text{kV}, V_{B2} = 80\text{kV}, and\ V_{B3} = 16\text{kV}$.

System Representation



Actual Values



Example 1: Solution

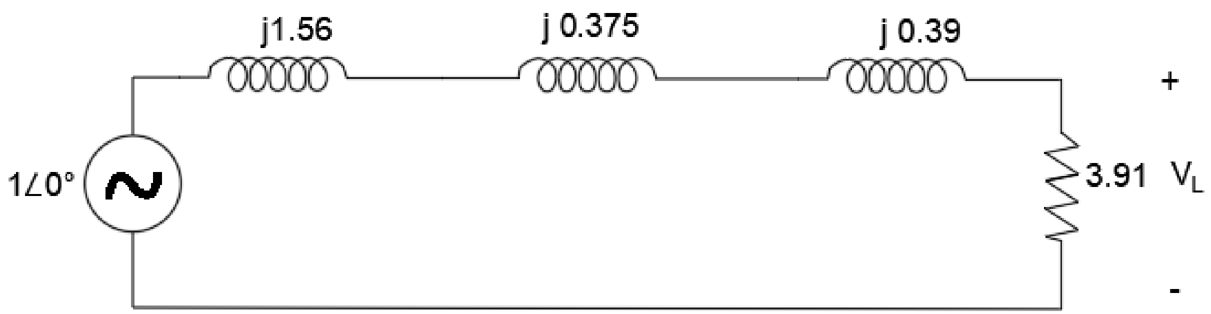
$$V_{B1} = 8\text{kV}; \quad V_{B2} = 80\text{kV}; \; V_{B3} = 16\text{k}V$$

$$Z_{B1} \qquad = \frac{(8\text{ k}V)^2}{100\text{MVA}} = 0.64\Omega.$$

$$Z_{B_2} \qquad = \frac{(80\text{ k}V)^2}{100MVA} = 64\Omega.$$

$$Z_{B3} \qquad = \frac{(16\text{ k}V)^2}{100\text{MVA}} = 2.56\Omega.$$

Therefore, we can convert the actual values diagram in p.u. diagram.



Contd…

$$I = \frac{1\angle 0^\circ}{3.91 + j2.327.} = 0.22\angle -30.8^\circ p.u.$$

$$V_L = 1\angle 0^\circ - 0.22\angle(-30.8^\circ) \times j2.327 = 0.859\angle -30.8^\circ\ p.u.$$

$$S_L = V_L \cdot I_L^* = 0.189\ p.u.$$

$$S_G = V_G \cdot I_G^* = 1\angle 0^\circ * 0.22\ \angle 30.8^\circ = 0.22\angle 30.8^\circ\ p.u$$

Contd…

Convert back to actual values.

$$V_L^{actual} = 0.859\angle(-30.8°) \times 16kV = 13.7\angle(-30.8°\ )\ kV$$

$$S_L^{actual} = 0.189\angle 0° \times 100MVA = 18.9\angle 0° MVA$$

$$S_G^{actual} = 0.22\ \angle 30.8° \times 100MVA = 22\angle 30.8° MVA$$

$$I_{2B} = \frac{100MVA}{80kV} = 1250\ A$$

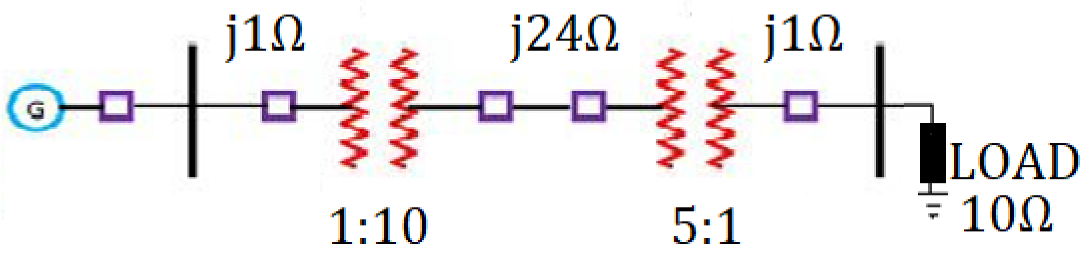
$$I_2^{actual} = 0.22\angle - 30.8° \times 1250\ A = 275\angle - 30.8° A$$

Three- Phase Per Unit Analysis

- Step 1: Pick a Three-phase base power for the entire system $S_B^{3\phi}$
- Step 2: Pick a base voltage for each different voltage level, $V_{B,\ LL}$
    - L-L Base $V_{B,LL}$
    - L-N Base $V_{B,LL/\sqrt{3}}$
- Step 3: Calculate Base Impedance, $Z_B = \dfrac{\left(\frac{V_{B,LL}}{\sqrt{3}}\right)^2}{\left(\frac{S_B^{3\phi}}{3}\right)} = \dfrac{V_{B,LL}^2}{S_B^{3\phi}}$
- Step 4: Calculate Base current, $I_B^{3\phi} = \dfrac{\frac{S_B^{3\phi}}{3}}{\frac{V_{B,LL}}{\sqrt{3}}} = \dfrac{S_B^{3\phi}}{\sqrt{3}V_{B,LL}}$
- Step 5: Convert actual values to p.u.

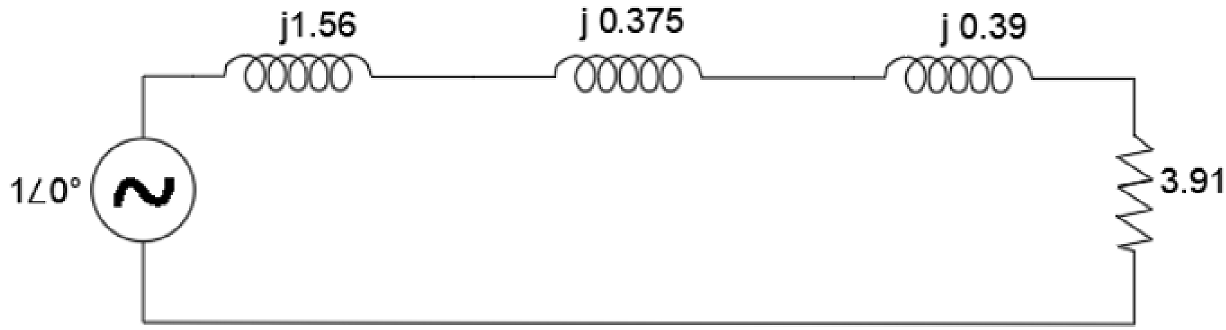
Example 2:

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ base power of 300 MVA, and line to line base voltages of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line-to-line voltage is 13.8 kV.



$V_{B1} = 13.8kV, V_{B2} = 138kV \; and \; V_{B3} = 27.6 \; kV$

Example 2: Solution



Convert to per unit as before. Note the system is exactly the same as example 1!

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle - 30.8^\circ \; p.u. \; (not\ amps)$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle - 30.8^\circ \times 2.327\angle 90^\circ = 0.859\angle - 30.8^\circ \; p.u.$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \; p.u.$$

Contd…

$$S_G = 1.0\angle 0° \times 0.22\angle 30.8° = 0.22\angle 30.8°\ p.u.$$

Again, analysis is exactly the same

Difference appear when we convert back to actual values

$$V_L^{actual} = 0.859\angle(-30.8°) \times 27.6kV = 23.8\angle(-30.8°\ )\ kV$$

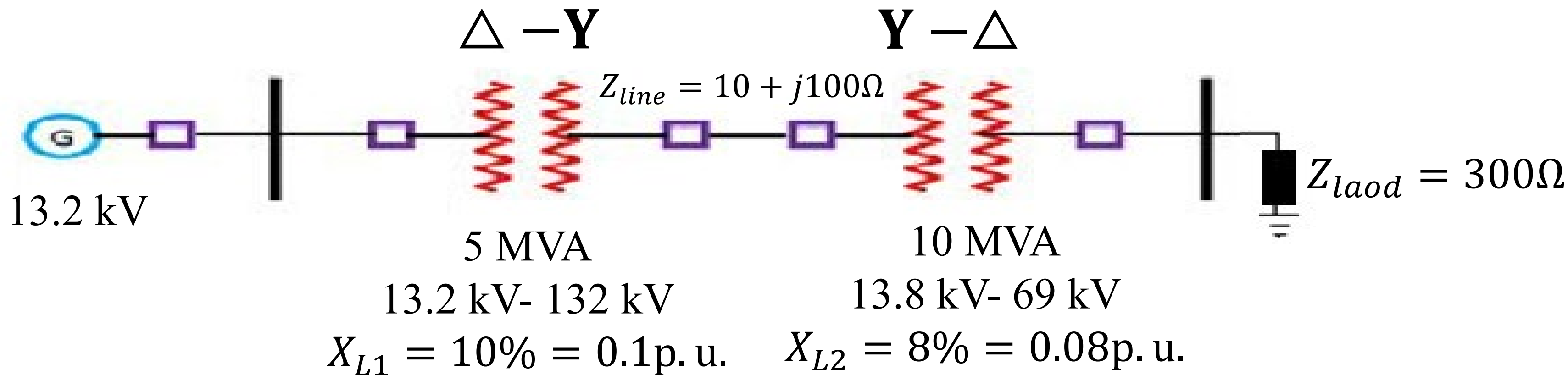
$$S_L^{actual} = 0.189\angle 0° \times 300MVA = 0.567\angle 0° MVA$$

$$S_G^{actual} = 0.22\ \angle 30.8° \times 300MVA = 0.66\angle 30.8° MVA$$

$$I_B^{Middle} = \frac{300MVA}{\sqrt{3}\ * 138kV} = 1250\ A$$ *(same currents)*

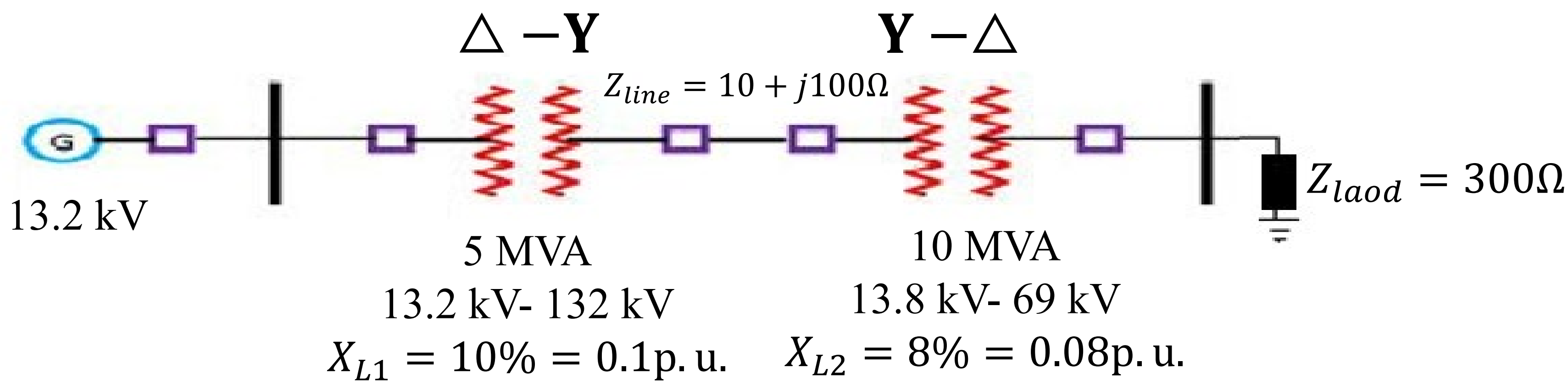
$$I_{Middle}^{actual} = 0.22\angle -30.8° \times 1250\ A = 275\angle -30.8° A$$

Example 3



Q. Find the equivalent p.u values of each component.

Example 3: Solution



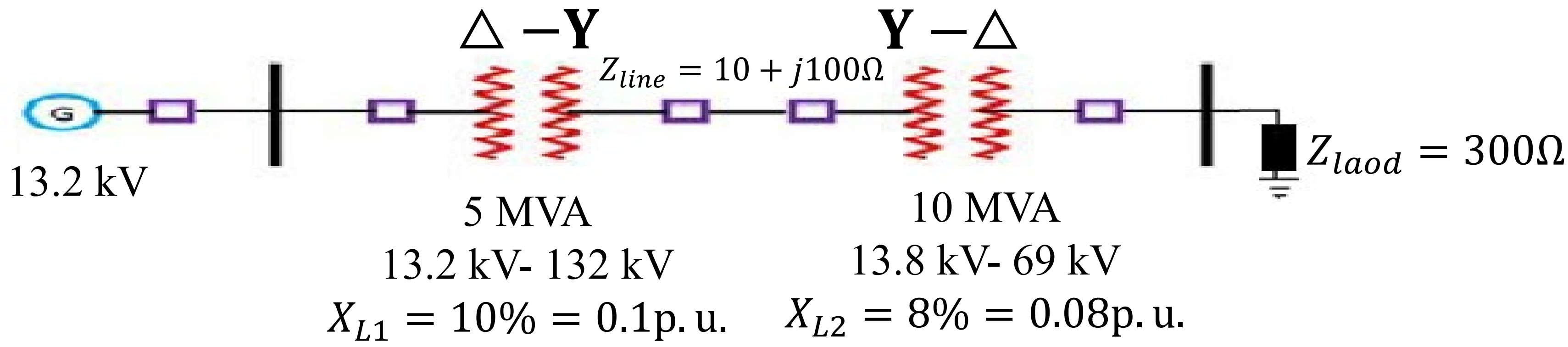
$$S_B^{3\phi} = 10 \text{ MVA}$$

$$V_{3B}^{\text{L−L}} = 69 \text{ kV}$$

$$V_{2B}^{\text{L−L}} = 138 \text{ kV}$$

$$V_{1B}^{\text{L−L}} = 138 \text{ kV} \times \frac{13.2\text{k}V}{132\text{k}V} = 13.8\text{kV}$$

Example 3: Solution



$$S_B^{3\phi} = 10 \text{ MVA}$$

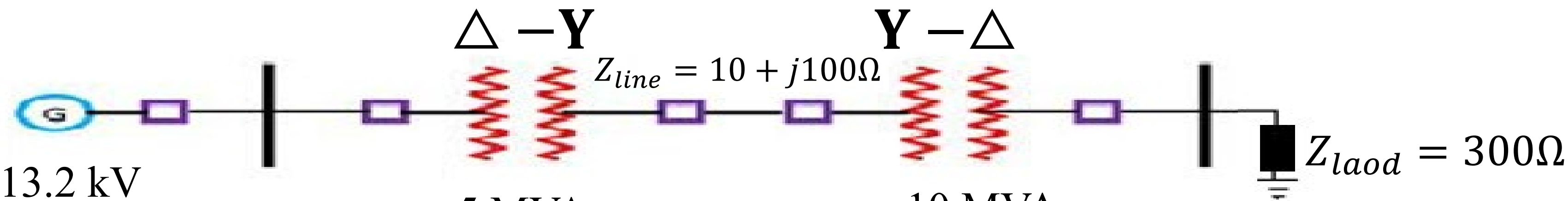
$V_{3B}^{L-L} = 69 \text{ kV}$ $\quad V_{2B}^{L-L} = 138 \text{ kV}$ $\quad V_{1B}^{L-L} = 13.8\text{kV}$

$$Z_{3B} = \frac{\left(V_{3B}^{L-L}\right)^2}{S_B^{3\phi}} = \frac{69^2}{10} = 476\Omega$$

$$Z_{2B} = \frac{\left(V_{2B}^{L-L}\right)^2}{S_B^{3\phi}} = \frac{138^2}{10} = 1904\Omega$$

$$Z_{1B} = \frac{\left(V_{1B}^{L-L}\right)^2}{S_B^{3\phi}} = \frac{13.8^2}{10} = 19.04\Omega$$

Example 3: Solution



5 MVA
13.2 kV- 132 kV
$X_{L1} = 10\% = 0.1\text{p.u.}$

10 MVA
13.8 kV- 69 kV
$X_{L2} = 8\% = 0.08\text{p.u.}$

$$S_B^{3\phi} = 10 \text{ MVA}$$

$V_{3B}^{\text{L}-\text{L}} = 69 \text{ kV}$ $V_{2B}^{\text{L}-\text{L}} = 138 \text{ kV}$ $V_{1B}^{\text{L}-\text{L}} = 13.8\text{kV}$

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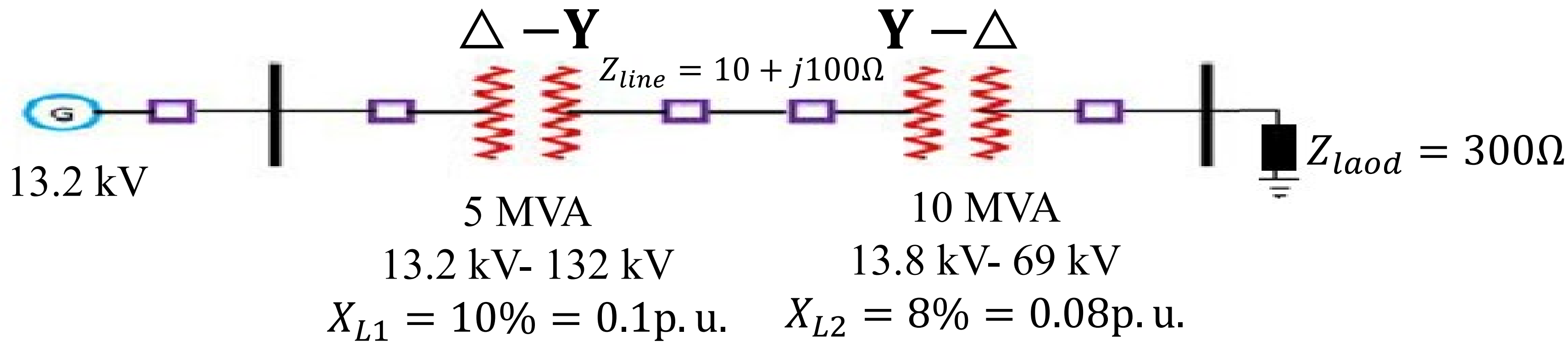
$$Z_{1B} = \frac{\left(V_{1B}^{\text{L}-\text{L}}\right)^2}{S_B^{3\phi}} = \frac{13.8^2}{10} = 19.04\Omega$$

Example 3: Solution

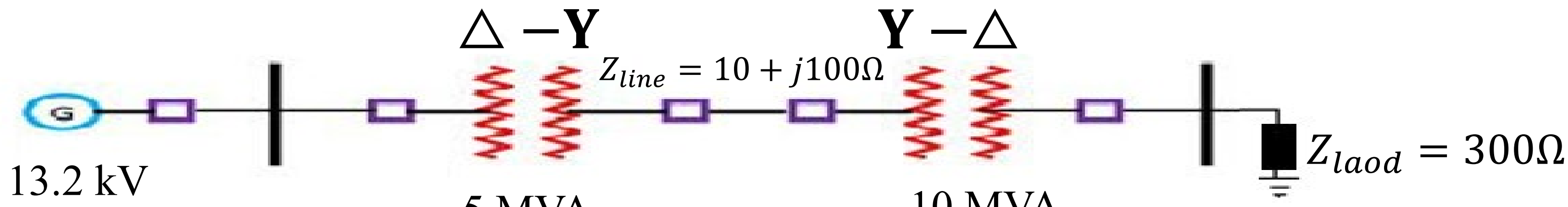


$$Z_{load} = \frac{350\Omega}{476\Omega} = 0.63 \text{ p.u.}$$

$$Z_{line} = \frac{10 + j100\Omega}{1904\Omega} = 0.005 + j0.05 \text{ p.u.}$$

$$E_s = \frac{13.2kV}{13.8 \text{ kV}} = 0.96 \text{ p.u.}$$

Example 3: Solution



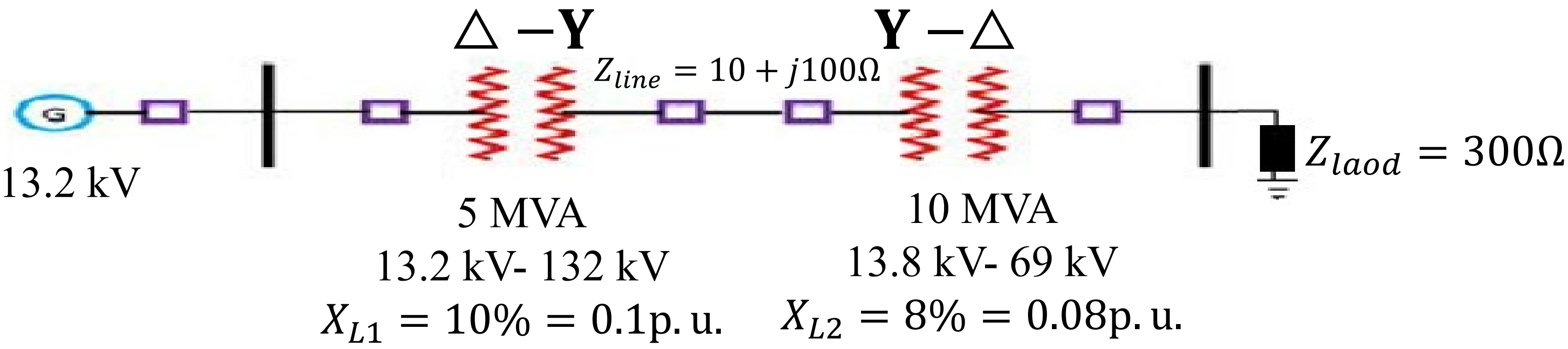
5 MVA
13.2 kV- 132 kV
$X_{L1} = 10\% = 0.1\text{p. u.}$

10 MVA
13.8 kV- 69 kV
$X_{L2} = 8\% = 0.08\text{p. u.}$

$$X_{\mathrm{L2}} = \frac{0.08 \times \frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}}{Z_{3B}} = \frac{0.08 \times \frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}}{\frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}} = 0.08$$

$$X_{\mathrm{L1}} = \frac{0.1 \times \frac{(13.2\text{kV})^2}{5\text{ MVA}}}{Z_{1B}} = \frac{0.1 \times \frac{(13.2)^2}{5}}{\frac{(13.8)^2}{10}} = 0.183\text{ p. u}$$

Example 3: Solution



$$X_{\mathrm{L1}} = \frac{0.1 \times \frac{(13.2\mathrm{kV})^2}{5\ \mathrm{MVA}}}{Z_{1B}} = \frac{0.1 \times \frac{(13.2)^2}{5}}{\frac{(13.8)^2}{10}} = 0.183\ \mathrm{p.u}$$

Alternatively, we can find X_{L1} as,

$$X_{\mathrm{L1}} = \frac{0.1 \times \frac{(132\mathrm{kV})^2}{5\ \mathrm{MVA}}}{Z_{2B}} = \frac{0.1 \times \frac{(132\ \mathrm{kV})^2}{5\ \mathrm{MVA}}}{\frac{(138)^2}{10}} = 0.183\ \mathrm{p.u}$$

Thank You!