EE 303 Energy Systems and Power Electronics

Power Flow

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Power Flow Analysis

- The *power flow problem* is a very well-known problem in the field of power systems engineering, where voltage magnitudes and angles for one set of buses are desired, given that voltage magnitudes and power levels for another set of buses are known.

- A *power flow solution procedure* is a numerical method that is employed to solve the power flow problem.

- The *power flow solution* contains the voltages and angles at all buses, and from this information, we may compute the real and reactive generation and load levels at all buses and the real and reactive flows across all circuits.

- The above terminology is often used with the word "load" substituted for "power," i.e., load flow problem, load flow solution procedure, load flow program, and load flow solution.

Terminologies

- Bulk high voltage transmission systems are always comprised of three phase circuits.
- However, under balanced conditions (the currents in all three phases are equal in magnitude and phase separated by 120^o), we may analyze the three-phase system using a per-phase equivalent circuit consisting of the 'A-phase' and the 'neutral conductor'.
- Per-unitization of a per-phase equivalent of a three phase, balanced system results in the per-unit circuit.
- It is the per-unitized, per-phase equivalent circuit of the power system that we use to formulate and solve the power flow problem.

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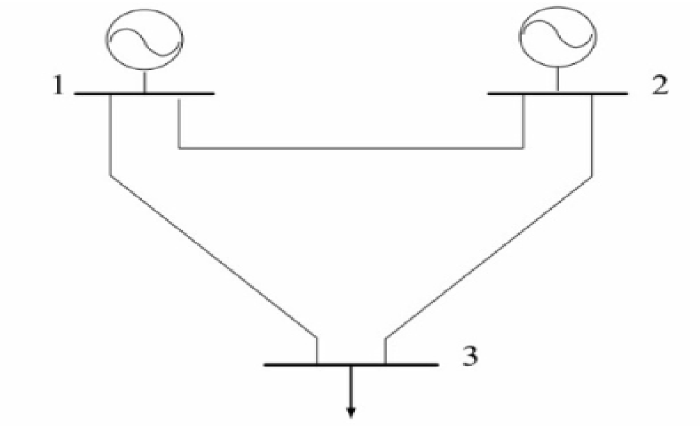
- It is the per-unitized, per-phase equivalent circuit of the power system that we use to formulate and solve the power flow problem.
- It is convenient to represent power system networks using the so-called one-line diagram (per-phase equivalent), but without the neutral conductor.

Terminologies

- The figure represents the one-line diagram of 3-Bus System.
- Minimum set to define a network is voltages.



- Given a set of power injections & voltages, for all buses find voltage angle and magnitudes.

Terminologies

- For each bus *i,* there are four possible variables P_i , Q_i , $|V_i|$, θ_i
- Based on what is known/unknown, we can classify buses into three categories.
  - PV Bus – Generator Bus
  - PQ Bus – Load Bus
  - Slack (Swing) Bus – Reference Bus
- Slack / Swing Bus is a mathematical artifact. Note: Slack bus is also a Generator Bus.
- An *injection* is the power, either real or reactive, that is being injected into or withdrawn from a bus by an element.
- We define a positive injection as one where power is flowing from the element into the bus (i.e., into the network); a negative injection is then when power is flowing from the bus (i.e., from the network) into the element.

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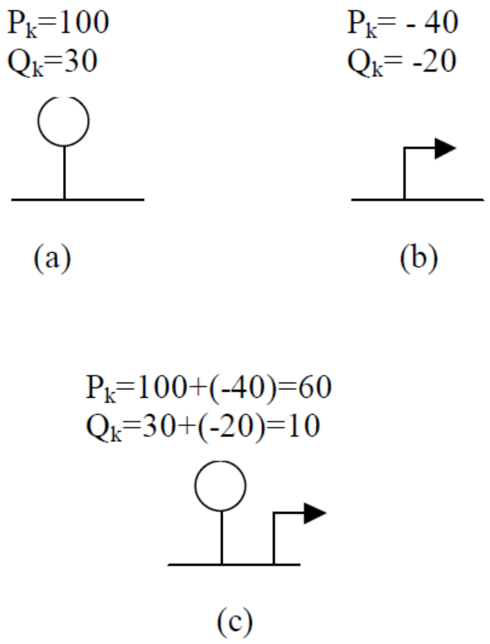


Illustration of (a) positive injection, (b) negative injection, and (c) net injection

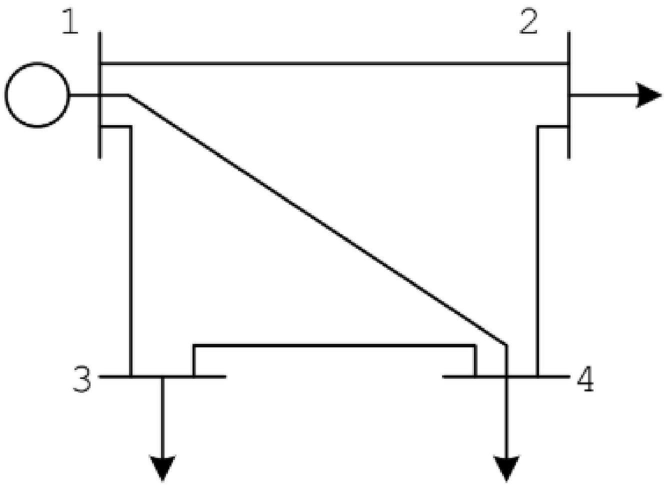
Terminologies

	Known Value	Unknown Value
PQ (Load Bus)	P, Q	$\|V\|, \theta$
PV (Gen Bus)	$P, \|V\|$	Q, θ
Slack (Swing Bus)	$\|V\|, \theta$	P, Q

Note:

N: number of buses in a network
N_G: number of Generator Buses

Swing Bus: 1
PV Bus: $N_G - 1$
PQ Bus: $N - N_G$



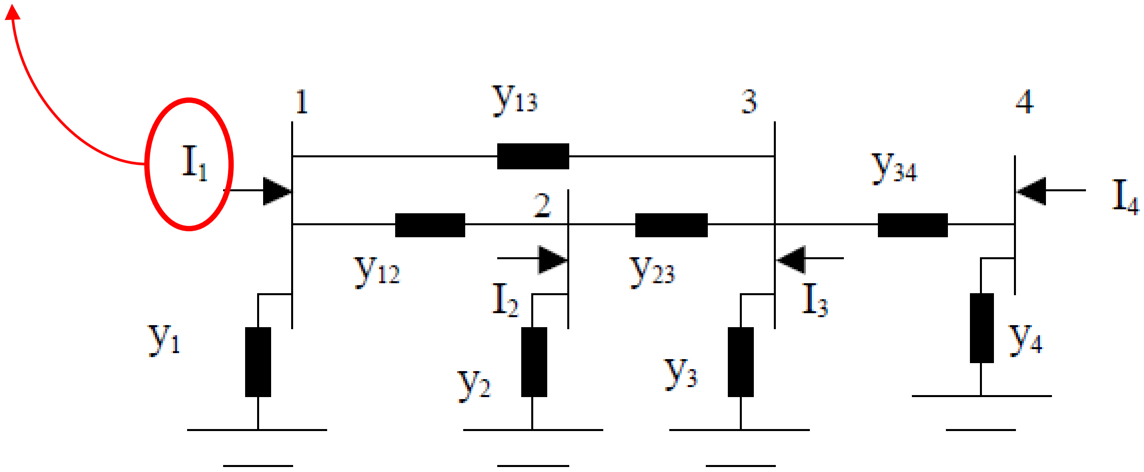
Network Matrix – Admittance Matrix

$$Z = r + jX;$$

Where, $y = \frac{1}{z}$; is called admittance.

$y = g + jb$; $'g'$ is called conductance and $'b'$ is called susceptance.

$I = Y.V$; where I is 'net current injection'.



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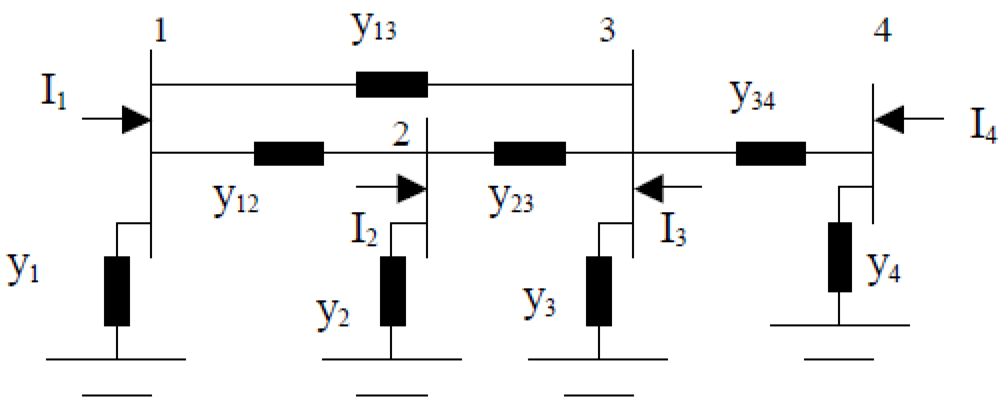


Using KCL in above figure, we get,

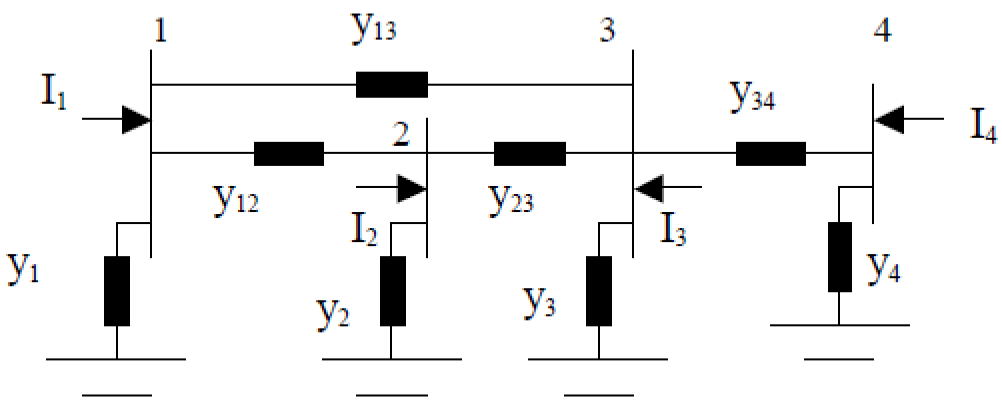
$$I_1 = (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13} + V_1 y_1$$

$$I_1 = (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13} + (V_1 - V_4)y_{14} + V_1 y_1$$ Here, $(y_{14} = 0)$

In general form, we can write,

$$I_1 = v_1(y_1 + y_{12} + y_{13} + y_{14}) + v_2(-y_{12}) + v_3(-y_{13}) + v_4(-y_{14})$$

Contd…



Similarly, we can develop the current injections at buses 2,3, and 4 as:

$$I_2 = V_1(-y_{21}) + V_2(y_2 + y_{21} + y_{23} + y_{24}) + V_3(-y_{23}) + v_4(-y_{24})$$

$$I_3 = v_1(-y_{31}) + v_2(-y_{32}) + v_3(y_3 + y_{31} + y_{32} + y_{34}) + v_4(-y_{34})$$

$$I_4 = v_1(-y_{41}) + v_2(-y_{42}) + v_3(-y_{43}) + v_4(y_4 + y_{41} + y_{42} + y_{43})$$

Contd...

In matrix form, the equations of I_1, I_2, I_3 and I_4 can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ y_4 \end{bmatrix}$$

$(\because I = \mathrm{Y} \cdot \mathrm{V})$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$, is an admittance matrix.

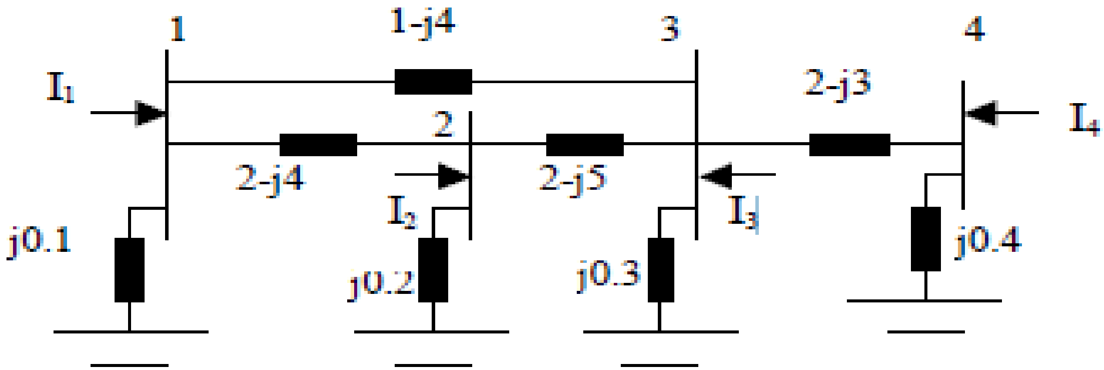
Note: $y_{ki} = y_{ik}$

Contd…

Observations:

1. The Y matrix is symmetric, i.e., $Y_{ij} = Y_{ji}$
2. The off-diagonal elements are the negative of the admittance connecting bus i & bus j, i.e. $y_{ij} = -y_{ji}$
3. The diagonal elements are the sum of the admittances connecting to bus i plus the self-admittance of bus i.

Example: Find Admittance Matrix.

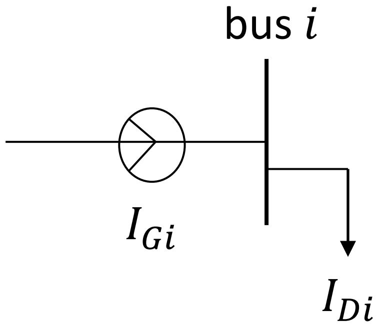


The admittance matrix is:

$j0.1 + 1 - j4 + 2 - j4$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 3 - j7.9 & -2 + j4 & -1 + j4 & 0 \\ -2 + j4 & 4 - j8.8 & -2 + j5 & 0 \\ -1 + j4 & -2 + j5 & 5 - j11.7 & -2 + j3 \\ 0 & 0 & -2 + j3 & 2 - j2.6 \end{bmatrix}$$

Power Flow Equations



Using KCL, the net injection at bus i is,

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik.}V_k$$

Power Flow Equations

$$I_i = \sum_{k=1}^{n} Y_{ik} \cdot V_k$$

$$I_1^* = \left(\sum_{k=1}^{n} Y_{ik} \cdot V_k \right)^*$$

$$V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^*$$

$$S_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

Net Power Injection

Power Flow Equations

$$S_i = P_i + jQ_i$$

$$= V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$= \sum_{k=1}^{n} |V_i||V_k|e^{j\theta_{ik}}(Gik - jB_{ik})$$

$$Y_{ik}^*$$

$$V_i \cdot V_k^* = |V_i|e^{j\theta_i} . |V_k|e^{-j\theta_k} = |V_i|\,|V_k|e^{j\theta_{ik}}$$

$$S_i = \sum_{k=1}^{n} |V_i||V_k|(\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Notes:

- $V_i = |V_i|e^{j\theta_i} = |V_i|\angle\,\theta_i$
- $\theta_{ik} = \theta_i - \theta_k$
- $Y_{ik} = G_{ik} + jB_{ik}$
- $e^{j\theta} = \cos\theta + j\sin\theta$

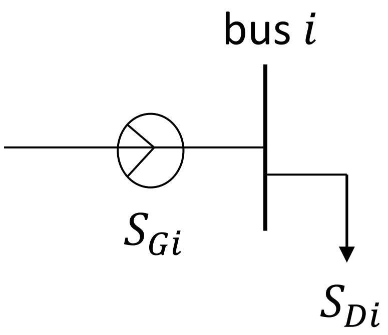
Power Flow Equations

$$S_i = P_i + jQ_i$$

$$S_i = \sum_{k=1}^{n} |V_i||V_k|(\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{G_i} - P_{D_i}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{G_i} - Q_{Di}$$



Here, P_i and Q_i are net power injections at bus i. These equations are known as power flow equations

Power Flow Equations

How to solve the Power Flow Equations??

Power Flow Equations

How to solve the Power Flow Equations??

The Power Flow equations are solved by using **iterative methods**. The most commonly used iterative methods are:

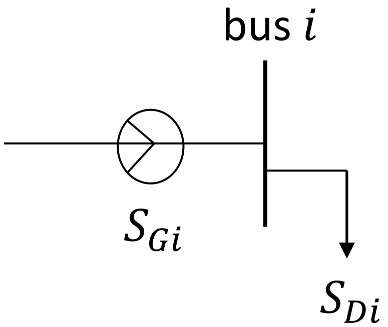
1. Gauss Seidel Method
2. Newton-Raphson Method

Power Flow Equations

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{G_i} - P_{D_i}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{G_i} - Q_{D_i}$$



Here, P_i and Q_i are net power injections at bus i.

& $\theta_{ik} = \theta_i - \theta_k$

$Y_{ik} = G_{ik} + jB_{ik}$

Gauss Iteration

- In Gauss method, we need to rewrite our equation as: $x = h(x)$.

- To iterate, we first make an initial guess of $x, x^{(0)}$ and solve.

- $x^{(v+1)} = h\left(x^{(v)}\right)$; until we find a fixed point. $|\Delta x^{(v)}| < \varepsilon$.

where, $\Delta x^{(v)} = x^{(v+1)} - x^{(v)}$

Example

Solve $x - \sqrt{x} - 1 = 0$. Use Gauss method.

Solution:

Rearranging the equation,

$$X^{(v+1)} = 1 + \sqrt{X^{(v)}}$$

Initial Guess: $X^{(0)} = 1$

v	$X^{(v)}$
0	1
1	$1 + \sqrt{1} = 2$
2	$1 + \sqrt{2} = 2.41421$
3	2.55538
4	2.59805
5	2.61185
6	2.61612
7	2.61744
8	2.61785
9	2.61798

$|X^{v+1} - X^{v}| < \varepsilon$

Power Flow : Gauss Iteration Method

$$S_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

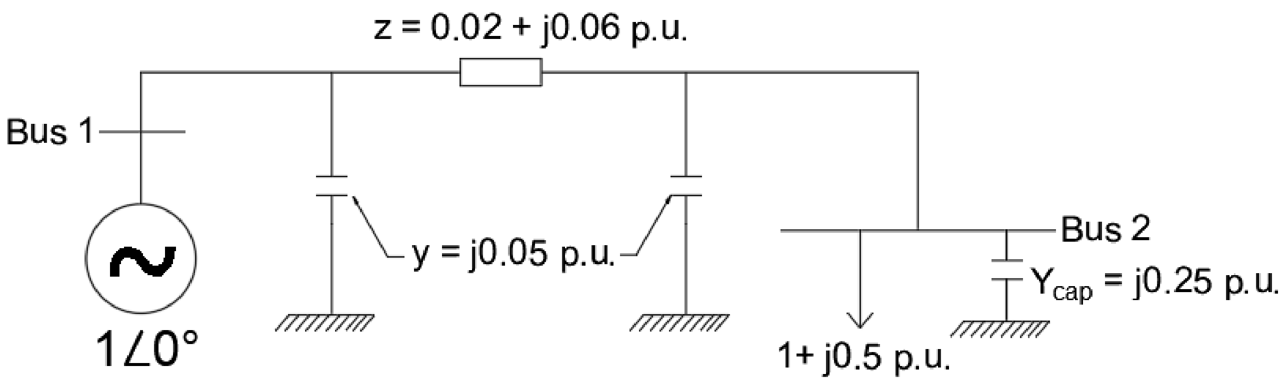
$$S_i^* = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k$$

$$Y_{ii} V_i = \frac{S_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{S_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Example



Solution:

Bus 1 is a swing bus

$$Y = \frac{1}{Z} = \frac{1}{0.02 + j0.06} = 5 - j15$$

$$Y_{\text{Bus}} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$$

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

Contd…

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1, k\neq i}^{n} Y_{ik}V_k\right)$$

$Y_{21}\,V_1$

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - Y_{21}V_1\right)$$

We have, $Y_{22} = 5 - j14.70$; $V_1 = 1\angle 0°$; $S_2 = -1 - j0.5$ p. u.

$$V_2 = \frac{1}{5 - j14.70}\left(\frac{-1 + j0.5}{V_2^*} - (-5 + j5)(1\angle 0°)\right)$$

Contd…

$$V_2 = \frac{1}{5 - j14.70}\left(\frac{-1 + j0.5}{V_2^*} - (-5 + j5)(1\angle 0°)\right)$$

$V_2^{(0)} = 1\angle 0°$ (flatstart)

ϑ	$V_2^{(v)}$
0	$1 + j0.$
1	$0.9671 + j0.0568$
2	$0.9624 - j0.0553$
3	$0.9622 - j0.0556$
4	$0.9622 - j0.0556.$

$\therefore v_2 = 0.9622 - j0.05 = 0.9638\angle - 3.3°$

Contd…

$\therefore v_2 = 0.9622 - j0.05 = 0.9638\angle -3.3°$

Now,

$$S_i^* = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2)$$

$$S_1^* = 1\angle 0[(5 - j14.95) + (-5 + j15) * (0.9638\angle -3.3°)]$$

$$\therefore S_1 = 1.023 + j0.239 \text{ p.u.}$$

Gauss Method with Multiple Bus

$$V_i^{(v+1)} = \frac{1}{Y_{ii}}\left(\frac{S_i^*}{V_i^{(\vartheta)*}} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k^{(\vartheta)}\right)$$

$$V_i(v+1) = h_i\left(V_1^{(\vartheta)}, V_2^{(\vartheta)}, \dots, V_n^{(v)}\right)$$

Considering, V_1 slack/swing Bus.

$$\left.\begin{aligned}
V_2^{(v+1)} &= h_2\left(V_1, V_2^{(v)}, V_3^{(v)}, \dots V_n^{(v)}\right) \\
V_3^{(v+1)} &= h_3\left(V_1, V_2^{(v)}, V_3^{(v)}, \dots, V_n^{(v)}\right) \\
&\vdots \\
V_n^{(v+1)} &= h_n\left(V_1, V_2^{(v)}, V_3^{(v)}, \dots V_n^{(v)}\right)
\end{aligned}\right\}$$

Note, these are **Gauss** Iterations

Contd…

But after we've determined $V_i^{(v+1)}$, we have a better estimate of its voltage, so it makes sense to use this new value. This approach is known as the Gauss-Seidel iteration.

Gauss - Seidel Iteration: **Immediately use the new voltage estimates**

$$V_3^{(v+1)} = h_3\left(V_1, V_2^{(v+1)}, V_3^{(v)}, \ldots V_n^{(v)}\right)$$
$$\vdots \qquad\qquad \vdots$$
$$V_n^{(v+1)} = h_n\left(V_1, V_2^{(v+1)}, V_3^{(v+1)}, \ldots V_n^{(v)}\right)$$

Advantages/ Disadvantages of Gauss Method

Advantages

- Each iteration is relatively fast
- Computational burden is relatively low
- easy to program.

Disadvantages

- Has the tendency to miss solutions, especially for large systems
- Need to compute using complex numbers.

Newton – Raphson Method

General form: Find an " $\hat{x}$ " such that

$$f(\hat{x}) = 0$$

1. For each guess of $\hat{x}, x^{(v)}$, define,

$$\boldsymbol{\Delta} x^{(v)} = \hat{x} - x^{(v)}$$

2. Represent $f(\hat{x})$ by a Taylor series.

$$f(\hat{x}) = f\left(x^{(v)}\right) + \frac{df\left(x^{(v)}\right)}{dx}\Delta x^{(v)} + \frac{1}{2}\frac{d^2 f\left(x^{(v)}\right)}{dx^2}(\Delta x^{(v)})^2 + \text{higher order terms}$$

3. Linearization:

$$f(\hat{x}) = 0 \approx f\left(x^{(v)}\right) + \frac{df\left(x^{(v)}\right)}{dx}\Delta x^{(v)}$$

4. $\Delta x^{(v)} = -\left[\frac{df\left(x^{(v)}\right)}{dx}\right]^{-1} f\left(x^{(v)}\right)$

5. $x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$

Example

Use $N - R$ to solve $f(x) = x^2 - 2 = 0$.

Solution,

$$\Delta x^{(v)} = -\left[\frac{df\left(x^{(v)}\right)}{dx}\right]^{-1} f\left(x^{(v)}\right)$$

$$\Delta x^{(v)} = -\left[2x^{(v)}\right]^{-1}\left(x^{(v)^2} - 2\right)$$

$$\Delta x^{(v)} = -\left[\frac{1}{2x^{(v)}}\right]\left(x^{(v)^2} - 2\right)$$

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$$

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(v)}}\right]\left(\left(x^{(v)}\right)^2 - 2\right)$$

Contd…

Use $N - R$ to solve $f(x) = x^2 - 2 = 0$.

Initial guess: $x^{(0)} = 1$

Therefore,

$\boldsymbol{v}$	$\boldsymbol{x^{(v)}}$
0	1
1	1.5
2	1.41667
3	1.41422

Multi – Variable N-R method

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

- Define the solution $\hat{x}$, $\quad f(\hat{x}) = 0.$
- Define $\Delta x = \hat{x} - x$
- Expand $f_i(x)$ using Taylor series

$$f_1(\hat{x}) = f_1(x) + \frac{\partial f_1(x)}{\partial x_1}\Delta x_1 + \frac{\partial f_1(x)}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_1(x)}{\partial x_n}\Delta x_n + \text{higher order terms}$$

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$$f_n(\hat{x}) = f_n(x) + \frac{\partial f_n(x)}{\partial x_1}\Delta x_1 + \frac{\partial f_n(x)}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_n(x)}{\partial x_n}\Delta x_n + \text{higher order terms}$$

Contd…

In Matrix Form:

$$f(\hat{x}) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \cdots & \frac{\partial f_2(x)}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \frac{\partial f_n(x)}{\partial x_2} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

n x n matrix of partial derivatives
Also called as Jacobian Matrix (J(x))

Contd…

So,

$$\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix} + J(x) \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = 0$$

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_\mathrm{n} \end{bmatrix} = -[J(x)]^{-1} \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

Example:

Solve for $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $f(x) = 0$, where

$$f_1(x) = 2x_1^2 + x_2^2 - 8 = 0$$

$$f_2(x) = x_1^2 - x_2^2 + x_1 x_2 - 4 = 0$$

Solution,

$$J(x) = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} \end{bmatrix}$$

$$J(x) = \begin{bmatrix} 4x_1 & 2x_2 \\ 2x_1 + x_2 & x_1 - 2x_2 \end{bmatrix}$$

Contd…

$$\begin{bmatrix}\Delta x_1\\ \Delta x_2\end{bmatrix} = -\begin{bmatrix}4x_1 & 2x_2\\ 2x_1+x_2 & x_1-2x_2\end{bmatrix}^{-1}\begin{bmatrix}f_1(x_1)\\ f_2(x_2)\end{bmatrix}$$

Now, Initial Guess, $x^{(0)} = \begin{bmatrix}1\\1\end{bmatrix}$

$$x^{(1)} = x^{(0)} + \left[\Delta x^{(1)}\right]$$

$$x^{(1)} = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}4 & 2\\ 3 & -1\end{bmatrix}^{-1}\begin{bmatrix}-5\\-3\end{bmatrix} = \begin{bmatrix}2.1\\1.3\end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix}2.1\\1.3\end{bmatrix} - \begin{bmatrix}8.4 & 2.6\\ 5.5 & -0.5\end{bmatrix}^{-1}\begin{bmatrix}2.51\\1.45\end{bmatrix} = \begin{bmatrix}1.8284\\1.2122\end{bmatrix}$$

$$\begin{bmatrix}f_1\left(x^{(2)}\right)\\ f_2\left(x^{(2)}\right)\end{bmatrix} = \begin{bmatrix}0.1556\\0.0900\end{bmatrix}$$

Load Flow Analysis: N-R Method

$$x = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ |v_2| \\ \vdots \\ |v_n| \end{bmatrix}$$

Assume bus 1 is the slack bus.

$$|v_1|\angle\theta_1 = 1\angle 0°$$

$$f(x) = \begin{bmatrix} P_2 - P_{G2} + P_{D2} \\ \vdots \\ P_n - P_{Gn} + P_{Dn} \\ Q_2 - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k))$$

$$\sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k))$$

Contd…

Set $v = 0$; make initial guess of x (flat start: $1\angle 0^\circ$).

while $\|f(x^{(v)})\| > \varepsilon$ do

$$x(v+1) = x^{(v)} - J\left(x^{(v)}\right)^{-1} + f\left(x^{(v)}\right)$$

$$v = v + 1$$

End while

Note:

Jacobian elements are calculated by differentiating each function $f_i(x)$ with respect to each variable.

For example: if $f_i(x)$ is the bus $'i'$ real power equation,

$$f_i(x) = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) - P_{Gi} + P_{Di}$$

Contd…

- $\frac{\partial f_i(x)}{\partial \theta_i}$?? , when $i = k$

$$f_i(x) = |v_i||v_i|(G_{ii} cos\, 0 + B_{ii} sin\, 0) - P_{Gi} + P_{Di}$$

$$= |v_i|^2 G_{ii} - P_{Gi} + P_{Di}$$

$$\frac{\partial f_i(x)}{\partial \theta_i} = 0$$

- $\frac{\partial f_i(x)}{\partial \theta_i}$?? When $i \neq k$.

$$f_i(x) = \sum_{k=1, k\neq i}^{n} |V_i||V_k|(G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)) - P_{Gi} + P_{Di}$$

$$\frac{\partial f_i(x)}{\partial \theta_i} = |V_i||V_k|(-G_{ik} \sin \theta_{ik} + B_{ik} \cos \theta_{ik})$$

Contd…

When j $\neq i$

$$\frac{\partial f_i(x)}{\partial \theta_j} = |V_i||V_j|\left(-G_{ij}\sin\theta_{ij} + B_{ij}\cos\theta_{ij}\right)$$

Thank You!