Resilient Distribution System

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Outline

• Motivation and Introduction
• Research Activities
  • Subtopic 1: Optimal line hardening strategy for distribution systems
  • Subtopic 2: Resilience-oriented design of distribution systems
• Research Publications
• Reference
Motivation: Impacts of Extreme Weather Events

- Climate Change
  - The probability of occurrence
  - The intensity
  - (Hurricane, ice-storm, flood, etc)

- Example: Hurricane Irma in September 2017
  - Left 6.7 million Floridians without power - 65% of all customers in Florida [1]
  - Its overall damage cost reached to approximately $50 billion [2]
Motivation: Current Situation of Distribution Systems

• Most existing distribution systems are designed and maintained for normal weather conditions

• The classic reliability principles cannot guarantee the lights on under extreme weather events

• U.S. power grids are now old and outdated

• Utilities upgrade grids based on experiences, patrols, and observations

As power engineers, how can we improve grid resilience to survive from extreme weather events?
Introduction: The Resilience of Distribution System

- A distribution system is considered to be resilient if it is able to anticipate, absorb, adapt to, and/or rapidly recover from a disruptive event [6].

Fig.1. A general system performance curve of a distribution system following an extreme weather event

- Event prevention stage: Resistant capability
- Damage propagation stage: Absorptive and adaptive capacity
- Restoration stage: Recovery capability
Introduction: The Resilience Enhancement Measures

- Two resilience goals of distribution systems [7]:
  - System adaptation (to reduce the impact of future events)
  - System survivability (to maintain an adequate functionality during and after the event)

- Resilience enhancement measures:
  - Topological and structural upgrades of the utility’s infrastructures
    - Upgrading distribution poles to stronger classes
    - Installing automatic switches
    - Installing back-up distributed generators (DGs)
  - “Smart” control-based actions
    - Network reconfiguration
    - DG rescheduling
    - Defensive islanding
    - Microgrid-assisted control actions
    - Priority-based load shedding

- We focus on exploring effects of ROD measures on system resilience with the consideration of operation response
Subtopic 1: Optimal line hardening strategy for distribution systems

- Problem Statement
- Literature Review
- Methodology
- Tri-level Robust Optimization Model
- Mathematical Formulation
- Solution Method
- Case Study
- Conclusion
Problem Statement

• The extreme weather events and their negative effects are uncertain.
  • Occurrence and traveling path
  • Damage (line failure)

• There are two major questions on distribution grid hardening[1]:
  • How to prioritize distribution lines for hardening with limited budget
  • What hardening measure should be applied to each line

• We propose an optimal hardening strategy to enhance the resilience of power distribution networks against extreme weather events, considering the time-varying uncertainty of the extreme weather events and the failure probabilities of hardened distribution lines
Literature Review

- Salman et al. in [8] proposed targeted hardening strategy to improve distribution system reliability
- Kuntz et al. in [9] proposed a vegetation management scheduling algorithm (optimal time and location)
- Yuan et al. in [10] proposed a robust optimization model
- Utilities: experiences, patrols and observation
- Drawbacks:
  - only consider a single hardening strategy
  - assume the hardened components will have zero failure probability
Methodology

• Use tri-level robust optimization to enhance the resilience of distribution networks against extreme weather events.
  • Three hardening measures
    • Upgrading distribution poles with higher classes
    • Vegetation Management
    • The combination of both
  • Use a polyhedral set to represent damage uncertainty considering the failure probabilities of hardened lines
Tri-level Robust Optimization Model

• **First stage (Defender):** identify hardening strategies
  
  Minimize: Hardening Investment Cost
  
  Determine:
  
  Distribution lines to be hardened

  The First Level
  System planner

• **Second stage (Attacker):** find the worst weather scenario
  
  Maximize: The Damage
  
  Determine:
  
  The failures of distribution lines

  The Second Level
  Extreme weather events

• **Third stage (Defender):** minimize load shedding
  
  Minimize: Load Shedding Cost
  
  Determine:
  
  Load shedding

  The Third Level
  System Operator
Mathematical Formulation

Objective:
Minimize the hardening investment and the projected load shedding cost under worst weather scenarios

\[
\min_{x \in \chi} \left\{ C^I(x) + \max_{u \in \mathcal{U}(x)} \min_{o \in \mathcal{O}(u)} C^S(o) \right\}
\]

s.t.:

First stage Variable:
\[ x^k_{i,j} \quad \text{whether k-th hardening strategy is selected (1) or not at line (i, j)} \]

First stage Constraints:

\[ C^I(x) = \sum_{(i,j) \in \Omega_B} \sum_{k \in \Omega_x} c^k_{i,j} x^k_{i,j} \leq B_L \quad \text{Hardening investment budget constraint} \]

\[ \chi = \left\{ x \mid \sum_{k \in \Omega_x} x^k_{i,j} = 1, \forall (i,j) \in \Omega_B, x^k_{i,j} \in \{0, 1\} \right\} \quad \text{Hardening measure selection constraint} \]
Second stage decision variables:

\[ z^k_{ij,t} \] : whether line \( ij \) hardened by the \( k \)th hardening strategy is failed (1) or not (0) at time \( t \)

\[ u_{ij,t} \] : line status at time \( t \): damaged (1) or not (0)

Second stage constraints:

\[
\begin{align*}
U &= \left\{ u \left| \sum_{(i,j) \in \Omega_B} (-\log_2 p_{ij,t}^k) z^k_{ij,t} \leq W, \quad \forall k \in \Omega_x, t \in T \right. \right. \\
&\quad \sum_{i \in T} z^k_{i,j,t} \leq x^k_{ij}, \quad \forall k \in \Omega_x, (i,j) \in \Omega_B, t \in T \\
&\quad \sum_{t} z^k_{ij, st} \leq 1, \quad \forall k \in \Omega_x, (i,j) \in \Omega_B \\
&\quad u_{ij,t} \leq \sum_{st=t-T_R} z^k_{ij, st}, \quad \forall k \in \Omega_x, (i,j) \in \Omega_B, t \in T
\end{align*}
\]

Uncertainty budget constraint:
- A line with lower failure probability takes up more uncertainty budget if it fails
- If the failure probability of a line is zero, then it takes up infinitely large uncertainty budget.

Hardening strategy constraint:
- The failure of a line being hardened by a specific strategy can only occur if that strategy is selected in the first stage.

Line failure time limit:
- Line failure only occurs once during the extreme weather event

Repair time constraint:
- If the line \( ij \) starts to be out of service at time \( st \), it remains failed until being repaired.
Mathematical Formulation

The third stage decision variables:

\[ V_{i,t}, P_{i,t}^g, Q_{i,t}^g, P_{ij,t}, Q_{ij,t}, \rho_{i,t} \]

The third stage constraints:

\[ c^S(o) = \sum_{i,t} c^i_t \rho_{i,t} P_{i,t}^L, \forall i \in \Omega_L, t \in T \]

Load shedding cost

\[ \mathcal{O}(u) = \left\{ \begin{array}{l}
\sum_{\{i|(i,j)\in \Omega_B\}} P_{ij,t} = \sum_{\{i|(i,j)\in \Omega_B\}} P_{ji,t} - P_{i,t}^g - (1 - \rho_{i,t}) P_{i,t}^L, \forall i \in \Omega_N, t \in T
\end{array} \right\} \]

\[ \lambda_{i,t}^1 \]

Power balance constraint

\[ 0 \leq P_{ij,t} \leq (1 - u_{ij,t}) P_{ij}^{\max}, \forall (i,j) \in \Omega_B, t \in T \]

\[ \lambda_{ij,t}^3 \]

Line flow limits

\[ 0 \leq Q_{ij,t} \leq (1 - u_{ij,t}) Q_{ij}^{\max}, \forall (i,j) \in \Omega_B, t \in T \]

\[ \lambda_{ij,t}^4 \]

Power flow constraints

\[ \left| V_{j,t} \right| \leq \left| V_{i,t} \right| \frac{R_{ij} P_{ij,t} + X_{ij} Q_{ij,t}}{V_0} + u_{ij,t} M_1, \forall i \in \Omega_N, t \in T \]

\[ \lambda_{i,t}^6 \]

DG output limits

\[ 0 \leq P_{i,t}^g \leq P_{i,t}^{g,\max}, \forall i \in \Omega_G, t \in T \]

\[ \lambda_{i,t}^7 \]

Voltage limit

\[ 0 \leq Q_{i,t}^g \leq Q_{i,t}^{g,\max}, \forall i \in \Omega_G, t \in T \]

\[ \lambda_{i,t}^8 \]

Load shedding ratio limit

\[ \left| V_{i,t} \right|_{\text{min}} \leq \left| V_{i,t} \right| \leq \left| V_{i,t} \right|_{\text{max}}, \forall i \in \Omega_N, t \in T \]

\[ \lambda_{i,t}^{10} \]
Solution Method—Problem Reformulation

• **Problem Formulation**

  Tri-level Formulation

  \[
  \min_{x \in \chi} \left\{ C^I(x) + \max_{u \in U(x)} \min_{o \in O(u)} C^S(o) \right\} \quad \text{Reformulation}
  \]

  \[\text{s.t.:} \quad C^I(x) \leq B_L\]

  **Bi-level Formulation**

  Upper-level problem \( \mathcal{H}(x) \): Select optimal hardening measures for the most critical line

  \[
  \min_{x \in \chi} C^I(x) \quad \text{s.t.} \quad x \in \chi
  \]

  \[C^I(x) \leq B_L\]

  \[\sum_{(i,j) \in \Omega_B} (-\log_2 p^k_{i,j,t}) z^k_{i,j,t} \geq \mathcal{W}, \forall k \in \Omega_x, t \in \mathcal{T}\]

  Lower-level problem \( \mathcal{R}(x) \): Identify critical lines

  \[
  \max_{u \in U(x)} \min_{o \in O(u)} C^S(o)
  \]

• **Max-min Problem \( \mathcal{R}(x) \) Reformulation**

  \[
  \min_{o \in O(u)} C^S(o) \quad \text{Dual} \quad \max \mathcal{D}^S(\lambda) \quad \text{KKT} \quad \text{LPCC} \quad \max_{u \in U} \mathcal{D}^S(\lambda) \quad \text{Big-M} \quad \text{MIP:} \quad \mathcal{R}(x)
  \]

  \[\text{s.t. KKT optimility constraints}\]
Solution Method-Greedy Searching Algorithm

• The selection of hardening strategies is coupled with the uncertainty set of out-of-service lines.

**Step 0:** Initialization. Set the worst extreme weather condition parameters and \( s = 0 \). Calculate each line’s failure probability without hardening \( p_{ij,t}^0 \).

**Step 1:** Solve \( \mathcal{R}(x^0) \) without hardening and let \( (\rho^0, u^0, z^0) \) denote its optimal solution.

**Step 2:** Obtain the initial critical line set \( \Gamma^0 \) whose failures have severe impacts on load shedding according Step 1’s solution.

**Step 3:** Update \( s \leftarrow s + 1 \). Calculate \( p_{ij,t}^k, \forall k \in \Omega_x, (i,j) \in \Gamma^s \). Solve \( \mathcal{H}(x^s) \), and select the most critical line from \( \Gamma^s \) to be hardened. Use the hardening strategy with the minimum cost to harden that line.

**Step 4:** Solve \( \mathcal{R}(x^s) \) and let \( (\rho^s, u^s, z^s) \) denote the optimal solution. Update critical lines in \( \Gamma^s \).

**Step 5:** If the investment budget reaches the limit, the algorithm ends; otherwise go to Step 3.
Case Study: A Modified Electric Power Research Institute (EPRI) Test Circuit

- This system has a 74-mile primary circuit that supplies 3885 customers.
- There are 68 lines and 69 nodes in the primary network.
- The total load demand at peak is 30.43MW
- The total load shedding cost is $51,832,148.26 before hardening
Case Study 1: With/Without Hardening

Optimal Hardening Plans for Category-4 Hurricane

<table>
<thead>
<tr>
<th>No</th>
<th>Hardened Line</th>
<th>Strategy</th>
<th>Hardening Cost ($)</th>
<th>Load Shedding Cost ($)</th>
<th>Total Failed Lines</th>
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<tbody>
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<td>1</td>
<td>L24-25</td>
<td>1</td>
<td>2,437.13</td>
<td>46,872,116.24</td>
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<tr>
<td>2</td>
<td>L33-38</td>
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<td>3,595.40</td>
<td>40,367,134.84</td>
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<td>L22-23</td>
<td>1</td>
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<td>36,937,089.47</td>
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<tr>
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<td>L15-16</td>
<td>2</td>
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<td>30,945,260.46</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>L12-13</td>
<td>2</td>
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<td>13,819,127.47</td>
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<tr>
<td>6</td>
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<td>7</td>
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<td>9,341,072.20</td>
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<td>334.43</td>
<td>8,233,018.56</td>
<td>8</td>
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</tbody>
</table>
Case Study 2: Sensitivity Analysis

- For all the worst-case hurricanes, the load shedding costs are proportionally decreasing with respect to the increasing of hardening budgets.
- A more severe hurricane results in higher load shedding costs and requires larger hardening investments.
Conclusion

• A new approach is proposed for hardening distribution systems to protect against extreme weather events.
• The problem is formulated as a tri-level mixed-integer linear program, and then reformulated as a bi-level model.
• The proposed model is tested on a modified EPRI test circuit.
• Numerical results show that the proposed model can assist utilities to identify optimal hardening strategies to mitigate systems’ vulnerability to extreme weather.
Subtopic 2: Resilience-Oriented Design of Distribution Systems

- Problem Statement
- Literature Review
- Research Objective
- Stochastic Decision Process of ROD Problem
- Mathematical Formulation of ROD Problem
- Solution Algorithm
- Case Study
- Conclusion
Problem Statement

- How to optimally apply ROD measures to prevent distribution system from extensive damages caused by extreme weather events

- Some spatial-temporal correlations exist among ROD decisions, extreme weather events, and system operations
  - Occurrence, intensity and traveling paths of events are uncertain
  - Physical infrastructure damage statuses are affected by both extreme weather event and ROD decisions (decision dependent uncertainty)
  - ROD decisions affect system recovery and the associated outage/repair costs

- A time varying interaction exists between structural damages and electric outage propagation

- Difficult to capture the entire failure-recovery-cost process of distribution systems during and after an extreme weather event.

- How to optimally apply ROD measures to prevent distribution system from extensive damages caused by extreme weather events
## Literature Review

<table>
<thead>
<tr>
<th>Ref</th>
<th>Uncertainty Consideration</th>
<th>Measures</th>
<th>Model/Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>[10]</td>
<td>• Use a polyhedral set to represent damage uncertainty</td>
<td>• line hardening</td>
<td>• Robust optimization/column-and-constraint generation algorithm</td>
</tr>
</tbody>
</table>
| [11] | • Use failure probabilities of distribution lines to represent damage uncertainty set | • Pole hardening  
• Vegetation management  
• Combination of both | • Tri-level robust optimization/greedy algorithm |
| [12] | • Use failure probabilities of overhead lines and underground gas pipelines to generate line damage uncertainty set | • Line hardening | • Tri-level robust optimization/column-and-constraint generation algorithm |
| [13] | • Use fragility model to generate line damage uncertainty | • Line hardening  
• DG placement  
• Switch Installation | • Two-stage stochastic program/a scenario-based variable neighborhood decomposition search algorithm |
| [14] | • Use fragility model to generate line damage uncertainty | • Line hardening (replace overhead line with underground line)  
• MGs  
• Networked MGs | • Two-stage stochastic program/a decomposition-based heuristic algorithm |
| [15] | • Use fragility model to generate line damage uncertainty  
• Model repair time uncertainty  
• Consider load demand uncertainty | • Line hardening  
• DG placement  
• Switch Installation | • Two-stage stochastic program/Progressive hedging algorithm |
Research Objective

• Develop a new modeling and solution methodology for the ROD of distribution systems against extreme weather events
  • Develop a stochastic decision process to describe the spatio-temporal correlations of ROD decisions and uncertainties
  • Formulate a two-stage stochastic mixed-integer linear program (SMILP) to capture the impacts of ROD decisions and uncertainties on system’s responses to extreme weather events
  • Design solution algorithm for solving the above problems
Stochastic Decision Process of ROD Problem

• Overview
• ROD Measures
• Uncertainty Modeling
Overview

- ROD problem is modeled as a two-stage stochastic decision process:
  - Planner makes ROD decisions
  - The operation uncertainties are resolved during the extreme weather event
  - Operator makes the operation decisions
ROD Measures

- Hardening poles:
  - Strengthening vulnerable components
  - 6 pole types
  - Pole stress (1 > 2 > 3 > 4 > 5 > 6)
- Installing backup DGs
  - Increasing adequacy of power supply
- Adding sectionalizers
  - Increasing topological flexibility

Fig. 1. Pole types
Uncertainty Modeling

- Consider three groups of random variables that have direct impacts on the evolution of the system operation state
  - Line damage status
  - Repair cost
  - Load demand

Fig. 1. The structure of uncertainty space: independent observable random variables/processes (highlighted in red) + deterministic casual connections (parameterized by the first-stage decision).
(a) Line Damage Status Uncertainty

\[ u_{12}(t) \ldots u_{ij}(t) \ldots u_{ij,t} = \sum_{k \in \Omega_K} x^{h}_{ij,k} c^{s}_{ij,k,t}, \forall (i, j) \in \Omega_B, t \in T \]

\[ \zeta_{ij,1}(t) \zeta_{ij,2}(t) \zeta_{ij,3}(t) \zeta_{ij,4}(t) \zeta_{ij,5}(t) \zeta_{ij,6}(t) \]

\[ T_{ij}^{R} \]

\[ H_{ij,1}^{3}(t) \ldots H_{ij,n}^{3}(t) \ldots H_{ij,m_{ij}}^{3}(t) \]

\[ N_{ij,3}^{D} \]

\[ G_{ij,n}^{3}(t) \]

\[ R_{ij,n}^{3} \]

\[ S_{ij,n}^{3}(t) \]

\[ F_{b,ij,n}^{3} \]

\[ v_{ij,n}(t) \]

\[ \Delta v_{ij,n}(t) \]

\[ A_a \]

\[ \delta_c \]

\[ \nu \]

\[ u_{tr} \]

\[ r_f \]

\[ v \]

\[ F_b \]

\[ T^{R} \]

\[ = \text{Pole resistance (R) – Wind load (S)} \]

\[ \text{Damaged pole counter} \]

\[ \text{Structural limit state function (G)} \]

\[ \text{Damaged pole counter} \]

\[ \text{Pole hardening decision} \]
(b) Repair Cost Uncertainty

\[
c_{ij}^r = \sum_{k \in \Omega_k} x_{ij,k}^r \chi_{ij,k}, \forall (i, j) \in \Omega_B
\]

Pole hardening decision

\[
\chi_{ij,1} X_{ij,2} X_{ij,3} X_{ij,4} X_{ij,5} X_{ij,6}
\]

Pole repair cost

\[
N_{ij,3}^D \chi_{ij,3}^p
\]

Structural limit state function (G)

= Pole resistance (R) – Wind load (S)

Repair cost

\[ c^r \]

Wind speed

\[ \nu \]

Wood fiber stress

\[ F_b \]

Wood fiber stress

\[ F_{b,ij,n} \]

Wind speed

\[ \Delta v_{ij,n}(t) \]

\[ v_{ij,n}(t) \]

\[ \bar{v}(t) \]

\[ \delta_c \]

\[ r_f \]

\[ A_a \]

\[ v_{tr} \]
(c) Load Demand Uncertainty

\[ P_i^L(t) = \tau_i^P \cdot M^P(t + t_0^L), \forall i \in \Omega_L, t \in T_H \]

\[ Q_i^L(t) = \tau_i^Q \cdot M^q(t + t_0^L), \forall i \in \Omega_L, t \in T_H. \]

\[ \tau_i^P \sim N(\bar{P}_i, (0.02\bar{P}_i)^2), \forall i \in \Omega_L \]

\[ \tau_i^Q \sim N(\bar{Q}_i, (0.02\bar{Q}_i)^2), \forall i \in \Omega_L \]

Fig. 1. Load demand uncertainty

Fig. 2. Load profile shape at the substation (root node)
Mathematic Formulation of ROD Problem

- Overview
- First-stage Problem
- Second-stage Problem
Overview

- **Investment Stage**: identify the optimal ROD decisions
- **Operation Stage**: achieve self-healing operation
  - need a mathematical formulation to model the full power outage propagation process
  - need an analytical optimization to sectionalize a distribution network into multiple self-supplied MGs while maintaining their radial network typologies
First-Stage Formulation

\[ \min \left( C_1^I(x^h) + C_1^I(x^g) + C_1^I(x^c_0) + \omega_H \mathbb{E}_\xi \phi(x, \xi) \right) \]

s.t.

Objective:
Minimize the ROD investment cost and the expected cost of the second stage in realized extreme weather events

First stage ROD variables:
- \( x^h_{i,j,k} \): whether hardening line \((i, j)\) (1) or not (0)
- \( x^c_1_{i,j,i} \): whether adding a sectionalizer at the end \(i\) of line \((i, j)\) (1) or not (0)
- \( x^g_i \): whether installing DG at node \(i\) (1) or not (0)

First stage constraints:
- Hardening strategy limit: \( \sum_{k \in \Omega_K} x^h_{i,j,k} = 1, \forall (i, j) \in \Omega_B \)
- DG number limit: \( \sum_{i \in \Omega_N} x^g_i \leq N_G \)
- Switch installation constraint: \( x^c_0 + x^c_{i,j,n} = x^c_{i,j,n}, \forall (i, j) \in \Omega_B, n \in \{i, j\} \)
- Expected cost of the second stage: \( \mathbb{E}_\xi \phi(x, \xi) \approx \sum_{s \in S} p_T(s) \phi(x, s) \)
Second-Stage Problem: Technique Outline (1)

- Model the power outage propagation constraints
  - Sectionalizers or breakers only exist in certain line sections
  - Customers at the nodes that directly connected to the damaged line will be out of service
- Propagation process:
  - Add a virtual node in the middle of each branch
  - Apply a symmetric fault to the virtual node if the line is damaged
  - Set the voltage feasible region: \( \{0\} \cup [V_{\min}, V_{\max}] \)
  - Fully curtail a load when its voltage magnitude is zero
  - Set loading limits to all branches and penalize load shedding amount in the objective

Fig. 1. The illustrative example for isolating a fault
Second-Stage Problem: Technique Outline (2)

• Model radiality constraints for each energized network
  • To reduce potential operation issues and facilitate system back to normal operation
  • Graph Theorem [16]: A forest of $N$ nodes has exactly $N - N_c$ edges, where $N_c$ is the number of connected network components.
• How to obtain $N_c$ in the distribution system
  • Calculate $N_c$ by counting the degree of freedom of voltage angles
  • Formulate a virtual DC optimal power flow (VDCOPF) sub-problem to obtain this degree of freedom
    • The optimal solution of this sub-problem satisfies that the virtual loads in the same energized island are nearly equally distributed at active nodes
    • Each energized island has and only has an active node with zero angle
• The radiality constraint is satisfied iff the number of active branches equals the total number of active nodes minus the number of active nodes with zero angles
Second-Stage Formulation

Objective
• Minimize the cost of the loss of load, DG operation, and damage repair in a realized extreme weather event given ROD decisions

\[ \phi(x, s) = \min \sum_{i \in \Omega_N} \sum_{t \in T_H^s} c_i^L y_{i,t}^r s P_{i,t}^L s \Delta t + \sum_{i \in \Omega_N} \sum_{t \in T_H^s} c_i^C P_{i,t}^C s \Delta t + \sum_{(i,j) \in \Omega_B} c_{ij}^r s \]

Constraints
• Distribution system operation constraints
  1) Line damage status constraint
  2) Line repair cost constraint
  3) Line’s on-off status constraints (controlled by switch’s on-off status)
  4) Line flow limits (controlled by line’s on-off status)
  5) Linearized AC power flow equations (Dist-Flow)
  6) DG capacity limits
• Fictitious faulting logic constraints (model outage propagation)
  1) Virtual node power injection constraints
  2) Voltage magnitude limits
  3) Load shedding ratio limit
• Radiality constraints
• Zero Angle indicator constraint (indicating a node with zero angle)
• The minimality condition of VDCOPF sub-problem (obtain the degree of freedom of voltage angle)
• **Key Points:**

- Fictitious faulting logic constraints + Distribution system operation constraints in 1)-3) + Penalty cost of load shedding in objective:
  - isolate damaged lines while minimizing the de-energized network parts
  - make network constraints such as power flow automatically adapt to the topology after reconfiguration
- Radiality Constraints + Zero angle indicator constraint + VDCOPF sub-problem
  - can keep each energized network radial
- Information passing:
  
  Second-stage problem $\xrightarrow{\text{Line’s on-off status and DG on-off status}}$ Optimal virtual voltage angle $\xrightarrow{\text{VDCOPF sub-problem}}$
Second-Stage Constraints

• Distribution system operation constraints

1) Line damage status constraint
2) Line repair cost constraint
3) Line’s on-off status constraints
4) Line flow limits
5) Linearized AC power flow (DistFlow) equations
6) DG capacity limits

Binary variables:

- $u_{i,j,t}^s$: Line damage status
- $y_{i,j,t}^c$: Sectionizer on-off status
- $w_{i,j,t}^o$: Line on-off status

\[ u_{i,j,t}^s = \sum_{k \in \Omega_K} x_{i,j,k}^h c_{i,j,k}^s, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H^s \]

\[ c_{i,j}^c = \sum_{k \in \Omega_K} x_{i,j,k}^h x_{i,j,k}^s, \forall (i, j) \in \Omega_B \]

\[ y_{i,j,t}^c \leq x_{i,j}^c, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H^s \]

\[ x_{i,j}^c + y_{i,j,t}^c + 2w_{i,j,t}^o > 2, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H^s \]

\[ w_{i,j,t}^o + y_{i,j,t}^c \leq 1, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H^s \]

\[ y_{i,j,t}^c, w_{i,j,t}^o \in \{0, 1\}, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H^s \]

\[ -w_{i,j,t}^o \leq \frac{x_{i,j}^c - y_{i,j,t}^c}{V_0} \leq w_{i,j,t}^o \leq \frac{x_{i,j}^o}{V_0} \]

\[ \sum_{(j, (i, j) \in \Omega_B)} x_{i,j}^c y_{i,j,t}^c w_{i,j,t}^o \leq 1 \]

\[ \sum_{(j, (i, j) \in \Omega_B)} Q_{i,j}^s = Q_{i,j}^s (1 - y_{i,j,t}^c) \leq \Omega_N, t \in \mathcal{T}_H^s \]

\[ V_{i,t} = \frac{R_{i,t}^P + P_{i,t}^o}{V_0} + X_{i,t}^c Q_{i,t}^s - (1 - w_{i,j,t}^o) M_1, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \]

\[ 0 \leq P_{i,t}^g \leq p_{i}^{g_{\text{max}}}, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \]

\[ 0 \leq Q_{i,t}^g \leq q_{i}^{g_{\text{max}}}, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \]
Second-Stage Constraints

• Fictitious faulting logic constraints

  1) Virtual node power injection constraints

  \[-u_{i,j,t}^s M_2 \leq \sum_{k \in \{i,j\}} P_{kf_{i,j},t}^s + \varepsilon_1 \cdot V_{i,t}^s \leq u_{i,j,t}^s M_2, \forall (i,j) \in \Omega_B, \; f_{ij} \in \Omega_{NF}, \; t \in \mathcal{T}_H^s \]

  \[-u_{i,j,t}^{s,m} M_2 \leq \sum_{k \in \{i,j\}} Q_{kf_{i,j},t}^s \leq u_{i,j,t}^{s,m} M_2, \forall (i,j) \in \Omega_B, \; f_{ij} \in \Omega_{NF}, \; t \in \mathcal{T}_H^s \]

  \[w_{i,t}^{m,s} V_i^{\min} \leq V_{i,t} \leq w_{i,t}^{m,s} V_i^{\max}, \forall i \in \Omega_{NF}, \; t \in \mathcal{T}_H^s \]

  2) Voltage magnitude limits

  \[u_{i,j,t}^{s,m} + w_{f_{ij},t}^{m,s} \leq 1, \forall (i,j) \in \Omega_B, f_{ij} \in \Omega_{NF}, \; t \in \mathcal{T}_H^s \]

  \[w_{i,t}^{m,s} \in \{0,1\}, \forall i \in \Omega_{NF}, \; t \in \mathcal{T}_H^s \]

  3) Load shedding ratio limit

  \[1 - w_{i,t}^{m,s} \leq y_{i,t}^{r,s} \leq 1, \forall i \in \Omega_N, \; t \in \mathcal{T}_H^s \]

• Radiality constraints

  1) Radiality constraint

  \[\sum_{(i,j) \in \Omega_{BF}} w_{i,j,t}^{b,s} = \sum_{i \in \Omega_{NF}} w_{i,t}^{m,s} - \sum_{i \in \Omega_{NF}} w_{i,t}^{a,s} \]

  2) Active branch identification constraint

  \[w_{i,j,t}^{o,s} + w_{i,t}^{m,s} - 1 \leq w_{i,j,t}^{b,s} \leq 0.5 w_{i,j,t}^{o,s} + 0.5 w_{i,t}^{m,s}, \forall i \in \Omega_{NF}, (i,j) \in \Omega_{BF}, \; t \in \mathcal{T}_H^s \]

  \[w_{i,t}^{a,s} , w_{i,j,t}^{b,s} \in \{0,1\}, \forall i \in \Omega_{NF}, (i,j) \in \Omega_{BF}, \; t \in \mathcal{T}_H^s \]

• Zero angle indicator constraint

  \[w_{i,t}^{a,s} - 1 \leq \frac{1}{2|\Omega_{NF}|} (\mu_{d,i,t}^{s} - 1 + \varepsilon_3) \leq w_{i,t}^{a,s}, \forall i \in \Omega_N, \; t \in \mathcal{T}_H^s \]

Binary variables:

- $w_{i,t}^{m,s}$ active node
- $w_{i,t}^{b,s}$ active branch
- $w_{i,t}^{a,s}$ active node with zero voltage angle
The minimality condition of VDCOPF sub-problem

To realize that a connected network component (healthy island) has one and only one degree of freedom of voltage angle under the condition of full DC power flow equations

\[
\left( \mathcal{P}_{s,t}^{s,*}, \mathcal{P}_{l,t}^{s,*}, \theta_t^{s,*} \right) = \text{arg min}_{\mathcal{P}_{s,t}^{s,*}, \mathcal{P}_{l,t}^{s,*}, \theta_t^{s,*}} \left\{ \sum_{i \in \Omega_N} (\theta_i^{s,t} + \frac{\alpha}{2} (\mathcal{P}_{L,i,t}^{s})^2) \right\}
\]

\[
a: -(1 - w_{ij,t}^{o,s}) M_3 \leq \mathcal{P}_{ij,t}^{s} - S_0 B_{ij}^{t} \left( \theta_i^{s,t} - \theta_j^{s,t} \right) \\
\leq (1 - w_{ij,t}^{o,s}) M_3, \forall (i, j) \in \Omega_B \tag{\ref{VDCOPF-Min1}} \]

\[
b: -w_{ij,t}^{o,s} M_3 \leq \mathcal{P}_{ij,t}^{s} \leq w_{ij,t}^{o,s} M_3, \forall (i, j) \in \Omega_B \tag{\ref{VDCOPF-Min2}} \]

\[
s.t. \sum_{j \mid (i, j) \in \Omega_B} \mathcal{P}_{ij,t}^{s} - \mathcal{P}_{i,t}^{g,s} + \mathcal{P}_{L,i,t}^{s} = 0, \forall i \in \Omega_N \tag{\ref{VDCOPF-Min3}} \]

\[
d: -\theta_i^{s,t} \leq 0, \forall i \in \Omega_N \tag{\ref{VDCOPF-Min4}} \]

\[
e: -\mathcal{P}_{L,i,t}^{s} \leq 0, \forall i \in \Omega_N \tag{\ref{VDCOPF-Min5}} \]

\[\forall t \in \mathcal{T}_H \]

KKT optimality condition:

Primal feasibility

\[a: -(1 - w_{ij,t}^{o,s}) M_3 \leq \mathcal{P}_{ij,t}^{s} - S_0 B_{ij}^{t} \left( \theta_i^{s,t} - \theta_j^{s,t} \right) \leq (1 - w_{ij,t}^{o,s}) M_3, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H \]

\[b: -w_{ij,t}^{o,s} M_3 \leq \mathcal{P}_{ij,t}^{s} \leq w_{ij,t}^{o,s} M_3, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H \]

\[c: \sum_{j \mid (i, j) \in \Omega_B} \mathcal{P}_{ij,t}^{s} - \mathcal{P}_{i,t}^{g,s} + \mathcal{P}_{L,i,t}^{s} = 0, \forall i \in \Omega_N, t \in \mathcal{T}_H \]

Stationarity

\[\frac{\partial L}{\partial \mathcal{P}_{L,i,t}^{s,*}} \leq \alpha_L \mathcal{P}_{L,i,t}^{s,*} + \lambda_{c,i,t}^{s} - \mu_{c,i,t}^{s} = 0, \forall i \in \Omega_N, t \in \mathcal{T}_H \]

\[\frac{\partial L}{\partial \mathcal{P}_{ij,t}^{s,*}} \leq -\lambda_{a,i,j,t}^{s} + \lambda_{b,i,j,t}^{s} + \lambda_{c,i,t}^{s} - \lambda_{c,j,t}^{s} = 0, \forall (i, j) \in \Omega_B, t \in \mathcal{T}_H \]

\[\frac{\partial L}{\partial \theta_i^{s,t}} \leq \lambda_{a,i,t}^{s} B_{ij} S_0 + 1 - \mu_{d,i,t}^{s} = 0, \forall i \in \Omega_N, t \in \mathcal{T}_H \]

Complementary slackness and dual feasibility

\[0 \leq \mu_{d,i,t}^{s} \perp \theta_i^{s,t} > 0, \forall i \in \Omega_N, t \in \mathcal{T}_H \]

\[0 \leq \mu_{c,i,t}^{s} \perp \mathcal{P}_{L,i,t}^{s} > 0, \forall (i, j) \in \Omega_N, t \in \mathcal{T}_H \]

On-off line status

\[-(1 - w_{ij,t}^{o,s}) M_4 \leq \lambda_{a,i,j,t}^{s} \leq (1 - w_{ij,t}^{o,s}) M_4, \forall i \in \Omega_N, t \in \mathcal{T}_H \]

\[-w_{ij,t}^{o,s} M_4 \leq \lambda_{b,i,j,t}^{s} \leq w_{ij,t}^{o,s} M_4, \forall i \in \Omega_N, t \in \mathcal{T}_H \]
Dual Decomposition Algorithm

• A Compact Notation Form of ROD Model

\[
z = \min \left\{ c^T x + \sum_{s \in S} p_r(s)q^T y^{R,s} : (x, y^{R,s}) \in K^s, \forall s \in S \right\}
\]

(1)

where \( K^s = \left\{ (x, y^{R,s}) : Ax = b, T(s)x + W(s)y^{R,s} = h(s), \right\}, \forall s \in S \)

\( x \in \{0, 1\}, y^{R,s} = (y_B^s, y_C^s), y_B^s \in \{0, 1\}, y_C^s \geq 0, \forall s \in S \)

• To induce a scenario-based decomposable structure, the copies \( x^s \) of the first-stage variables \( x \) are introduced to create the following reformulation

\[
z = \min \left\{ \sum_{s \in S} p_r(s)(c^T x^s + q^T y^{R,s}) : x^1 = \cdots = x^{|S|}, (x^s, y^{R,s}) \in K^s, \forall s \in S \right\}
\]

(2)

• The Lagrangian relaxation with respect to the nonanticipativity constraint

\[
L(\mu) = \sum_{s \in S} L_s(\mu^s) = \sum_{s \in S} \min_{x^s, y^{R,s}} \left\{ p_r(s)(c^T x^s + q^T y^{R,s}) + \mu^s x^s : (x^s, y^{R,s}) \in K^s \right\}
\]

(3)

• The lower bound of the Lagrangian relaxation (Lagrangian Dual)

\[
z_{LD} = \max_{\mu} \left\{ \sum_{s \in S} L_s(\mu^s) : \sum_{s \in S} \mu^s = 0 \right\}
\]

(4)
Case Study

The IEEE 123-bus system is mapped into a coastal city in Texas.

- The repair cost of a single pole for 6 pole types is assumed to be the same $\chi_{i,j,1} = \cdots = \chi_{i,j,6} = \$4000$
- Consider the budget limitation, the total number of backup DGs is limited to be 5
- Basic load shedding cost is assumed to be $\$14/kWh$
- DG operation cost is assumed to be $\$8/kWh$
- 20 scenarios are randomly generated
- The total investment cost is $\$5, 048,000$

**TABLE II**

<table>
<thead>
<tr>
<th>#No.</th>
<th>Methods</th>
<th>Cost($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Upgrading pole class</td>
<td>6,000/pole</td>
</tr>
<tr>
<td>2</td>
<td>Adding transverse guys to pole</td>
<td>4,000/pole</td>
</tr>
<tr>
<td>3</td>
<td>The combination of upgrading and guying pole</td>
<td>10,000/pole</td>
</tr>
<tr>
<td>3</td>
<td>Installing a natural gas-fired CHPs as DG with 400kW capacity</td>
<td>1,000/kW</td>
</tr>
<tr>
<td>4</td>
<td>Adding an automatic sectionalizer</td>
<td>15,000</td>
</tr>
</tbody>
</table>

*Assume the span of two consecutive poles is 150 ft.

Fig.1. The optimal ROD methods implementation
Simulating a pole damage status in a hurricane

(a) Wind Speed (m/s)

(b) Wind Load on Pole (lbs-ft)

(c) Pole's Damage Status
Case 1: Comparison with and without ROD

- Compare the second stage cost from the hurricane hits the system to the point when all damaged lines are repaired

![Graph showing cost comparison](image)

Fig.1. The second stage cost comparison with and without ROD under different scenarios

- The expected second-stage cost with optimal ROD is 8.93% of that without ROD
Case 1: Comparison with and without ROD

- Compare the system resilience by the resilience curve, which can be expressed by the percentage of power-served (POPS(t)):

$$POPS(t) = \sum_{s \in S} p_r(s) \frac{\sum_{i \in \Omega_N} (1 - y_{i,t}) P_{L,s}^{i,t}}{\sum_{i \in \Omega_N} P_{L,s}^{i,t}}, \forall t \in T_H$$

- The system with optimal ROD has stronger surviving ability to withstand hurricane and faster recovery.
- DGs and automatic sectionalizers can contribute to mitigating the hurricane’s impact on the system.

![Fig.1. The system resilience curve comparison](image)

- The system with optimal ROD has stronger surviving ability to withstand hurricane and faster recovery.
- DGs and automatic sectionalizers can contribute to mitigating the hurricane’s impact on the system.
Case 2: The self-healing operation case

- To validate the novelty of our MILP formulation strategy to solve the challenges of self-healing operation

Fig. 1. System's self-healing operation at $t = 10$

Fig. 2. System's self-healing operation at $t = 21$
Case 3: Computational Results

Table 4.3 The solution quality statics for DD algorithm solving ROD problems

<table>
<thead>
<tr>
<th>#Scenario</th>
<th>Upper Bound</th>
<th>Lower Bound</th>
<th>Wall Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>674,286.3</td>
<td>628,434.8</td>
<td>67</td>
</tr>
<tr>
<td>10</td>
<td>729,310.1</td>
<td>671,694.6</td>
<td>115</td>
</tr>
<tr>
<td>20</td>
<td>1,057,962.1</td>
<td>976,499.1</td>
<td>156</td>
</tr>
</tbody>
</table>

- It is assumed the relative optimality gap is 8%.
Conclusions

• A new modeling and solution methodology for resilience-oriented design (ROD) of power distribution systems against extreme weather events is proposed

  • The spatial-temporal correlations among ROD decisions, uncertainty space, and system operations during and after extreme weather events are well explored and established

  • A two-stage stochastic mixed-integer model is proposed with the objective to minimize the investment cost in the first-stage and the expected costs of the loss of loads, repairs and DG operations in the second stage

  • A scenario-based dual composition algorithm is developed to solve the proposed model

  • Numerical studies on the 123-bus distribution system demonstrate the effectiveness of optimal ROD on enhancing the system resilience
Research Publications

Journal Papers


Conference Paper


Reference


Thank You!

Q & A