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Automatic Self-Adaptive Local Voltage Control Under Limited Reactive Power

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Outline

1. Background and Motivation
2. Literature Review
3. Analytical Illustration
4. Numerical Case Study
5. Conclusion and Future Work

The presentation is based on our work [1].

1. Background

A large-scale integration of distributed energy resources (DERs), e.g., photovoltaic (PV) generators and wind, in distribution networks.

It provides a variety of benefits to distribution networks, e.g., responding rapidly to near-term generation or reliability-related requirement

The uncertain and intermittent nature of DERs has posed new challenges to voltage regulations problems in distribution networks.

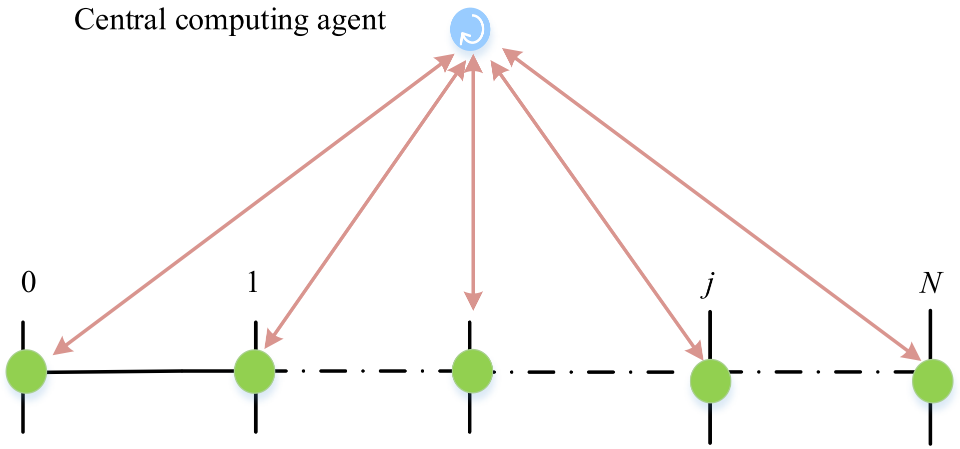
1. Motivation

- Over-/under- voltage problems in distribution systems.
- Rapid development of inverter-based technologies for DERs provides the potential of utilizing the inverter’s reactive power outputs (VAr) to manage voltage.
- An increasing deployment of measuring devices in distribution systems.

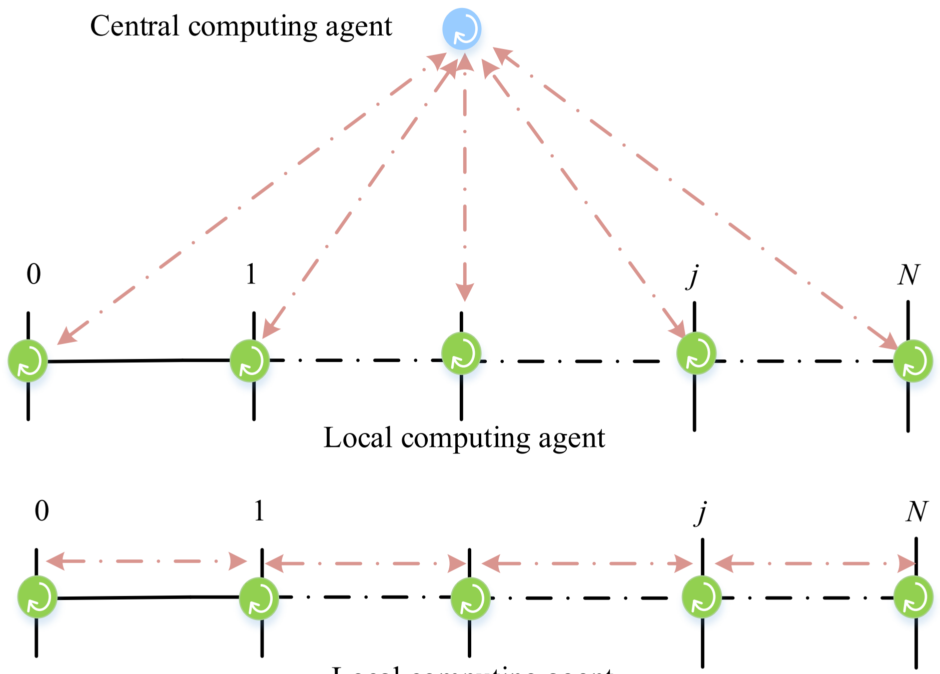
How to better perform Volt/VAr Control (VVC) in distribution networks by taking advantage of those devices?

2. Literature Review

Different VVC strategies



Centralized VVC [2]-[4]



Distributed VVC [5]-[10]

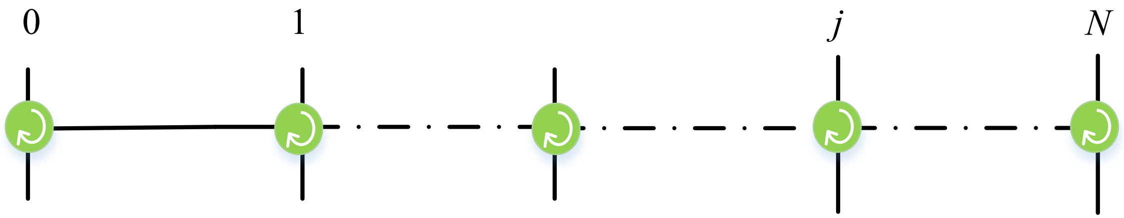
- ➢ Considerable communication and computation overload
- ➢ Not scalable

- ➢ Highly Rely on the reliable communication framework.

Central computing agent | Local non-computing agent | Local computing agent | single-round communication line | multi-round communication line

2. Literature Review

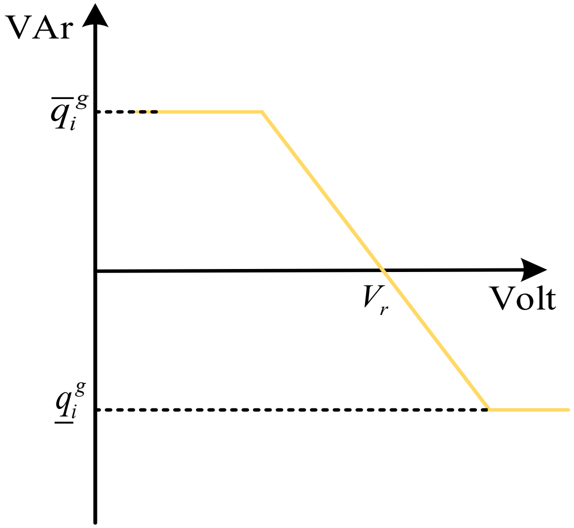
Different VVC strategies



Local computing agent

Local VVC [11]-[16]

- Rely on local information without requiring communication
- More practical and scalable



- Each local agent adjust its reactive power output based on its voltage measurement

2. Literature Review

- Classical Droop Control (CDC) [11]-[13]: $q_i(t+1) = \left[-a_i[V_i(t) - V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$
  - Constant slope and intercept
  - Stability and slow convergence problem
- Delayed Droop Control (DDC) [14]: $q_i(t+1) = (1-\alpha_i)q_i(t) + \alpha_i\left[-a_i[V_i(t) - V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$
  - Constant slope and time-varying intercept
  - Not easy to determine the delay parameter α_i
  - w/o the optimality analysis
- Gradient Projection-Based Droop Control (GPDC) [15]-[16]: $q_i(t+1) = \left[q_i(t) - a_i[V_i(t) - V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$
  - Constant slope and time-varying intercept
  - w/ the optimality analysis; slow convergence rate
- Scaled GPDC [16]: $q_i(t+1) = \left[q_i(t) - a_i d_i[V_i(t) - V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$
  - Constant slope and time-varying intercept
  - w/ the optimality analysis;
  - faster convergence rate than GPDC through tuning d_i

2. Literature Review

Control Type	Update	Description	Optimality
CDC [11]-[13]	$q_i(t+1)=\left[-a_i[V_i(t)-V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$	Constant slope, constant intercept	w/o analyses
DDC [14]	$q_i(t+1)=(1-\alpha_i)q_i(t)+\alpha_i\left[-a_i[V_i(t)-V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$	Constant slope, time-varying intercept	w/o analyses
GPDC [15]-[16]	$q_i(t+1)=\left[q_i(t)-a_i[V_i(t)-V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$	Constant slope, time-varying intercept	w/ analyses
SGPDC [16]	$q_i(t+1)=\left[q_i(t)-a_i d_i[V_i(t)-V_r]\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$	Constant slope, time-varying intercept	w/ analyses

2. Contributions to Date

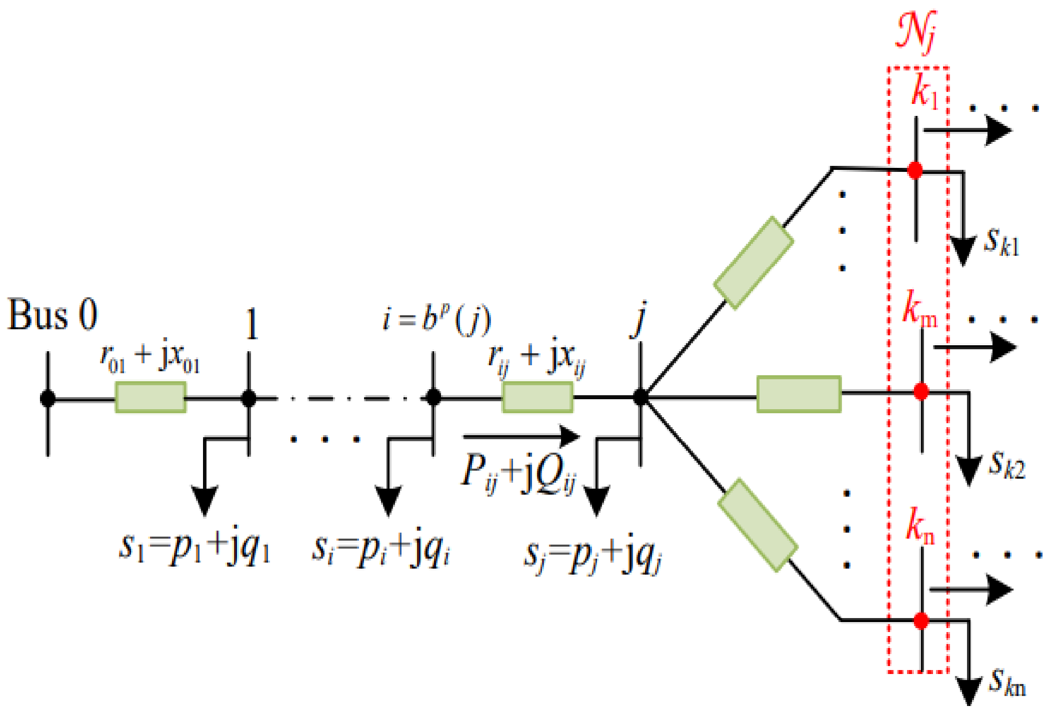
- This local voltage control is *automatic self-adaptive*, allowing each bus agent to locally and dynamically adjust its voltage droop function in accordance with time-varying system changes. This voltage droop function is associated with *both the bus-specific time-varying slope and intercept*, significantly increasing the diversity and flexibility of local voltage control.

- The time-varying slope and intercept are *locally and intelligently* updated by each bus agent merely based on its local voltage measurements without requiring communications, *where the closed-form expressions* of the bus-specific time-varying slope and intercept are analytically explored and presented.

- This automatic self-adaptive local voltage control exhibits *an accelerated convergence rate both theoretically and practically* in static scenarios, indicating a better tracking capability to follow time-varying changes in dynamic scenarios.

3. Problem Statement

Distribution Network



The nonlinear power flow:

$$P_{ij}-\sum_{k\in\mathcal{N}_j}P_{jk}=-p_j+r_{ij}\frac{P_{ij}^2+Q_{ij}^2}{V_i^2}$$

$$Q_{ij}-\sum_{k\in\mathcal{N}_j}Q_{jk}=-q_j+x_{ij}\frac{P_{ij}^2+Q_{ij}^2}{V_i^2}$$

$$V_i^2-V_j^2=2(r_{ij}P_{ij}+x_{ij}Q_{ij})-(r_{ij}^2+x_{ij}^2)\frac{P_{ij}^2+Q_{ij}^2}{V_i^2}$$

Compact form:

$$V=[V_i]_{i\in\mathcal{N}},\ p=[p_i]_{i\in\mathcal{N}},\ q=[q_i]_{i\in\mathcal{N}}$$

$$P=[P_{b^p(j)j}]_{(b^p(j),j)\in\mathcal{L}},\ Q=[Q_{b^p(j)j}]_{(b^p(j),j)\in\mathcal{L}}$$

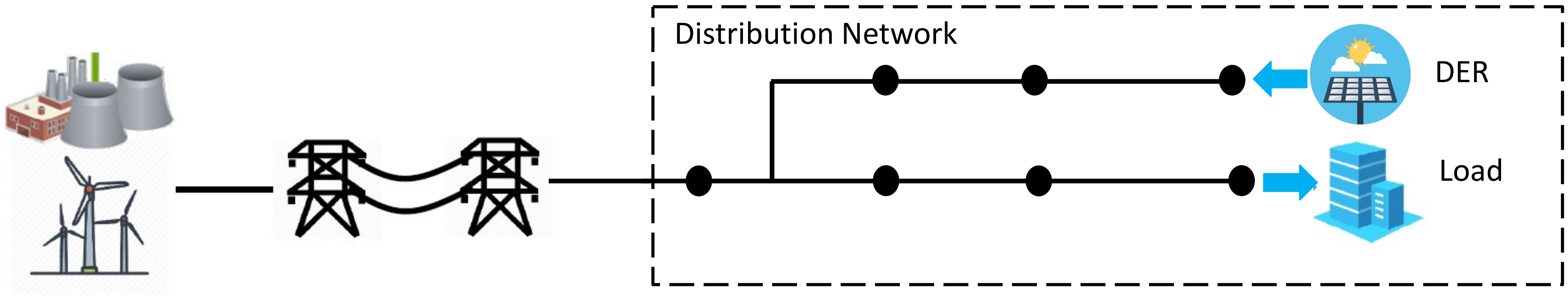
$$q=q^g-q^c$$

reactive power contributed by other loads

reactive power contributed by DERs

$$d=\{q^c,p\}\quad V=h(q^g,d)$$

3. Problem Statement



The VVC problem, based on *the nonlinear power flow*, can be presented as follows:

Minimize the voltage deviations

$$\min m(q^g) = \frac{1}{2} \| V - V_r \|_\Phi^2 = \frac{1}{2} (V - V_r)^T \Phi (V - V_r)$$

$$\text{s.t.}\ \underline{q}^g \le q^g \le \overline{q}^g$$

$$\boxed{V = h(q^g, d)}$$

→ Hard to solve

non-convex and non-linear

3. Problem Statement

Replace *the nonlinear power flow* by *the linearized power flow:*

$$P_{ij} - \sum_{k\in\mathcal{N}_j} P_{jk} = -p_j + r_{ij}\frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \quad \times$$

$$Q_{ij} - \sum_{k\in\mathcal{N}_j} Q_{jk} = -q_j + x_{ij}\frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \quad \times$$

$$V_i^2 - V_j^2 = 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) - (r_{ij}^2 + x_{ij}^2)\frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \quad \times$$

$$V_i^2 - V_j^2 \approx 2(V_i - V_j)$$

$$\Rightarrow$$

$$P_{ij} - \sum_{k\in\mathcal{N}_j} P_{jk} = -p_j$$

$$Q_{ij} - \sum_{k\in\mathcal{N}_j} Q_{jk} = -q_j$$

$$V_i - V_j = r_{ij}P_{ij} + x_{ij}Q_{ij}$$

Assumption 1: The loss is negligible compared to the line flow.

Assumption 2: Assume a relatively flat voltage profile, $V_i = 1, \forall i \in \mathcal{N}$

Compact Form: $V = h_l(q^g, d) = Aq^g + V^{par}(d)$ symmetric and positive-definite [16]

where $A = M^{-T}XM^{-1}$ *and* $V^{par}(d) = M^{-T}RM^{-1}p - Aq^c - V_0 M^{-T}m_0$

$\bar{M} = [m_0, M^T]^T \in \mathbb{R}^{(N+1)\times N}$: the incidence matrix of a radial distribution network.

R, X : diagonal matrices with diagonal entries being the resistance and reactance of line segments.

3. Problem Statement

The VVC problem, based on *the linearized power flow*, can be presented as follows:

$$\min m(q^g) = \frac{1}{2} \| V - V_r \|_{\Phi}^2$$

$$\text{s.t.}\ \underline{q}^g \le q^g \le \overline{q}^g$$

$$V = h_l(q^g, d) = Aq^g + V^{par}(d)$$

Define $f(q^g) = \frac{1}{2} \| h_l(q^g, d) - V_r \|_{\Phi}^2 = \frac{1}{2} \| Aq^g + V^{par}(d) - V_r \|_{\Phi}^2$

$g(q^g)$: the indicator function of $\underline{q}^g \le q^g \le \overline{q}^g$

Then, we have:

$$\min F(q^g) = f(q^g) + g(q^g)$$

3. GFGM-Based VVC

Definition: Approximation model of $F(q^g)$.

Given a symmetric positive-definite matrix L, we say $Q_L(q^g, y)$ is the quadratic approximation model of $F(q^g)$ at a given point y if $Q_L(q^g, y)$ satisfies:

$$F(q^g) = f(q^g) + g(q^g)$$

$$\leq Q_L(q^g, y) = f(y) + < \nabla f(y), q^g - y > + \frac{1}{2} \| q^g - y \|_L^2 + g(q^g)$$

where:

$$< \nabla f(y), q^g - y > = [\nabla f(y)]^T (q^g - y)$$

$$\| q^g - y \|_L^2 = (q^g - y)^T L (q^g - y)$$

Based on the above definition, the **generalized fast Gradient method (GFGM)** can be applied to solve the VVC problem.

Algorithm 1 GFGM-Based VVC

Initialization: Set the iteration time $k = 0$, and $\gamma(1) = 1$, $\boldsymbol{q}^g(0) = \boldsymbol{y}(1) = \boldsymbol{0}$.

For $k \geq 1$: Alternately update variables by the following steps (S1)-(S3) until convergence:

S1: Update $\boldsymbol{q}^g(k)$:

$$\boldsymbol{q}^g(k) = p_L(\boldsymbol{y}(k)) = \arg\min_{\boldsymbol{q}^g} Q_L(\boldsymbol{q}^g, y)$$

S2: Update $\gamma(k+1)$:

$$\gamma(k+1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

S3: Update $\boldsymbol{y}(k+1)$:

$$\boldsymbol{y}(k+1) = \boldsymbol{q}^g(k) + [\frac{\gamma(k) - 1}{\gamma(k+1)}] [\boldsymbol{q}^g(k) - \boldsymbol{q}^g(k-1)]$$

- ***How to ensure stability, convergence and optimality?***
- ***How to facilitate local implementation?***

3. Stability, Convergence, and Optimality

Proposition 1: Assume that $f(q^g)$ is convex and continuously differentiable and L is a symmetric positive-definite matrix. The condition that:

$$f(q^g) \le f(y) + < \nabla f(y), q^g - y > + \frac{1}{2} \| q^g - y \|_L^2$$

is equivalent to:

$$< \nabla f(q^g) - \nabla f(y), q^g - y > \le \| q^g - y \|_L^2$$

As long as the condition is satisfied:

$$< \nabla f(q^g) - \nabla f(y), q^g - y > \le \| q^g - y \|_L^2$$

then:

$Q_L(q^g, y)$ is the quadratic approximation model of $F(q^g)$ at a given point y.

3. Stability, Convergence, and Optimality

Proposition 2: Suppose $F(q^g) = f(q^g) + g(q^g)$ satisfies the following conditions:

- [P2.A] $g(q^g)$ is a convex function which may not be differentiable.
- [P2.B] $f(q^g)$ is convex and continuously differentiable.
- [P2.C] $Q_L(q^g, y)$ is the quadratic approximation model of $F(q^g)$.

Then the sequence $\{q^g(k)\}$, generated by Algorithm 1: GFGM-Based VVC, satisfies:

$$F(q^g(k)) - F(q^{g*}) \le \frac{2\| q^g(0) - q^{g*} \|_L^2}{(k+1)^2}, \forall k \ge 1$$

where q^{g*} is the optimal solution of the VVC problem.

From **Proposition 1**, we know:

$$< \nabla f(q^g) - \nabla f(y), q^g - y >= \| q^g - y \|_{A\Phi A}^2 \le \| q^g - y \|_L^2$$

then [P2.C] holds.

$L \succeq A\Phi A$, $L - A\Phi A$ *is semi-definite positive.* ➡ $\| q^g - y \|_{A\Phi A}^2 \le \| q^g - y \|_L^2$

3. Stability, Convergence, and Optimality

Proposition 3: Suppose $F(q^g) = f(q^g) + g(q^g)$ satisfies the following conditions:

- [P3.A] [P2.A]-[P2.C] hold.
- [P3.B] $g(q^g)$ is an indicator function, and for $\forall q^g, y \in R^N$, there exists a positive definite matrix H satisfying:

$$< \nabla f(q^g) - \nabla f(y), q^g - y > \ \geq \| q^g - y \|_H^2$$

Then the sequence $\{q^g(k)\}$, generated by Algorithm 1: GFGM-Based VVC, satisfies:

$$\| q^g(k) - q^{g*} \| \leq \frac{2 \| q^g(0) - q^{g*} \|_L}{(k+1)\sqrt{\sigma_{\min}(H)}}$$

where $\sigma_{\min}(\cdot)$ denotes the smallest eigenvalue.

$$f(q^g) = \frac{1}{2} \| h_l(q^g, d) - V_r \|_\Phi^2 = \frac{1}{2} \| Aq^g + V^{par}(d) - V_r \|_\Phi^2 \Rightarrow < \nabla f(q^g) - \nabla f(y), q^g - y > = \| q^g - y \|_{A\Phi A}^2$$

$\Rightarrow$ $< \nabla f(q^g) - \nabla f(y), q^g - y > = \| q^g - y \|_{A\Phi A}^2$ $\Rightarrow$ [P3.B] always holds with $H = A\phi A$

3. Stability, Convergence, and Optimality

$$\min m(q^g) = \frac{1}{2} \| h(q^g, d) - V_r \|_\Phi^2$$
$$\text{s.t.}\ \underline{q}^g \le q^g \le \overline{q}^g$$

Nonlinear power flow-based OPF

$$\min f(q^g) = \frac{1}{2} \| h_l(q^g, d) - V_r \|_\Phi^2$$
$$\text{s.t.}\ \underline{q}^g \le q^g \le \overline{q}^g$$

Linearized power flow-based OPF

Proposition 4: Let $\hat{q}^{g*}, m(\hat{q}^{g*})$ be the optimal solution and value of problem and $q^{g*}, f(\hat{q}^{g*})$ be the optimal solution and value of problem. Assume the following conditions hold:

- [P4.A] The error between the linearized power flow model and the exact nonlinear power flow model is bounded. That is, there exists a δ<∞ satisfying:
$$\| h(q^g, d) - h_l(q^g, d) \|_2 \le \delta, \text{where}\ \underline{\mathrm{q}}^{\mathrm{g}} \le \mathrm{q}^{\mathrm{g}} \le \overline{\mathrm{q}}^{\mathrm{g}}$$
- [P4.B] The error between the optimal objective values of problem ( ) and problem ( ) is bounded. That is, there exists a τ<∞ satisfying:
$$\left| m(\hat{q}^{g*}) - f(q^{g*}) \right| \le \tau$$
- [P4.C] [P2.A]-[P2.C] hold.

Then, it follows that:

$$m(q^g(k)) - m(\hat{q}^{g*}) \le \frac{1}{2} \| E \|_2^2 \, \delta^2 + \frac{2 \| q^g(0) - q^{g*} \|_L^2}{(k+1)^2} + \tau$$

where E, satisfying $E^T E = \Phi$, is a upper triangular matrix with real and positive entries

3. Local Implementation

Algorithm 1 GFGM-Based VVC

Initialization: Set the iteration time $k = 0$, and $\gamma(1) = 1$, $\boldsymbol{q}^g(0) = \boldsymbol{y}(1) = \boldsymbol{0}$.

For $k \geq 1$: Alternately update variables by the following steps (S1)-(S3) until convergence:

S1: Update $\boldsymbol{q}^g(k)$:

$$\boldsymbol{q}^g(k) = p_{\boldsymbol{L}}(\boldsymbol{y}(k)) = \arg\min_{\boldsymbol{q}^g} Q_{\boldsymbol{L}}(\boldsymbol{q}^g, y)$$

S2: Update $\gamma(k+1)$:

$$\gamma(k+1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

S3: Update $\boldsymbol{y}(k+1)$:

$$\boldsymbol{y}(k+1) = \boldsymbol{q}^g(k) + [\frac{\gamma(k) - 1}{\gamma(k+1)}][\boldsymbol{q}^g(k) - \boldsymbol{q}^g(k-1)]$$

Simultaneously and locally update

$$y_i(k+1) = q_i^g(k) + [\frac{\gamma(k)-1}{\gamma(k+1)}][q_i^g(k) - q_i^g(k-1)], \forall i \in \mathcal{N}$$

3. Local Implementation

$$q^g(k) = \arg\min_{q^g} Q_L(q^g, y)$$

$$= \arg\min_{\underline{q}^g \le q^g \le \overline{q}^g} < \nabla f(y(k)), q^g - y(k) > + \frac{1}{2} \| q^g - y(k) \|_L^2$$

$$= \arg\min_{\underline{q}^g \le q^g \le \overline{q}^g} \sum_{i=1}^{N} \{ \frac{\partial f(y(k))}{\partial y_i(k)} [q_i^g - y_i(k)] \} + \frac{1}{2} \| q^g - y(k) \|_L^2$$

For a diagonal positive matrix *L*,

$$\sum_{i=1}^{N} \frac{L_i}{2} [q_i^g - y_i(k)]^2$$

$$q^g(k) = \arg\min_{\underline{q}^g \le q^g \le \overline{q}^g} \sum_{i=1}^{N} \{ \frac{\partial f(y(k))}{\partial y_i(k)} [q_i^g - y_i(k)] + \frac{L_i}{2} [q_i^g - y_i(k)]^2 \}$$

naturally decomposable

naturally decomposable

$$q_i^g(k) = \arg\min_{\underline{q}_i^g \le q_i^g \le \overline{q}_i^g} \frac{\partial f(y(k))}{\partial y_i(k)} [q_i^g - y_i(k)] + \frac{L_i}{2} [q_i^g - y_i(k)]^2$$

3. Local Implementation

$$q_i^g(k) = \arg\min_{\underline{q}_i^g \le q_i^g \le \overline{q}_i^g} \frac{\partial f(y(k))}{\partial y_i(k)}[q_i^g - y_i(k)] + \frac{L_i}{2}[q_i^g - y_i(k)]^2, \forall i \in \mathcal{N}$$

Scalar

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i}\frac{\partial f(y(k))}{\partial y_i(k)}\right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

$$q^g(k) = [y(k) - L^{-1}\nabla f(y(k))]_{\underline{q}^g}^{\overline{q}^g}$$

Proposition 5: As L is a diagonal positive definite matrix, $q^{g(k)} = p_L(y(k))$ is equivalent to:

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i}\frac{\partial f(y(k))}{\partial y_i(k)}\right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

which can be expressed in a compact form:

$$q^g(k) = [y(k) - L^{-1}\nabla f(y(k))]_{\underline{q}^g}^{\overline{q}^g}$$

3. Local Implementation

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)} \right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

$$\nabla f(y(k)) = A\Phi[Ay(k) + c - V_r]$$

$$Ay(1) + c - V_r = Aq^g(0) + c - V_r = V(0) - V_r$$

$$Ay(k) + c - V_r = \left[1 + \frac{\gamma(k-1)-1}{\gamma(k)}\right][Aq^g(k-1) + c - V_r] - \frac{\gamma(k-1)-1}{\gamma(k)}[Aq^g(k-2) + c - V_r]$$

$$= \left[1 + \frac{\gamma(k-1)-1}{\gamma(k)}\right][V(k-1) - V_r] - \frac{\gamma(k-1)-1}{\gamma(k)}[V(k-2) - V_r], \; k \geq 2$$

3. Local Implementation

$$\Phi = A^{-1} \longrightarrow \nabla f(y(k)) = A\Phi[Ay(k)+c-V_r] = Ay(k)+c-V$$

$$= \begin{cases} V(0)-V_r, k=1 \\ [1+\frac{\gamma(k-1)-1}{\gamma(k)}][V(k-1)-V_r]-\frac{\gamma(k-1)-1}{\gamma(k)}[V(k-2)-V_r], k \geq 2 \end{cases}$$

$$\frac{\partial f(y(k))}{\partial y_i(k)} = \begin{cases} V_i(0)-V_r, k=1 \\ [1+\frac{\gamma(k-1)-1}{\gamma(k)}][V_i(k-1)-V_r]-\frac{\gamma(k-1)-1}{\gamma(k)}[V_i(k-2)-V_r], k \geq 2 \end{cases}$$

→ locally update

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i}\frac{\partial f(y(k))}{\partial y_i(k)}\right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

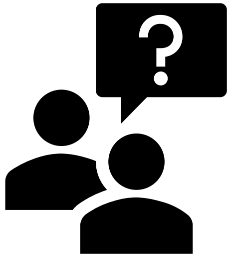
→ locally update

3. Local Implementation

- In short, as $\boldsymbol{\Phi} = \boldsymbol{A}^{-1}$ and $\boldsymbol{L}$ is a diagonal positive definite matrix, we can achieve ***the local implementation*** of Algorithm 1: GFGM-Based VVC.

- $L \succeq A\Phi A$ ensures ***the stability, convergence and optimality*** of Algorithm 1: GFGM-Based VVC.

$$How\ to\ determine\ L?$$



3. Local Implementation

We utilize the following convex semi-definite programming problem to determine $\boldsymbol{L}$:

$$\min tr(L) = \sum_{i=1}^{N} L_i \qquad (*)$$

$$s.t.\ L \succeq A, L = diag(L_1, L_2, ..., L_N)$$

As we choose $\boldsymbol{\Phi = A^{-1}}$ and $\boldsymbol{L}$, determined by (*), it facilitates the local implementation of Algorithm 1: GFGM-Based VVC while ensuring its ***stability, convergence and optimality.***

3. Local Implementation

Algorithm 1 GFGM-Based VVC

Initialization: Set the iteration time $k = 0$, and $\gamma(1) = 1$, $\boldsymbol{q}^g(0) = \boldsymbol{y}(1) = \boldsymbol{0}$.

For $k \geq 1$: Alternately update variables by the following steps (S1)-(S3) until convergence:

S1: Update $\boldsymbol{q}^g(k)$:

$$\boldsymbol{q}^g(k) = p_{\boldsymbol{L}}(\boldsymbol{y}(k)) = \arg\min_{\boldsymbol{q}^g} Q_{\boldsymbol{L}}(\boldsymbol{q}^g, y)$$

S2: Update $\gamma(k+1)$:

$$\gamma(k+1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

S3: Update $\boldsymbol{y}(k+1)$:

$$\boldsymbol{y}(k+1) = \boldsymbol{q}^g(k) + [\frac{\gamma(k) - 1}{\gamma(k+1)}][\boldsymbol{q}^g(k) - \boldsymbol{q}^g(k-1)]$$

local implementation

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i}\frac{\partial f(y(k))}{\partial y_i(k)}\right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

$$\frac{\partial f(y(k))}{\partial y_i(k)} = \begin{cases} V_i(0) - V_r, k = 1 \\ [1 + \frac{\gamma(k-1)-1}{\gamma(k)}][V_i(k-1) - V_r] \\ -\frac{\gamma(k-1)-1}{\gamma(k)}[V_i(k-2) - V_r], k \geq 2 \end{cases}$$

Simultaneously and locally update

$$y_i(k+1) = q_i^g(k) + [\frac{\gamma(k)-1}{\gamma(k+1)}][q_i^g(k) - q_i^g(k-1)], \forall i \in \mathcal{N}$$

3. Reinterpretation of GFGM: Modified Droop Control

$$\frac{\partial f(y(k))}{\partial y_i(k)} = \begin{cases} V_i(0) - V_r, k = 1 \\ [1 + \frac{\gamma(k-1)-1}{\gamma(k)}][V_i(k-1) - V_r] \\ -\frac{\gamma(k-1)-1}{\gamma(k)}[V_i(k-2) - V_r], k \geq 2 \end{cases}$$

$$y_i(k) = \begin{cases} q_i^g(0) = 0, k = 1 \\ q_i^g(k-1) + [\frac{\gamma(k-1)-1}{\gamma(k)}][q_i^g(k-1) - q_i^g(k-2)], k \geq 2 \end{cases}$$

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i}\frac{\partial f(y(k))}{\partial y_i(k)}\right]_{\underline{q}_i(k)}^{\overline{q}_i(k)}, \forall i \in \mathcal{N}$$

$$q_i^g(k) = \left[-a_i(k)[V_i(k-1) - V_r] + b_i(k)\right]_{\underline{q}_i^g}^{\overline{q}_i^g}, k \geq 1$$

$$\mu(k) = \begin{cases} 0, k = 1 \\ \frac{\gamma(k-1)-1}{\gamma(k)}, k \geq 2 \end{cases}$$

$$a_i(k) = \frac{1 + \mu(k)}{L_i}, k \geq 1$$

$$b_i(k) = \begin{cases} q_i^g(0), k = 1 \\ [1 + \mu(k)]q_i^g(k-1) - \mu(k)q_i^g(k-2) \\ + \frac{\mu(k)}{L_i}[V_i(k-2) - V_r], k \geq 2 \end{cases}$$

3. Reinterpretation of GFGM: Modified Droop Control

$$q_i^g(k) = \left[-a_i(k)[V_i(k-1) - V_r] + b_i(k)\right]_{\underline{q}_i^g}^{\overline{q}_i^g}, k \geq 1$$

modified droop control with bus-specific self-adaptive coefficients

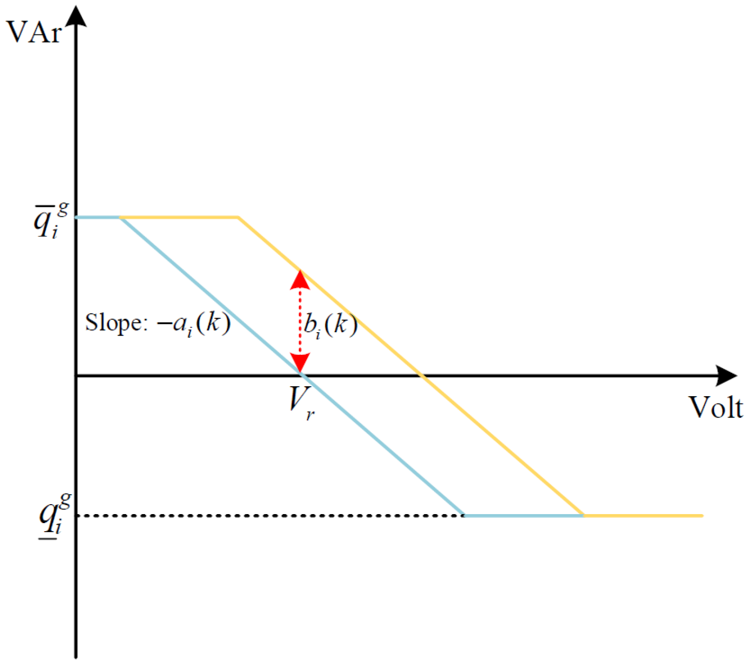
$$a_i(k) = \frac{1+\mu(k)}{L_i}, k \geq 1$$

Locally update

$$b_i(k) = \begin{cases} q_i^g(0), k = 1 \\ [1+\mu(k)]q_i^g(k-1) - \mu(k)q_i^g(k-2) \\ + \frac{\mu(k)}{L_i}[V_i(k-2) - V_r], k \geq 2 \end{cases}$$

Locally update

3. Reinterpretation of GFGM: Modified Droop Control



Algorithm 2 Automatic Self-Adaptive Local Voltage Control (ASALVC): Offline Implementation

Initialization: Set the iteration time $k = 0$. Each bus i sets $\gamma(1) = 1$, $q_i^g(0) = y_i(1) = 0$,

For $k \geq 1$: Each bus i alternately update variables by the following steps until convergence:

- Update $\mu(k)$, $a_i(k)$, $b_i(k)$, respectively.
- Update $q_i^g(k)$: $q_i^g(k) = \big[-a_i(k)[V_i(k-1) - V_r] + b_i(k)\big]_{\underline{q}_i^g}^{\overline{q}_i^g}$
- Update $\gamma(k+1)$:

$$\gamma(k+1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

The blue droop function: the droop control with the slope $-a_i(k)$.

The yellow droop function: the modified droop control.

The modified droop control (the yellow line segments) for bus i is translated from the blue droop function.

3. Online Implementation

$$\min m(q^g) = \frac{1}{2} \| V - V_r \|_{\Phi}^2$$

$$\text{s.t. } \underline{q}^g \le q^g \le \overline{q}^g$$

$$V = h_l(q^g, d) = Aq^g + V^{par}(d)$$

$d = \{q^c, p = p^g - p^c\}$ denotes the changes in the system.

What will happen if d is *time-varying*?

d might have changed before the decision/control variables converge in the offline implementation.

How? **Online implementation.**

Online implementation: *the decision/control variables are adjusted in real-time (for each iteration), based on the real-time feedback from operating statuses, to adapt to real-time changes in the environment.*

3. Online Implementation

Algorithm 3 Automatic Self-Adaptive Local Voltage Control (ASALVC): Online Implementation

For any bus i at time step t:

- Estimate VAr Limits: Locally update $\underline{q}_i^g$ and $\overline{q}_i^g$
- Reset $\gamma(t)$: If t mod T_γ =0, then set $\gamma(t) = 1$.
- Reset $\mu(t)$: If t mod T_γ =0, then set $\mu(t) = 0$; otherwise update $\mu(t)$ by $\frac{\gamma(t-1)-1}{\gamma(t)}$.
- Update $a_i(t)$, $b_i(t)$, respectively.
- Update $q_i^g(t)$: $q_i^g(t) = \left[-a_i(t)[V_i(t-1) - V_r] + b_i(t)\right]_{\underline{q}_i^g}^{\overline{q}_i^g}$, which is based on the real-time voltage measurement $V_i(t-1)$.
- Update $\gamma(t+1)$:

$$\gamma(t+1) = \frac{1 + \sqrt{1 + 4\gamma(t)^2}}{2}$$

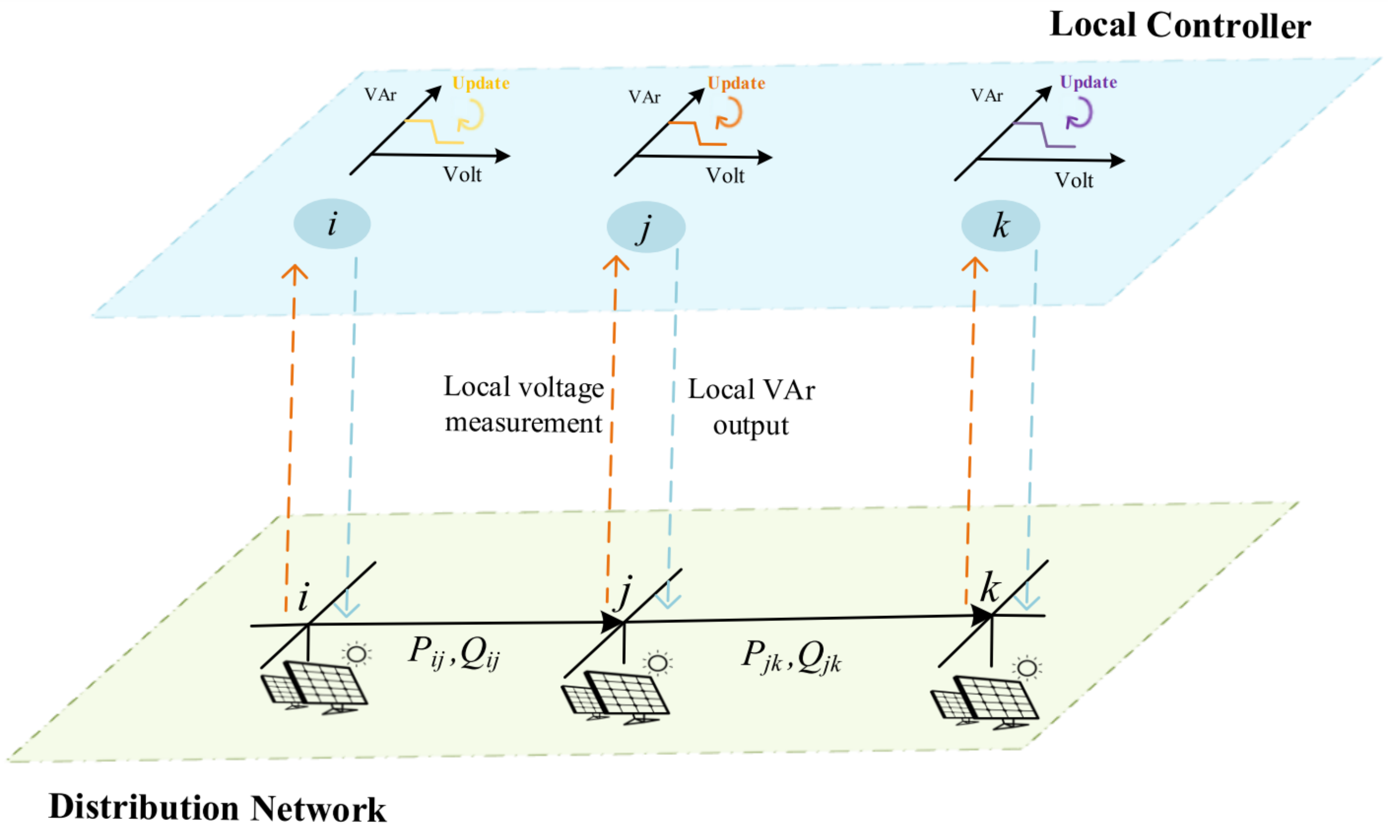
Updated based on the inverter capacities and instantaneous real power outputs of DERs

Reset $\gamma(t)$ and $\mu(t)$ every T_γ time steps.

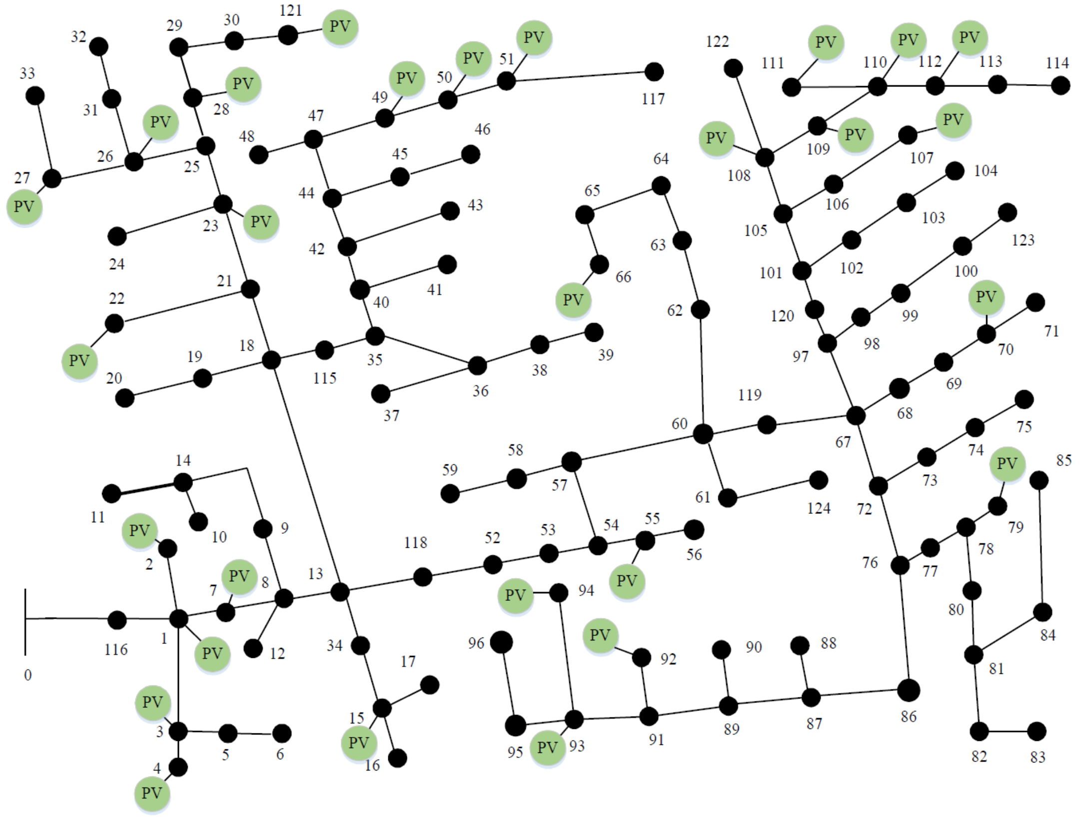
Note that the time-varying $\boldsymbol{d}$ can be reflected in the real-time voltage measurement $V_i(t-1)$.

3. Online Implementation

Each local DER agent adjusts its droop control function in real-time, based on its local voltage measurement, to determine its real-time VAr output.



4. Case Study

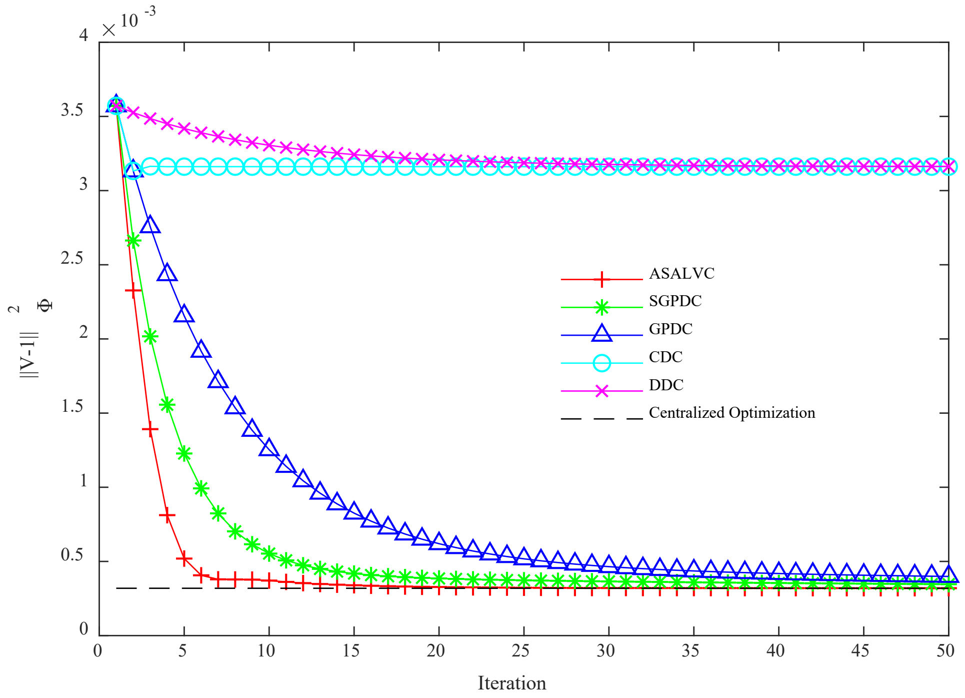


Modified single-phase IEEE 123-bus test system.

- **Static Scenario:**
  - ➢ Each bus has a constant load 1+j0.5 kVA.
  - ➢ Each PV inverter can supply or absorb at most 10 kVAr.

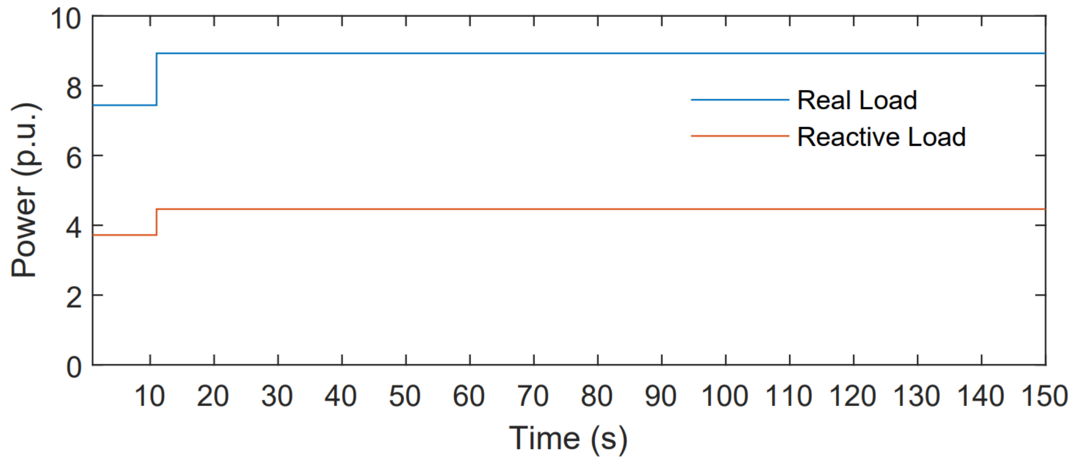
4. Case Study

Voltage mismatch error versus iteration for various controls under the static scenario

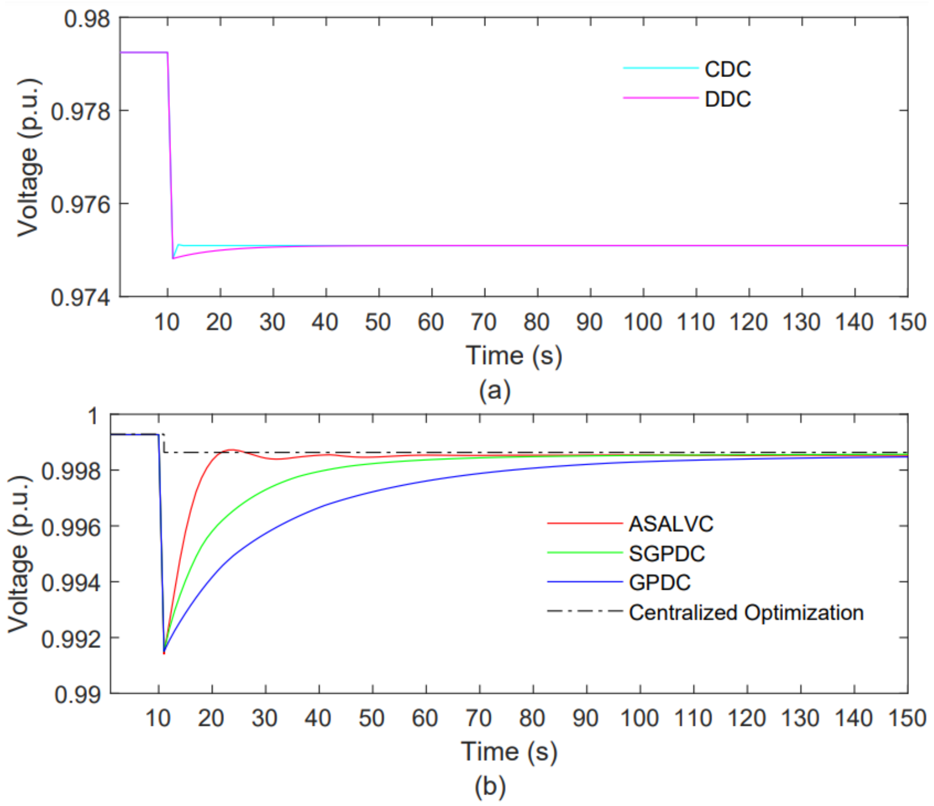


- The convergence outcomes of CDC and DDC fail to track the centralized optimization outcomes.
- The convergence outcomes of ALALVC, SGPDC, and GPDC closely track the centralized optimization outcomes.
- The ALALVC exhibits the best convergence performance.

4. Case Study



Aggregate load in the dynamic scenario with a sudden load change.



Voltage at bus 56 under the dynamic scenario with a sudden load change: (a) for the CDC and DDC; (b) for the GPDC, SGPDC, ASALVC, and centralized optimization.

Reactive power (Var) outputs are updated every second.

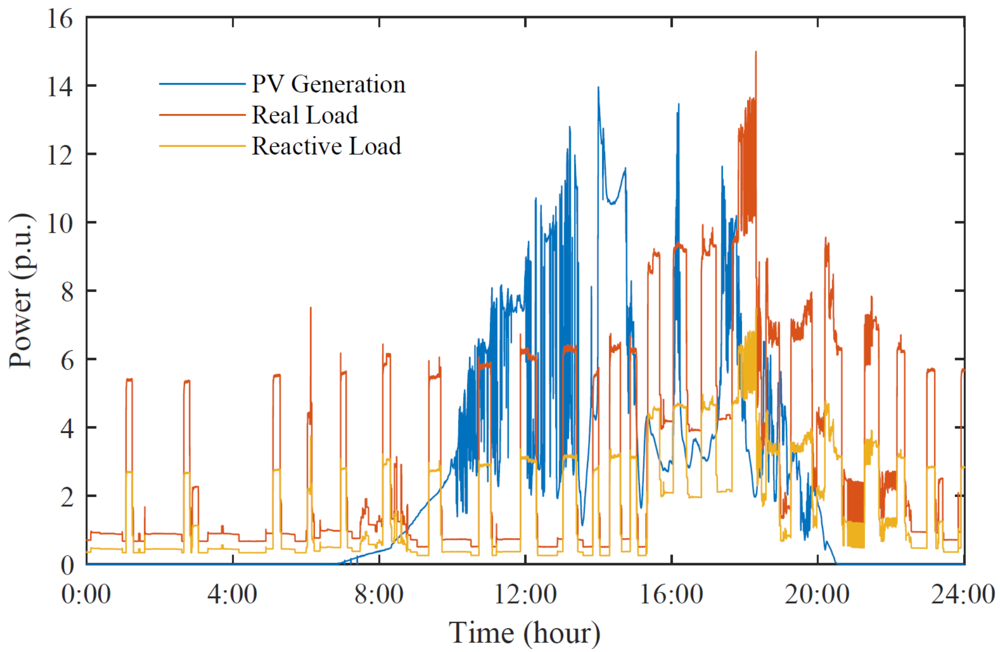
- **Dynamic Scenario with a sudden load change**

➢ The stable operating statuses, determined by the CDC and DDC, are both far away from the optimal operating statuses.

➢ The stable operating statuses, determined by the ASALVC, GPDC and SGPDC, are the same as the optimal operating statuses.

➢ The ALALVC exhibits its stronger capability to recover from a sudden disturbance.

4. Case Study



Aggregate load and PV generation with continuous fast system changes.

- **Dynamic Scenario with fast continuous system changes**

➢ The time span is one day.

➢ The time granularity is 6s. We also set $T_r = 6s$.

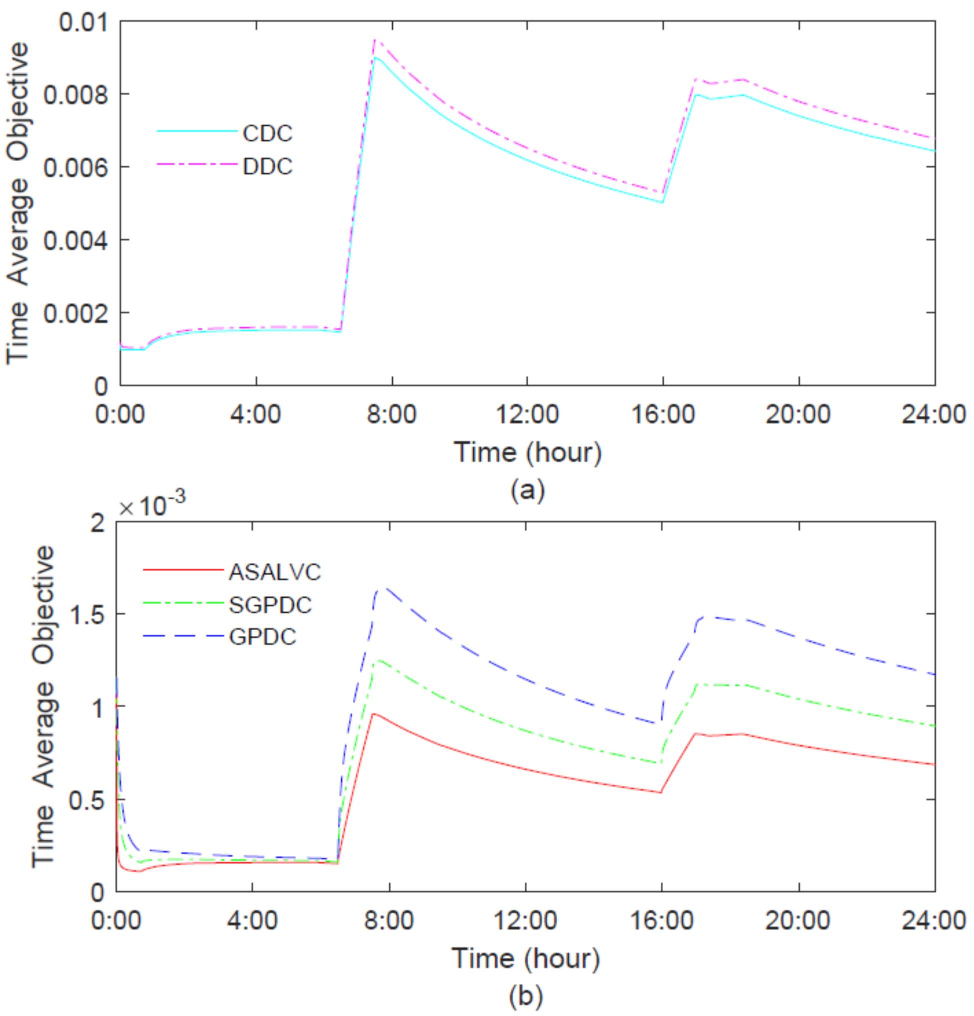
Voltage and capacity issues under the dynamic scenario with continuous fast system changes

	CDC	DDC	GPDC	SGPDC	ASALVC
Voltage issue	Yes	Yes	No	No	No
Capacity issue	No	No	No	No	No

➢ The CDC and DDC suffer from voltage violation problems under the dynamic scenario with continuous fast system changes.

➢ There are not capacity violation problems for all controls.

4. Case Study



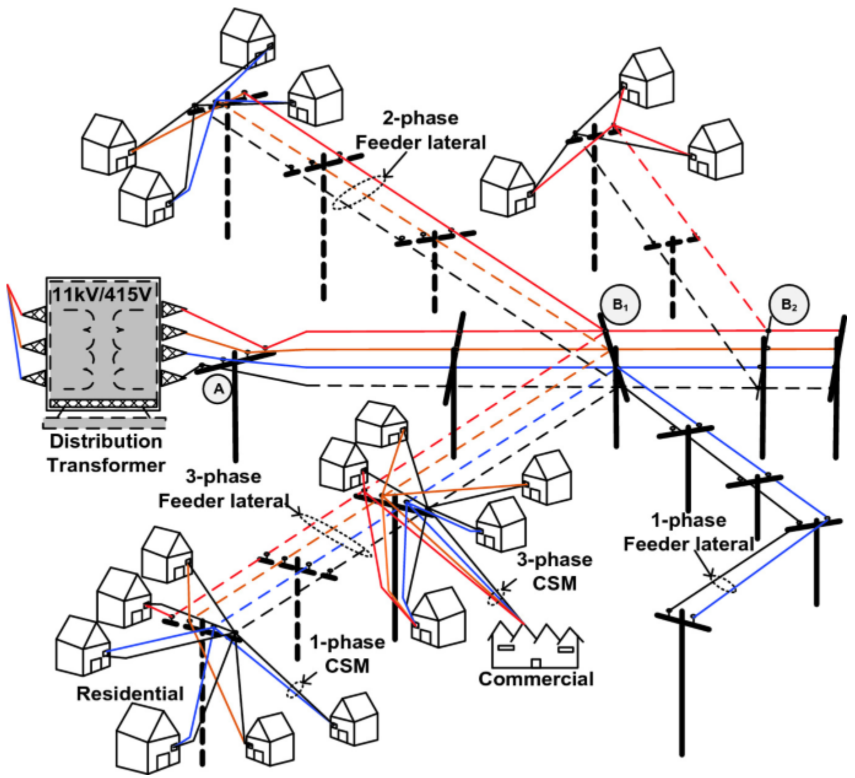
- The performances of the CDC and DDC are poor in the dynamic scenario.
- The ASALVC still exhibits the best performance compared to the GPDC and SGPDC in the dynamic scenario.
- The ASALVC is more capable of maintaining a flat network voltage profile in the time-varying environment.

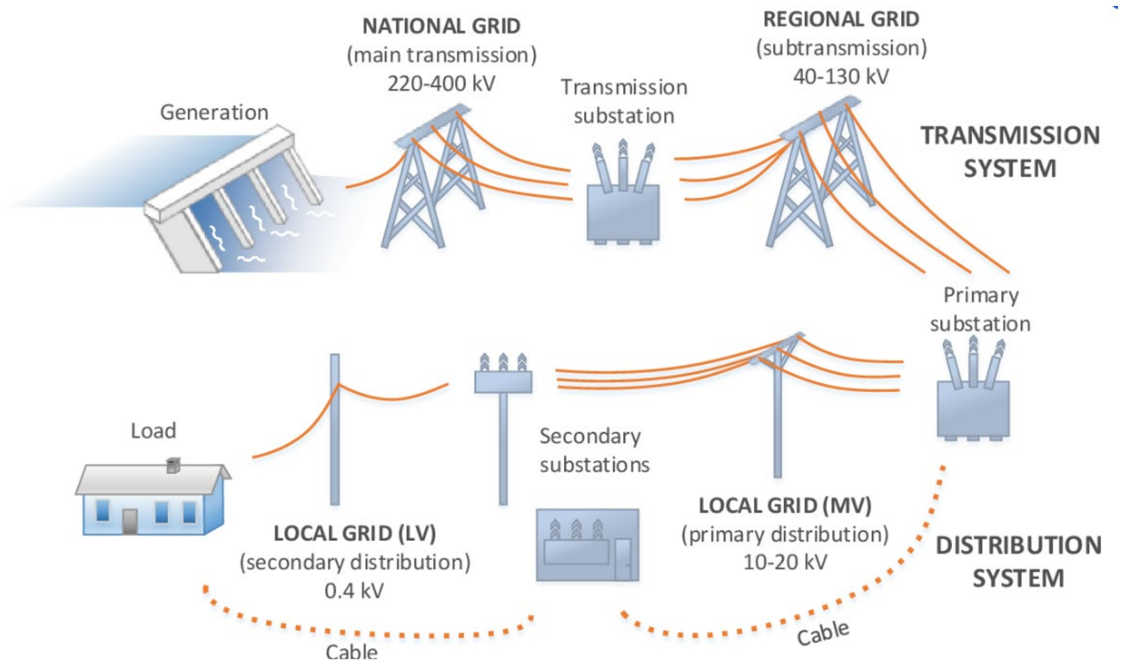
5. Conclusion

- We propose an ASALVC strategy, where each bus agent locally adjusts the VAr output of its DER based on its time-varying voltage droop function.
- This function is associated with the bus-specific time-varying slope and intercept, which can be dynamically updated merely based on the local voltage measurement.
- Stability, convergence and optimality properties of the ASALVC are analytically established.
- The ASALVC exhibits a great performance for both static and dynamic scenarios. It shows a strong capability to quickly recover from a sudden disturbance and a great tracking capability for continuous fast system changes.

5. Future Work

- Most customer-owned DERs are distributed across the secondary distribution network.
- Consider the secondary distribution network modeling, power flow, and its associated convexification.





Thank You!
Q & A