Resilient Distribution System

PI: Zhaoyu Wang, GRA: Shanshan Ma
Department of Electrical and Computer Engineering
Iowa State University

Outline

- Motivation and Introduction
- Research Activities
  - Subtopic 1: Optimal line hardening strategy for distribution systems
  - Subtopic 2: Resilience-oriented design of distribution systems
- Research Publications
- Reference

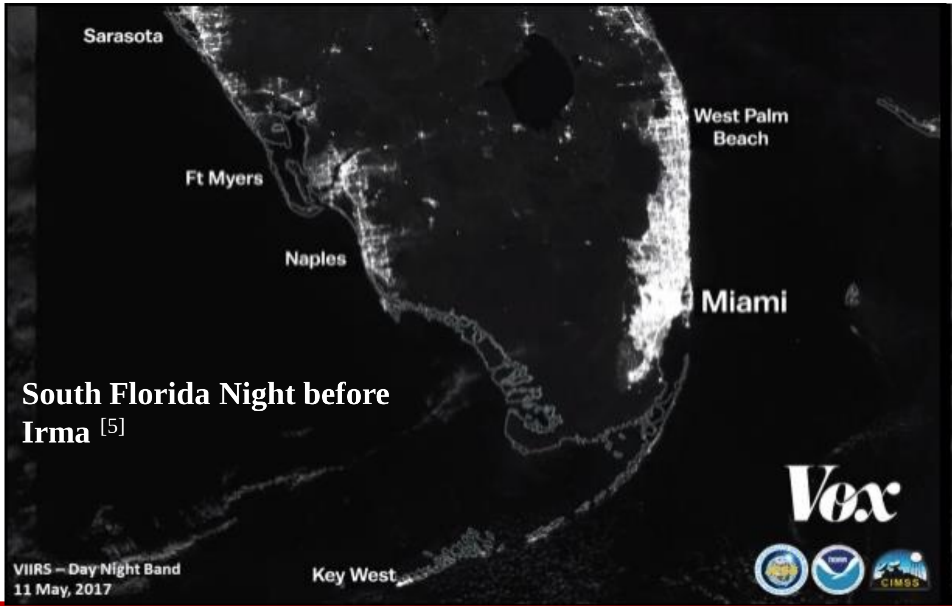
Motivation: Impacts of Extreme Weather Events

Extreme weather event | Distribution Grids

- Climate Change ➡ { The probability of occurrence ↑ ; The intensity ↑ (Hurricane, ice-storm, flood, etc) } ➡ { The failure frequency ↑ ; The power outages ↑ }
- Example: Hurricane Irma in September 2017
  - Left 6.7 million Floridians without power-65% of all customers in Florida [1]
  - Its overall damage cost reached to approximately $50 billion [2]

Power poles pulled down by Irma[3]

Substation flooded by Irma [4]



South Florida Night before Irma [5]

South Florida Night After Irma [5]

Motivation: Current Situation of Distribution Systems

- Most existing distribution systems are designed and maintained for normal weather conditions

- The classic reliability principles cannot guarantee the lights on under extreme weather events

- U.S. power grids are now old and outdated

- Utilities upgrade grids based on experiences, patrols, and observations

As power engineers, how can we improve grid resilience to survive from extreme weather events?

Introduction: The Resilience of Distribution System

- A distribution system is considered to be *resilient* if it is able to anticipate, absorb, adapt to, and/or rapidly recover from a disruptive event [6].

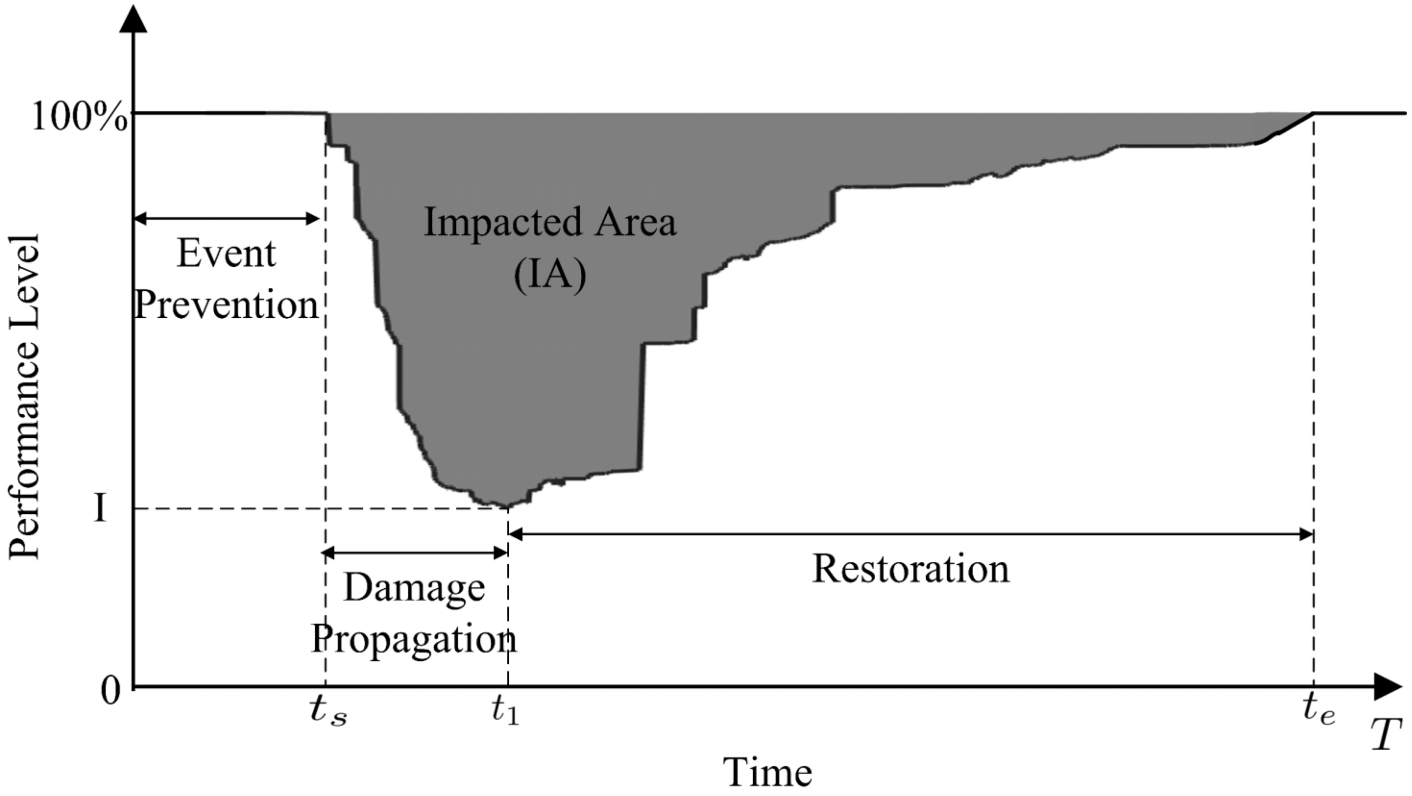


Fig.1. A general system performance curve of a distribution system following an extreme weather event

- Event prevention stage: Resistant capability
- Damage propagation stage: Absorptive and adaptive capacity
- Restoration stage: Recovery capability

Introduction: The Resilience Enhancement Measures

- Two resilience goals of distribution systems [7]:
  - System adaptation (to reduce the impact of future events)
  - System survivability (to maintain an adequate functionality during and after the event)
- Resilience enhancement measures:

Resilience-Oriented Design (ROD) Measures

- Topological and structural upgrades of the utility's infrastructures
  - Upgrading distribution poles to stronger classes
  - Installing automatic switches
  - Installing back-up distributed generators (DGs)

+

Resilience-Oriented Operational (ROO) Measures

- "Smart" control-based actions
  - Network reconfiguration
  - DG rescheduling
  - Defensive islanding
  - Microgrid-assisted control actions
  - Priority-based load shedding

- We focus on exploring *effects of ROD measures* on *system resilience* with the consideration of operation response

Subtopic 1: Optimal line hardening strategy for distribution systems

- Problem Statement
- Literature Review
- Methodology
- Tri-level Robust Optimization Model
- Mathematical Formulation
- Solution Method
- Case Study
- Conclusion

Problem Statement

- The extreme weather events and their negative effects are uncertain.
  - Occurrence and traveling path
  - Damage (line failure)
- There are two major questions on distribution grid hardening[1]:
  - How to prioritize distribution lines for hardening with limited budget
  - What hardening measure should be applied to each line
- We propose an optimal hardening strategy to enhance the resilience of power distribution networks against extreme weather events, considering the time-varying uncertainty of the extreme weather events and the failure probabilities of hardened distribution lines

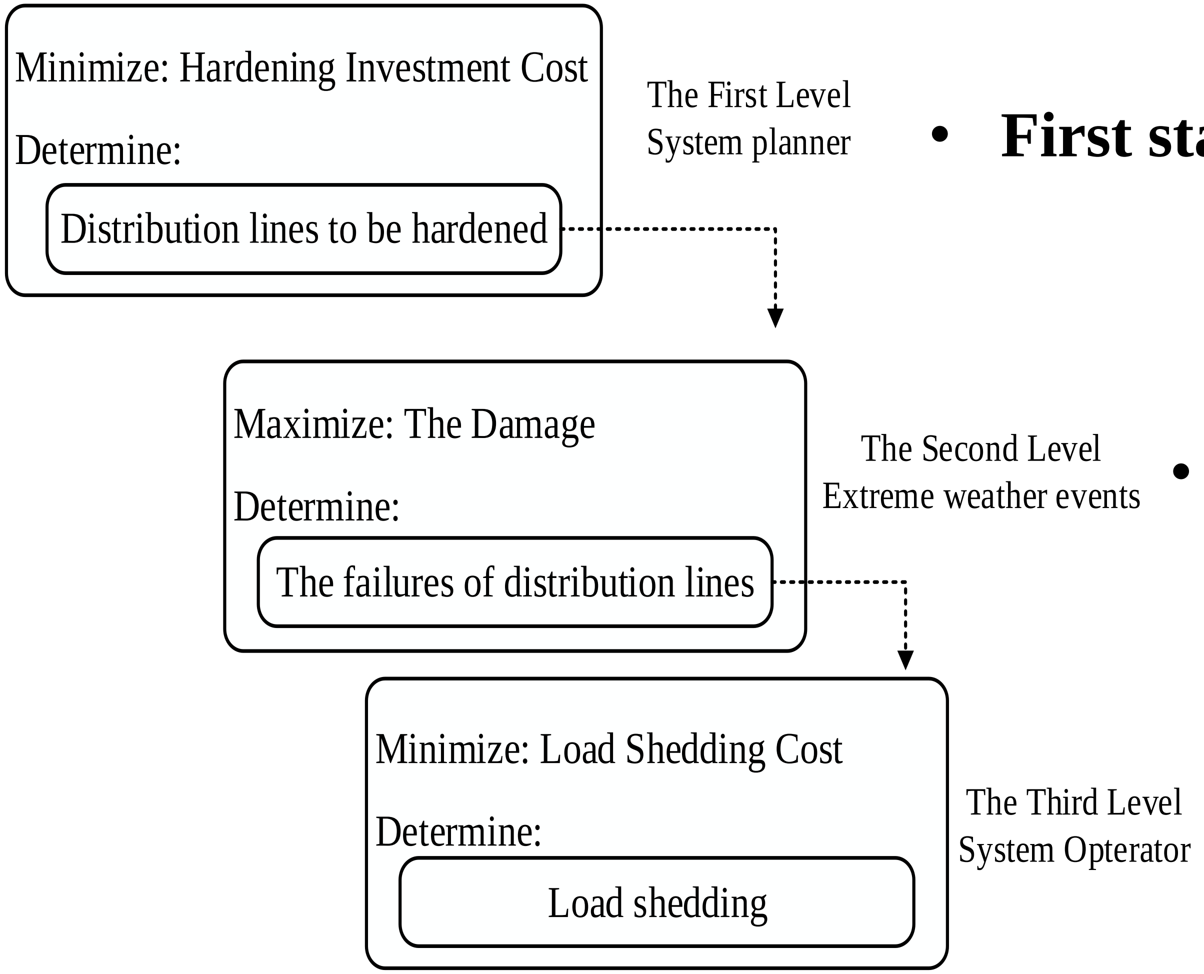
Literature Review

- Salman et al. in [8] proposed targeted hardening strategy to improve distribution system reliability
- Kuntz et al. in [9] proposed a vegetation management scheduling algorithm (optimal time and location)
- Yuan et al. in [10] proposed a robust optimization model
- Utilities: experiences, patrols and observation
- Drawbacks:
  - only consider a single hardening strategy
  - assume the hardened components will have zero failure probability

Methodology

- Use tri-level robust optimization to enhance the resilience of distribution networks against extreme weather events.
  - Three hardening measures
    - Upgrading distribution poles with higher classes
    - Vegetation Management
    - The combination of both
  - Use a polyhedral set to represent damage uncertainty considering the failure probabilities of hardened lines

Tri-level Robust Optimization Model



- **First stage (Defender):** identify hardening strategies
- **Second stage (Attacker):** find the worst weather scenario
- **Third stage (Defender):** minimize load shedding

Mathematical Formulation

First Stage Hardening Cost | Second Stage Line Damage | Third Stage Load Shedding Cost

$$\min_{x \in \chi} \left\{ \mathcal{C}^I(x) + \max_{u \in \mathcal{U}(x)} \min_{o \in \mathcal{O}(u)} \mathcal{C}^S(o) \right\}$$

Objective:
Minimize the hardening investment and the projected load shedding cost under worst weather scenarios

s.t.:

First stage Variable:

x_{ij}^k whether k-th hardening strategy is selected (1) or not at line *(i, j)*

First stage Constraints:

$$\mathcal{C}^I(x) = \sum_{(i,j)\in\Omega_B} \sum_{k\in\Omega_x} c_{ij}^k x_{ij}^k \leqslant B_L$$

Hardening investment budget constraint

$$\chi = \left\{ x \,\middle|\, \sum_{k\in\Omega_x} x_{ij}^k = 1, \forall(i,j) \in \Omega_B, x_{ij}^k \in \{0,1\} \right\}$$

Hardening measure selection constraint

Mathematical Formulation

Second stage decision variables:

$z_{ij,t}^{k}$: whether line *ij* hardened by the kth hardening strategy is failed (1) or not (0) at time t

$u_{ij,t}$: line status at time t: damaged (1) or not (0)

Second stage constraints:

$$\mathcal{U} = \left\{ u \middle| \sum_{(i,j)\in\Omega_B} (-\log_2 p_{ij,t}^{k}) z_{ij,t}^{k} \leqslant \mathcal{W}, \quad \forall k \in \Omega_x, t \in \mathcal{T} \right.$$

$$z_{ij,t}^{k} \leqslant x_{ij}^{k}, \forall k \in \Omega_x, (i,j) \in \Omega_B, t \in \mathcal{T}$$

$$\sum_{t\in\mathcal{T}} z_{ij,t}^{k} \leqslant 1, \forall k \in \Omega_x, (i,j) \in \Omega_B$$

$$u_{ij,t} \leqslant \sum_{st=t-T_R}^{t} z_{ij,st}^{k}, \forall k \in \Omega_x, (i,j) \in \Omega_B, t \in \mathcal{T}$$

Uncertainty budget constraint:

- A line with lower failure probability takes up more uncertainty budget if it fails
- If the failure probability of a line is zero, then it takes up infinitely large uncertainty budget.

Hardening strategy constraint:

- The failure of a line being hardened by a specific strategy can only occur if that strategy is selected in the first stage.

Line failure time limit:

- Line failure only occurs once during the extreme weather event

Repair time constraint:

- If the line *ij* starts to be out of service at time *st*, it remains failed until being repaired.

Mathematical Formulation

The third stage decision variables:

$|V_{i,t}| \quad P_{i,t}^{g} \quad Q_{i,t}^{g} \quad P_{ij,t} \quad Q_{ij,t} \quad \rho_{i,t}$

Operation Constraints

The third stage constraints:

$\mathcal{C}^S(o) = \sum_{i,t} c_i^L \rho_{i,t} P_{i,t}^L, \forall i \in \Omega_L, t \in \mathcal{T}$ Load shedding cost

$$
\mathcal{O}(u) = \left\{ o \middle| \begin{array}{ll}
\sum_{\{j|(i,j)\in\Omega_B\}} P_{ij,t} = \sum_{\{j|(i,j)\in\Omega_B\}} P_{ji,t} - P_{i,t}^{g} - (1-\rho_{i,t})P_{i,t}^{L}, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{1}] \\
\sum_{\{j|(i,j)\in\Omega_B\}} Q_{ij,t} = \sum_{\{j|(i,j)\in\Omega_B\}} Q_{ji,t} - Q_{i,t}^{g} - (1-\rho_{i,t})Q_{i,t}^{L}, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{2}] \\
0 \leqslant P_{ij,t} \leqslant (1-u_{ij,t})P_{ij}^{\max}, \forall (i,j) \in \Omega_B, t \in \mathcal{T} & [\lambda_{ij,t}^{3}] \\
0 \leqslant Q_{ij,t} \leqslant (1-u_{ij,t})Q_{ij}^{\max}, \forall (i,j) \in \Omega_B, t \in \mathcal{T} & [\lambda_{ij,t}^{4}] \\
|V_{j,t}| \leqslant |V_{i,t}| - \frac{R_{ij}P_{ij,t} + X_{ij}Q_{ij,t}}{V_0} + u_{ij,t}M_1, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{5}] \\
|V_{i,t}| - \frac{R_{ij}P_{ij,t} + X_{ij}Q_{ij,t}}{V_0} - u_{ij,t}M_1 \leqslant |V_{j,t}|, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{6}] \\
0 \leqslant P_{i,t}^{g} \leqslant P_{i,t}^{g,\max}, \forall i \in \Omega_G, t \in \mathcal{T} & [\lambda_{i,t}^{7}] \\
0 \leqslant Q_{i,t}^{g} \leqslant Q_{i,t}^{g,\max}, \forall i \in \Omega_G, t \in \mathcal{T} & [\lambda_{i,t}^{8}] \\
|V_i|^{\min} \leqslant |V_{i,t}| \leqslant |V_i|^{\max}, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{9}, \lambda_{i,t}^{10}] \\
0 \leqslant \rho_{i,t} \leqslant 1, \forall i \in \Omega_N, t \in \mathcal{T} & [\lambda_{i,t}^{11}]
\end{array} \right\}
$$

Power balance constraint

Line flow limits

Power flow constraints

DG output limits

Voltage limit

Load shedding ratio limit

Solution Method-Problem Reformulation

- Problem Formulation

Tri-level Formulation

$$\min_{x\in\chi}\left\{\mathcal{C}^I(x)+\max_{u\in\mathcal{U}(x)}\min_{o\in\mathcal{O}(u)}\mathcal{C}^S(o)\right\}$$

$$s.t.: \quad \mathcal{C}^I(x)\leqslant B_L$$

Reformulation $\Longrightarrow$

Bi-level Formulation

Upper-level problem $\mathcal{H}(x)$: Select optimal hardening measures for the most critical line

$$\min \mathcal{C}^I(x)$$
$$s.t. \quad x\in\chi$$
$$\mathcal{C}^I(x)\leqslant B_L$$
$$\sum_{(i,j)\in\Omega_B}(-\log_2 p^k_{ij,t})z^k_{ij,t}\geqslant\mathcal{W},\forall k\in\Omega_x,t\in\mathcal{T}$$

Lower-level problem $\mathcal{R}(x)$: Identify critical lines

$$\max_{u\in\mathcal{U}(x)}\min_{o\in\mathcal{O}(u)}\mathcal{C}^S(o)$$

- Max-min Problem $\mathcal{R}(x)$ Reformulation

$$\min_{o\in\mathcal{O}(u)}\mathcal{C}^S(o)\xRightarrow{\text{Dual}}\max\mathcal{D}^S(\lambda)\xRightarrow{\text{KKT}}\overset{\text{LPCC}}{\max_{u\in\mathcal{U}}\mathcal{D}^S(\lambda)}\xRightarrow{\text{Big-M}}\text{MIP: }\mathcal{R}(x)$$

$$s.t.\quad \text{KKT optimilaty constraints}$$

Solution Method-Greedy Searching Algorithm

- The selection of hardening strategies is coupled with the uncertainty set of out-of-service lines

Step 0: Initialization. Set the worst extreme weather condition parameters and $s = 0$. Calculate each line's failure probability without hardening $p^0_{ij,t}$.

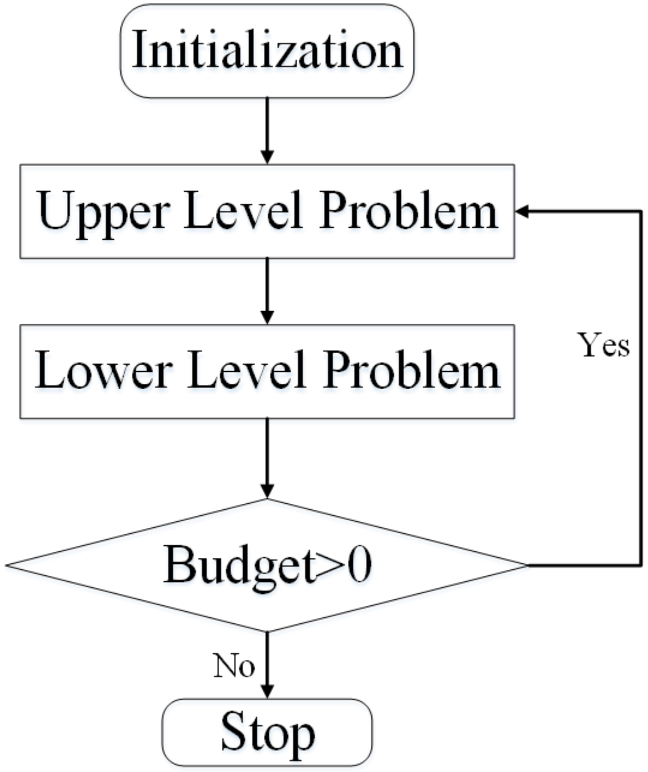
Step 1: Solve $\mathcal{R}(x^0)$ without hardening and let (ρ^0, u^0, z^0) denote its optimal solution.

Step 2: Obtain the initial critical line set Γ^0 whose failures have severe impacts on load shedding according **Step 1**'s solution.

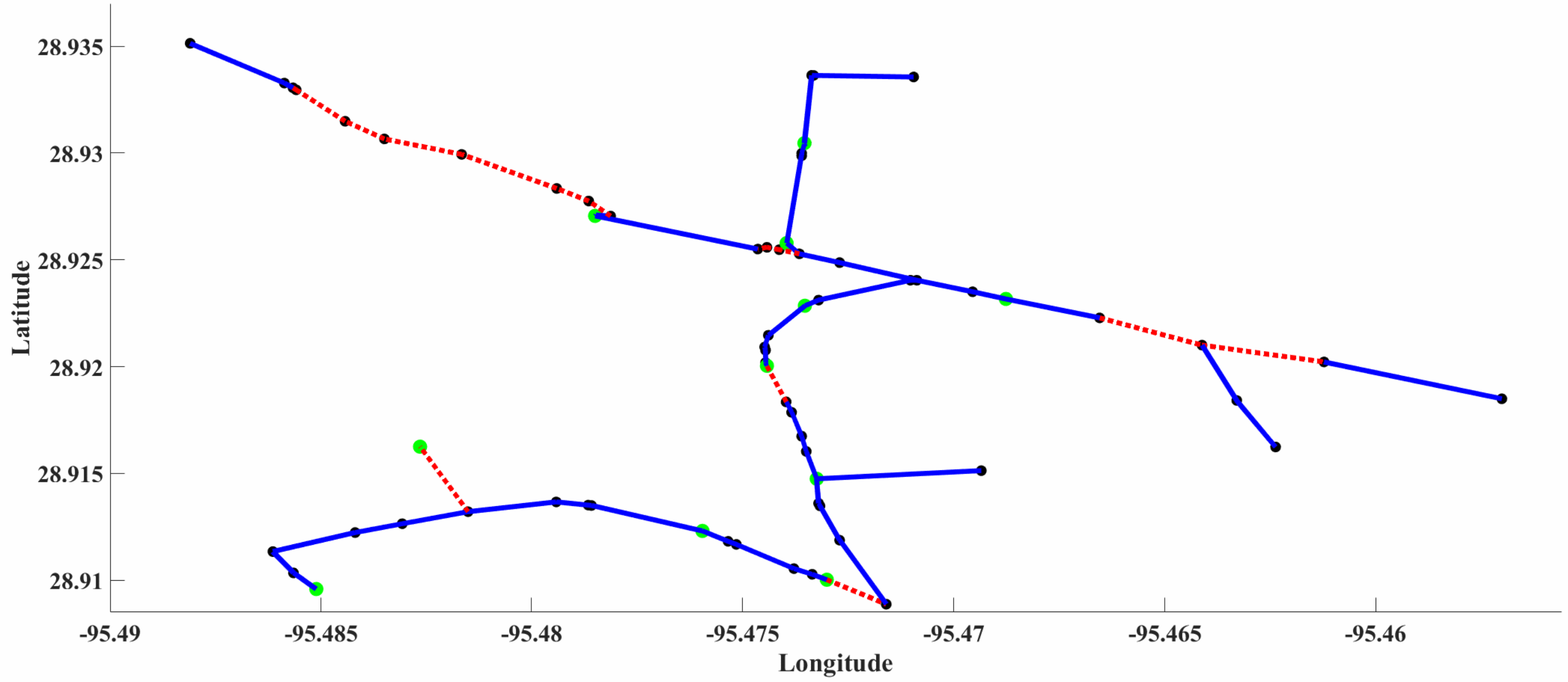
Step 3: Update $s \leftarrow s + 1$. Calculate $p^k_{ij,t}, \forall k \in \Omega_x, (i,j) \in \Gamma^s$. Solve $\mathcal{H}(x^s)$, and select the most critical line from Γ^s to be hardened. Use the hardening strategy with the minimum cost to harden that line.

Step 4: Solve $\mathcal{R}(x^s)$ and let (ρ^s, u^s, z^s) denote the optimal solution. Update critical lines in Γ^s .

Step 5: If the investment budget reaches the limit, the algorithm ends; otherwise go to **Step 3**.

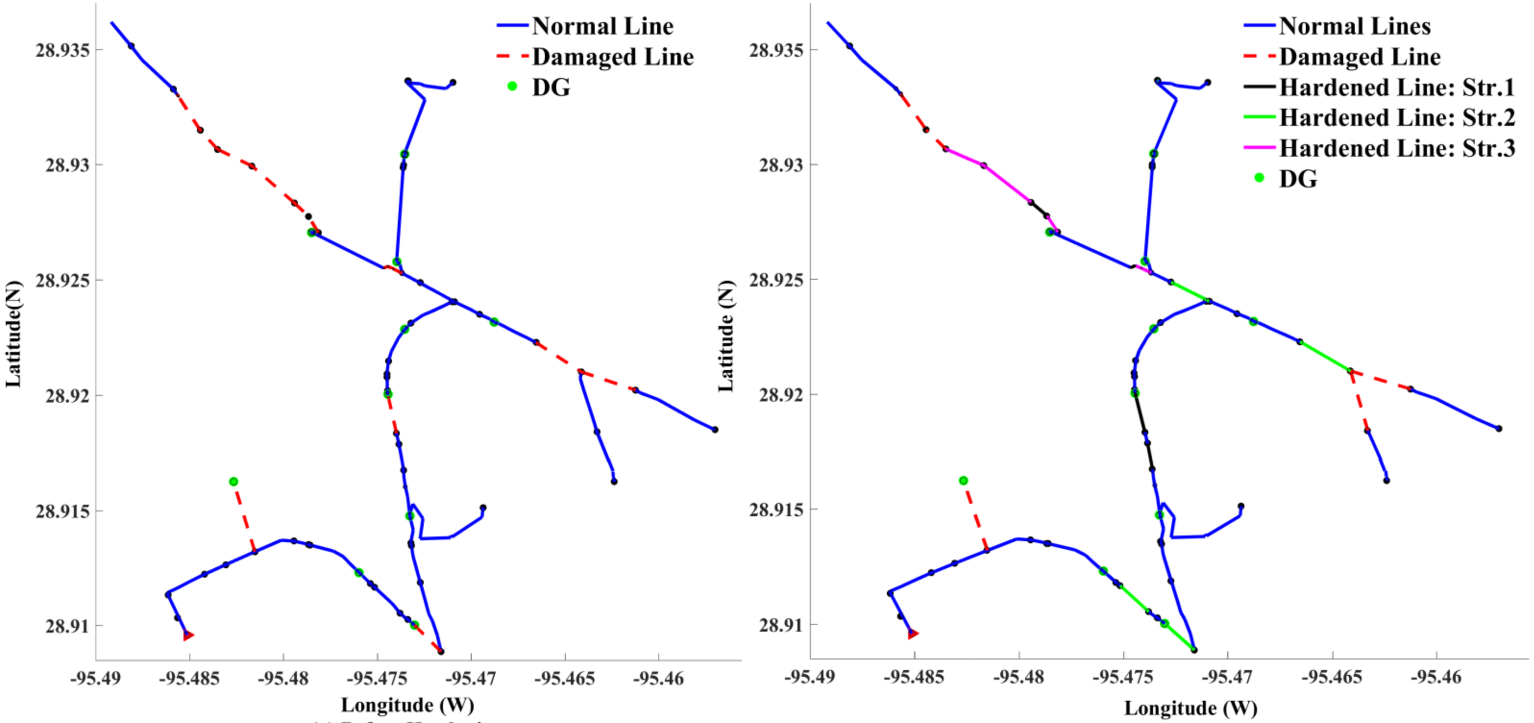


Case Study: A Modified Electric Power Research Institute (EPRI) Test Circuit



- This system has a 74-mile primary circuit that supplies 3885 customers.
- There are 68 lines and 69 nodes in the primary network.
- The total load demand at peak is 30.43MW
- The total load shedding cost is $51, 832, 148.26 before hardening

Case Study 1: With/Without Hardening



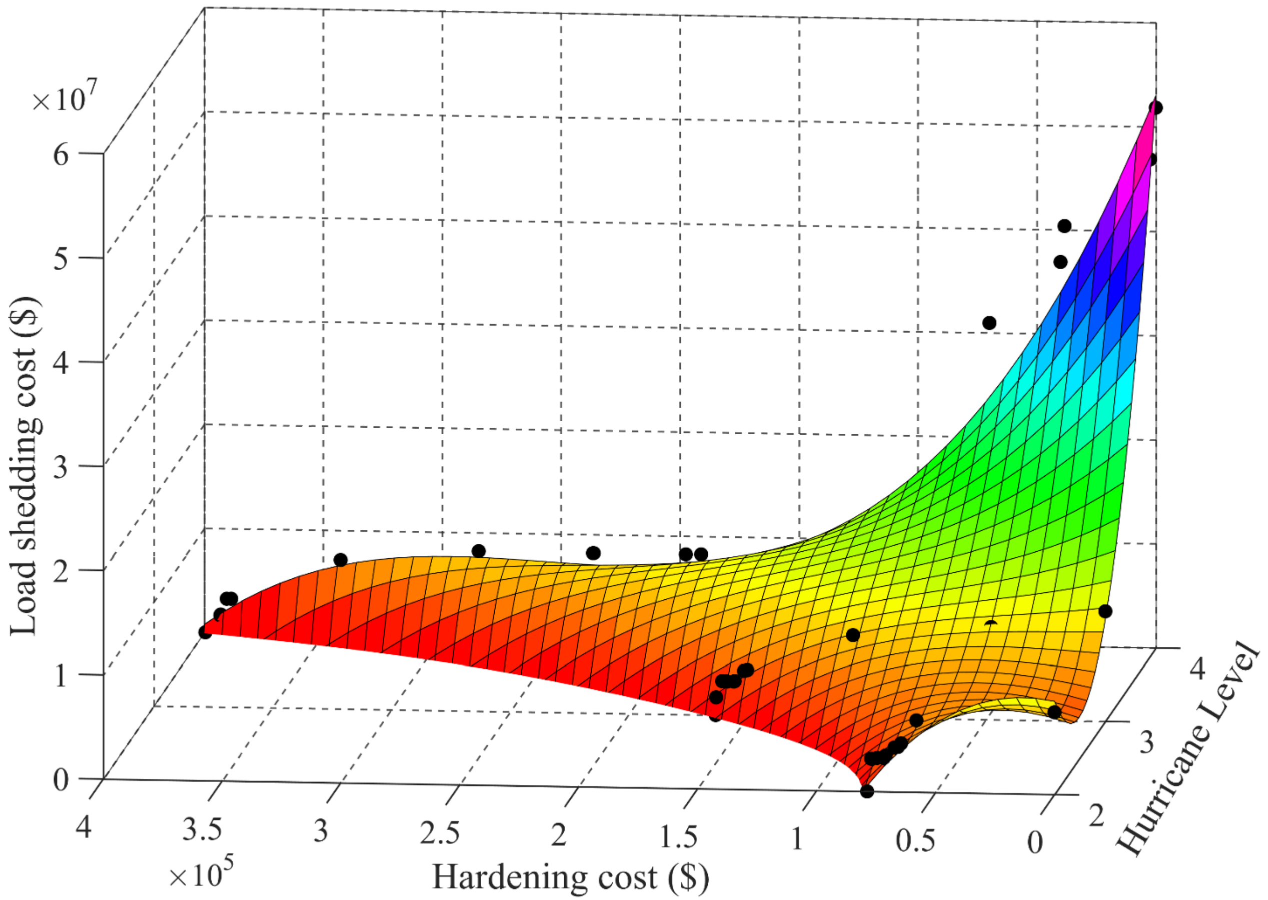
(a) Before Hardening

(b) After Hardening

OPTIMAL HARDENING PLANS FOR CATEGORY-4 HURRICANE

No	Hardened Line	Strategy	Hardening Cost ($)	Load Shedding Cost ($)	Total Failed Lines
1	L24-25	1	2,437.13	46,872,116.24	16
2	L33-38	2	3,5954.20	40,367,134.84	15
3	L22-23	1	1,589.79	36,937,089.47	15
4	L15-16	2	29,961.83	30,945,260.46	14
5	L12-13	2	29,961.83	13,819,127.47	15
6	L46-66	3	5,992.37	10,435,943.94	15
7	L39-47	3	12,695.40	9,709,752.35	14
8	L52-53	3	19,389.63	9,341,072.20	13
9	L53-54	1	1337.81	9,267,009.32	12
10	L54-55	3	51,787.92	8,264,433.90	11
11	L47-48	3	6,437.88	8,241,964.90	10
12	L55-56	3	38,714.86	8,233,018.56	9
13	L48-49	1	334.43	8,233,018.56	8

Case Study 2: Sensitivity Analysis



- For all the worst-case hurricanes, the load shedding costs are proportionally decreasing with respect to the increasing of hardening budgets.
- A more severe hurricane results in higher load shedding costs and requires larger hardening investments.

Conclusion

- A new approach is proposed for hardening distribution systems to protect against extreme weather events.
- The problem is formulated as a tri-level mixed-integer linear program, and then reformulated as a bi-level model.
- The proposed model is tested on a modified EPRI test circuit.
- Numerical results show that the proposed model can assist utilities to identify optimal hardening strategies to mitigate systems’ vulnerability to extreme weather.

Subtopic 2: Resilience-Oriented Design of Distribution Systems

- Problem Statement
- Literature Review
- Research Objective
- Stochastic Decision Process of ROD Problem
- Mathematical Formulation of ROD Problem
- Solution Algorithm
- Case Study
- Conclusion

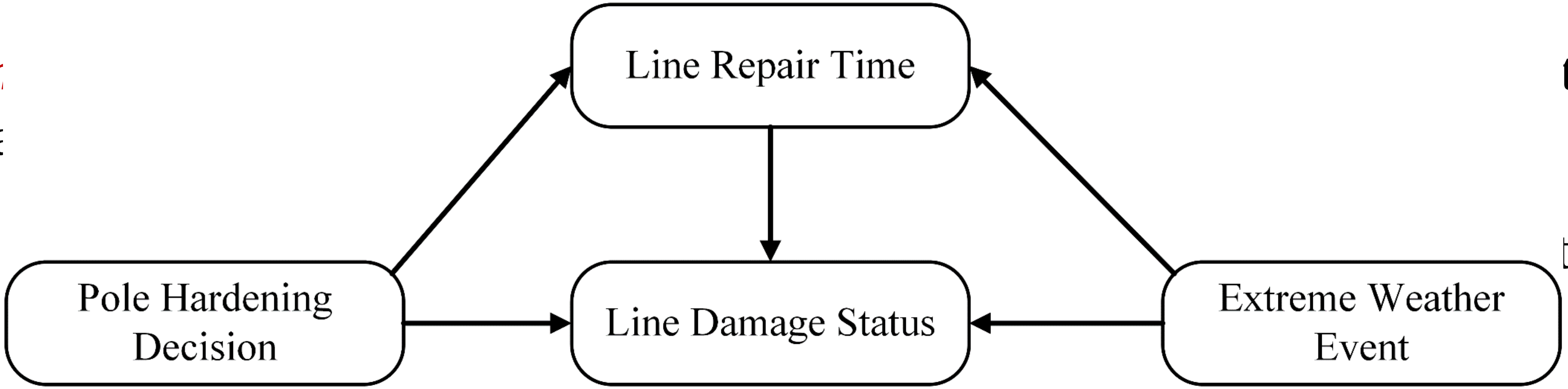
Problem Statement

- How to optimally apply ROD measures to prevent distribution system from extensive damages caused by extreme weather events

  - Some *spatial-temporal correlations* exist among ROD decisions, extreme weather events, and system operations
    - Occurrence, intensity and traveling paths of events are *uncertain*
    - Physical infrastructure damage statuses are affected by both extreme weather event and ROD decisions (decision dependent uncertainty)
    - ROD decisions affect system recovery and the associated outage/repair costs

  - A *tim* tage propa

    - tion



Literature Review

Ref	Uncertainty Consideration	Measures	Model/Algorithm
[10]	• Use a polyhedral set to represent damage uncertainty	• line hardening	• Robust optimization/column-and-constraint generation algorithm
[11]	• Use failure probabilities of distribution lines to represent damage uncertainty set	• Pole hardening • Vegetation management • Combination of both	• Tri-level robust optimization/greedy algorithm
[12]	• Use failure probabilities of overhead lines and underground gas pipelines to generate line damage uncertainty set	• Line hardening	• Tri-level robust optimization/column-and-constraint generation algorithm
[13]	• Use fragility model to generate line damage uncertainty	• Line hardening • DG placement • Switch Installation	• Two-stage stochastic program/a scenario-based variable neighborhood decomposition search algorithm
[14]	• Use fragility model to generate line damage uncertainty	• Line hardening (replace overhead line with underground line) • MGs • Networked MGs	• Two-stage stochastic program/a decomposition-based heuristic algorithm
[15]	• Use fragility model to generate line damage uncertainty • Model repair time uncertainty • Consider load demand uncertainty	• Line hardening • DG placement • Switch Installation	• Two-stage stochastic program/Progressive hedging algorithm

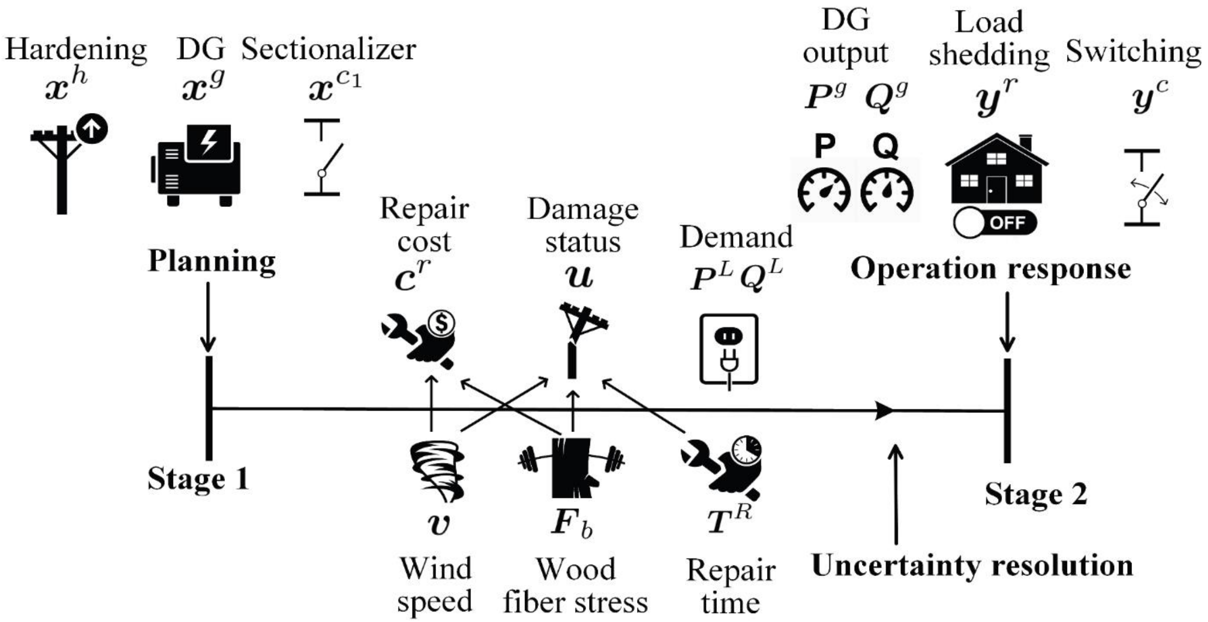
Research Objective

- Develop a new modeling and solution methodology for the ROD of distribution systems against extreme weather events
  - Develop a stochastic decision process to describe the spatio-temporal correlations of ROD decisions and uncertainties
  - Formulate a two-stage stochastic mixed-integer linear program (SMILP) to capture the impacts of ROD decisions and uncertainties on system's responses to extreme weather events
  - Design solution algorithm for solving the above problems

Stochastic Decision Process of ROD Problem

- Overview
- ROD Measures
- Uncertainty Modeling

Overview



- ROD problem is modeled as a two-stage stochastic decision process:
  - Planner makes ROD decisions
  - The operation uncertainties are resolved during the extreme weather event
  - Operator makes the operation decisions

ROD Measures

- Hardening poles:
  - Strengthening vulnerable components
  - 6 pole types
  - Pole stress (**1** > **2** > **3** > **4** > **5** > **6**)

- Installing backup DGs
  - Increasing adequacy of power supply

- Adding sectionalizers
  - Increasing topological flexibility

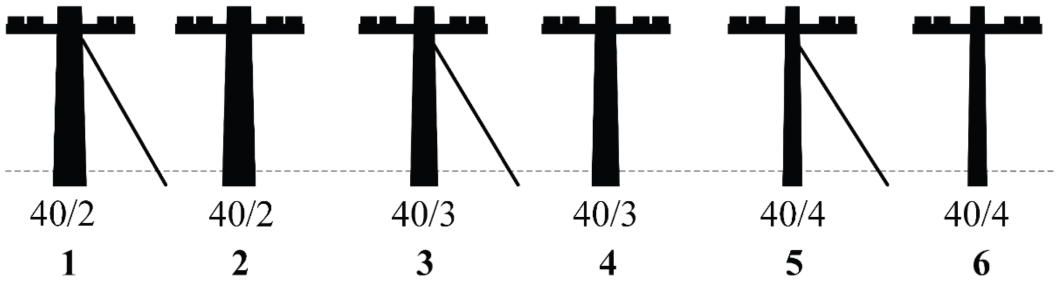


Fig.1. Pole types

Uncertainty Modeling

- Consider three groups of random variables that have direct impacts on the evolution of the system operation state
  - Line damage status
  - Repair cost
  - Load demand

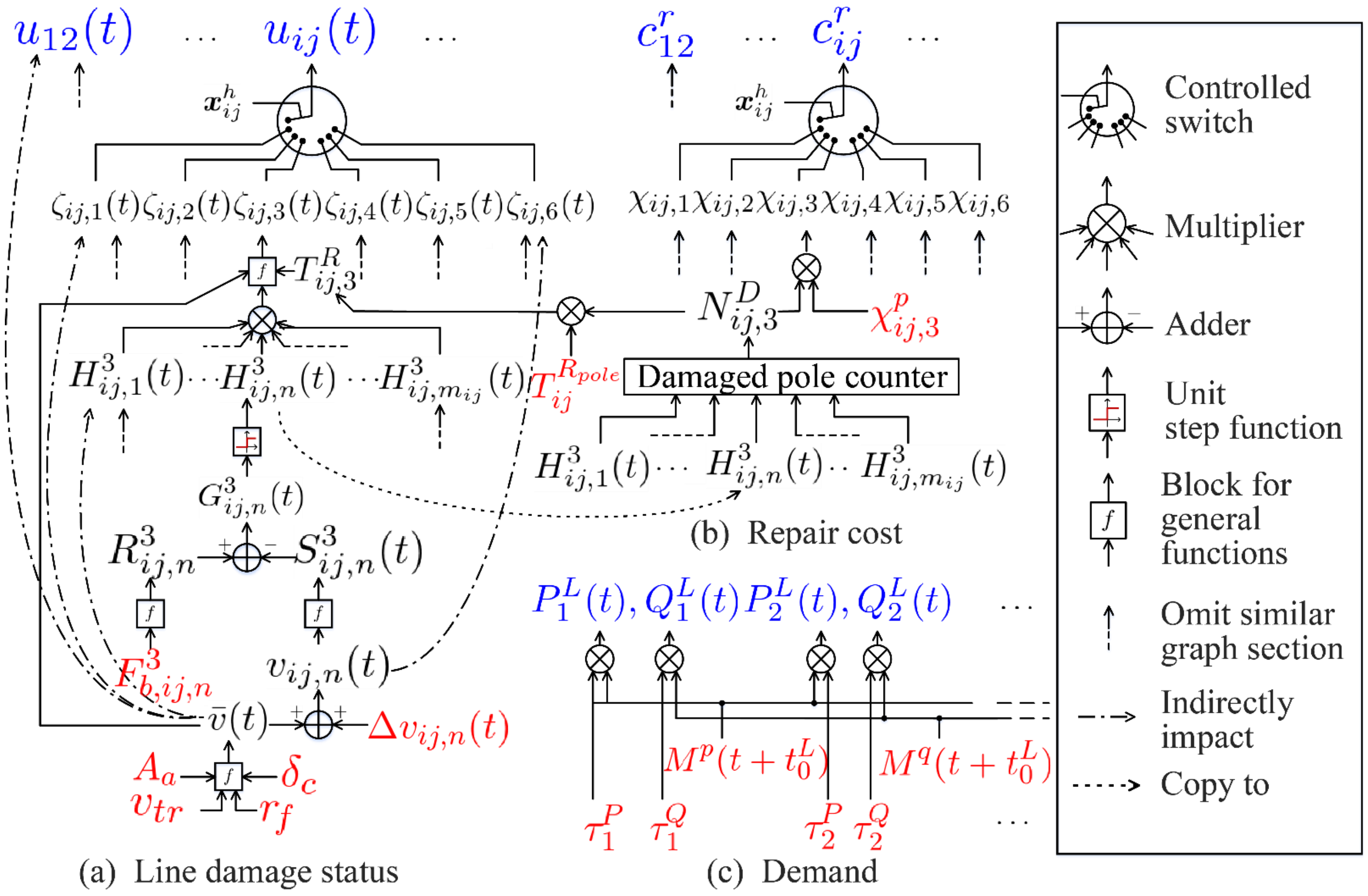
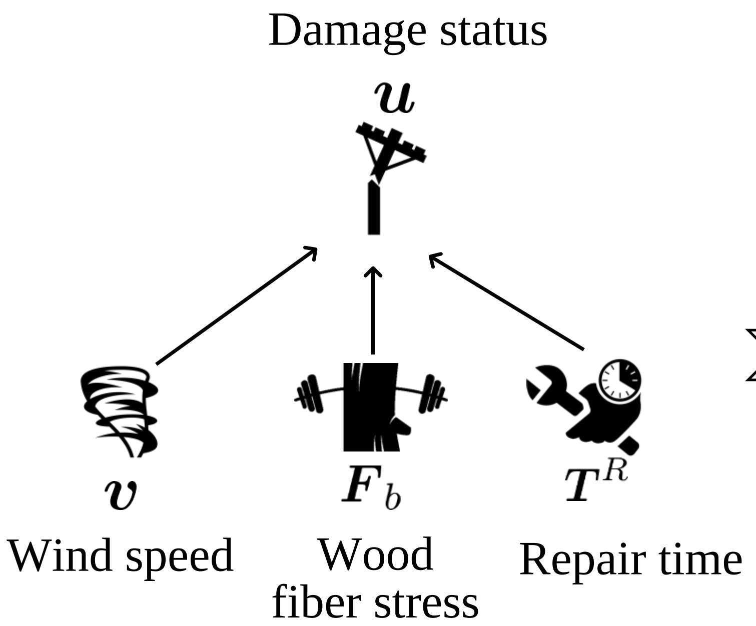
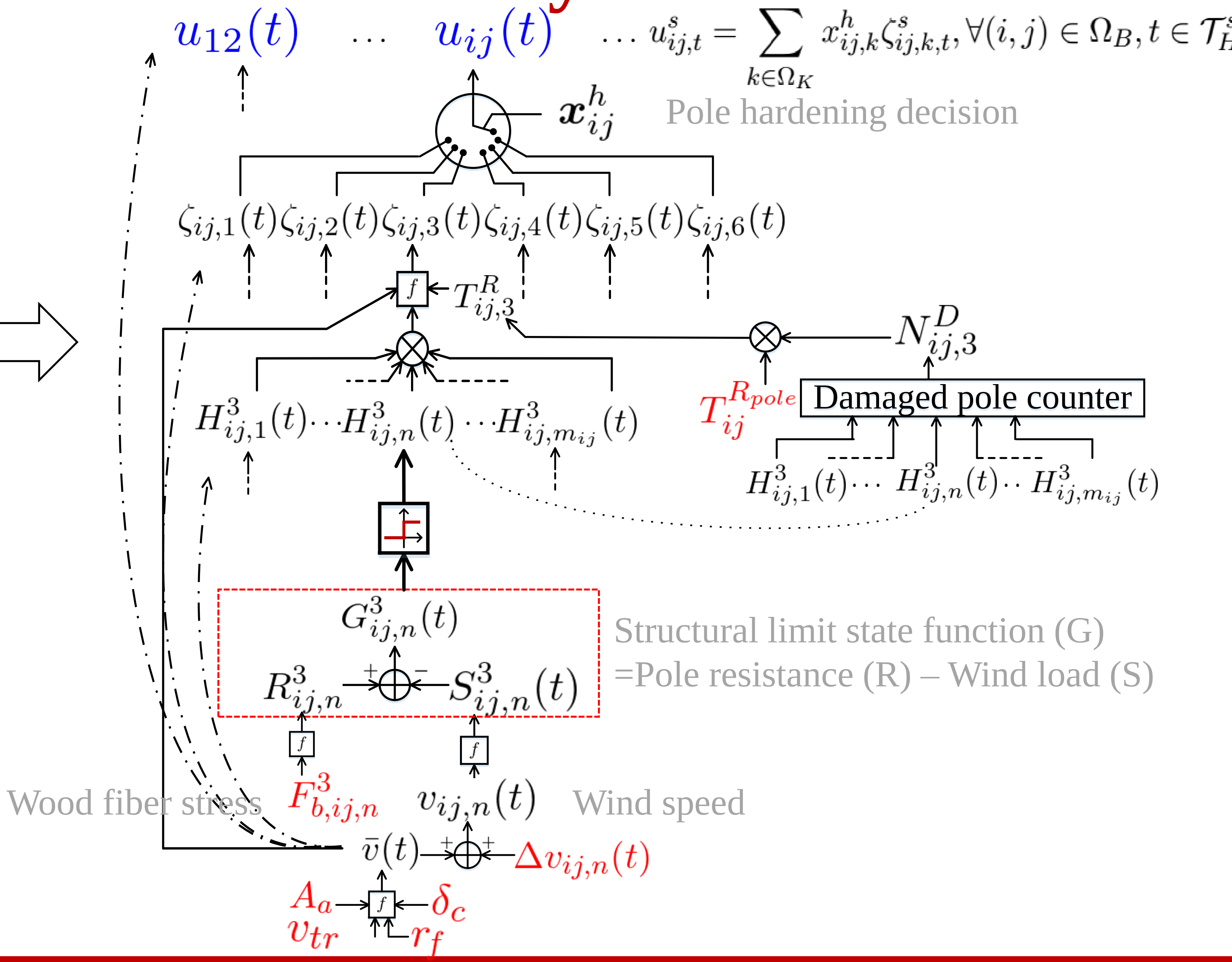


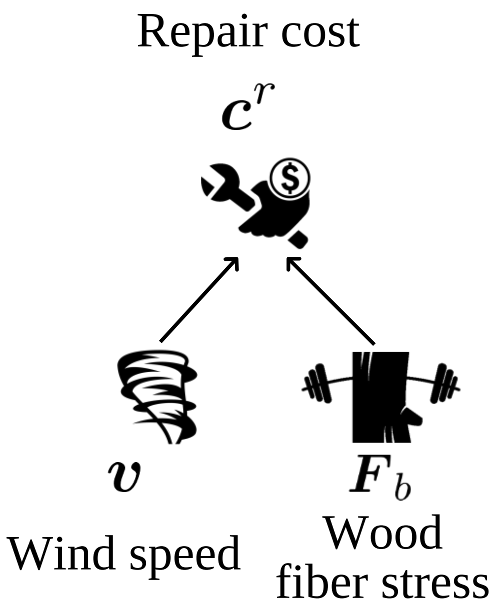
Fig.1. The structure of uncertainty space: independent observable random variables/processes (highlighted in red) + deterministic casual connections (parameterized by the first-stage decision).

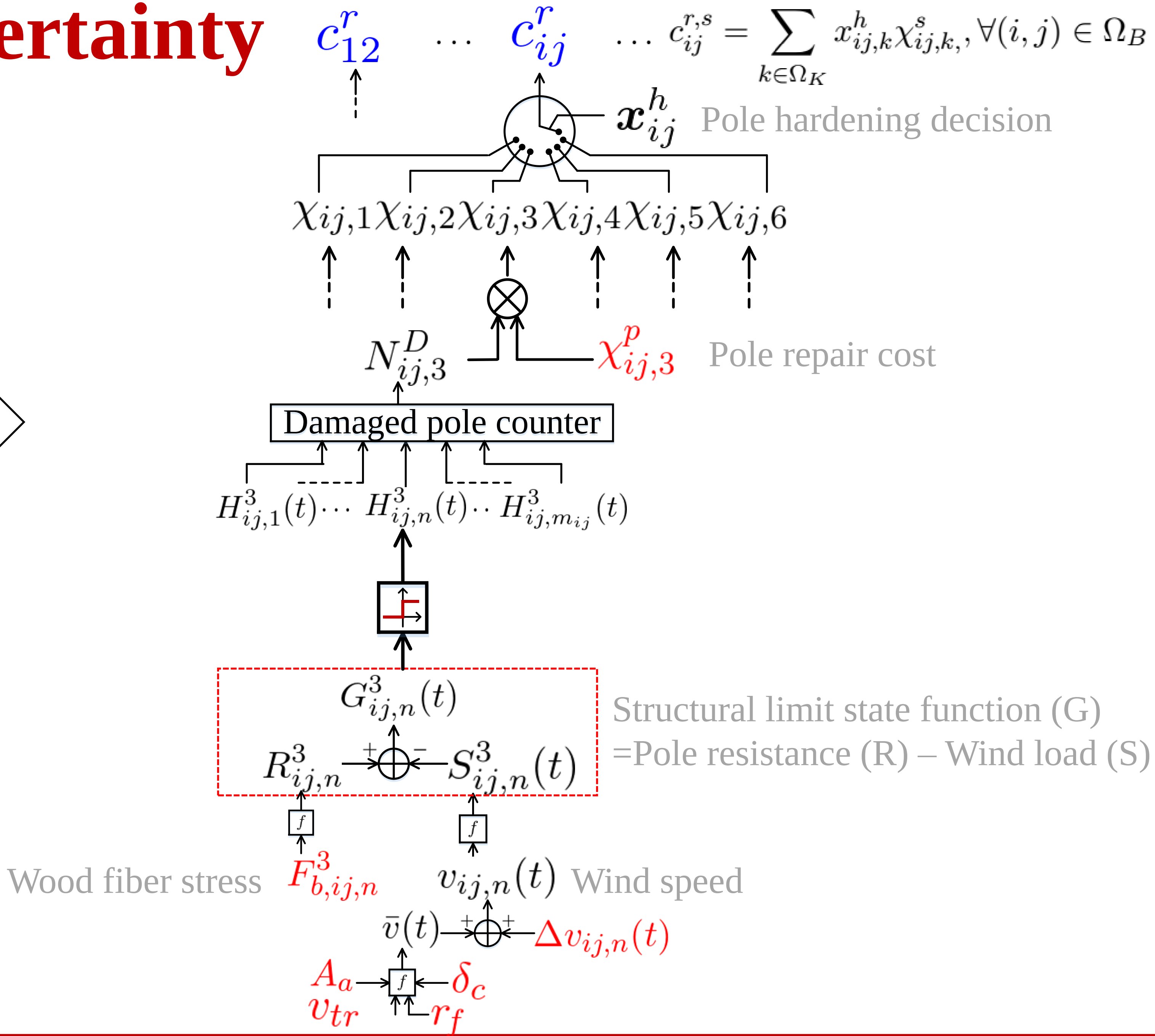
(a) Line Damage Status Uncertainty





(b) Repair Cost Uncertainty





(c) Load Demand Uncertainty

$$P_i^L(t) = \tau_i^P \cdot \boxed{M^p(t+t_0^L)}, \forall i \in \Omega_L, t \in \mathcal{T}_H$$

$$Q_i^L(t) = \tau_i^Q \cdot \boxed{M^q(t+t_0^L)}, \forall i \in \Omega_L, t \in \mathcal{T}_H.$$

$$\tau_i^P \sim N(\overline{P}_i, (0.02\overline{P}_i)^2), \forall i \in \Omega_L$$

$$\tau_i^Q \sim N(\overline{Q}_i, (0.02\overline{Q}_i)^2), \forall i \in \Omega_L$$

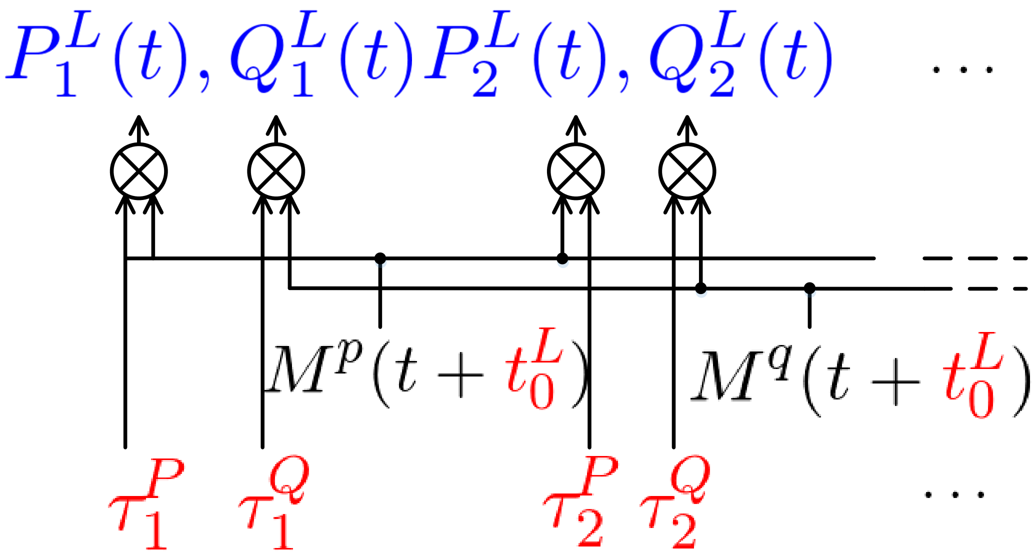


Fig.1. load demand uncertainty

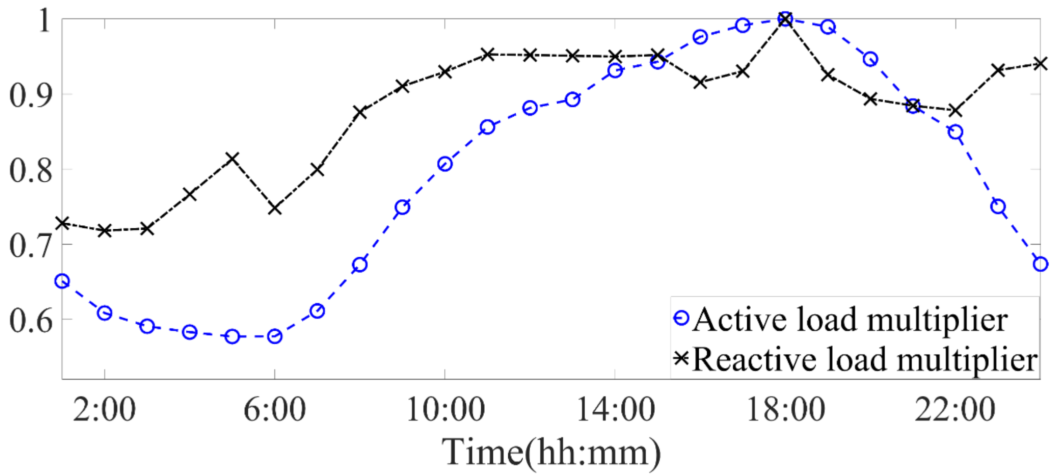
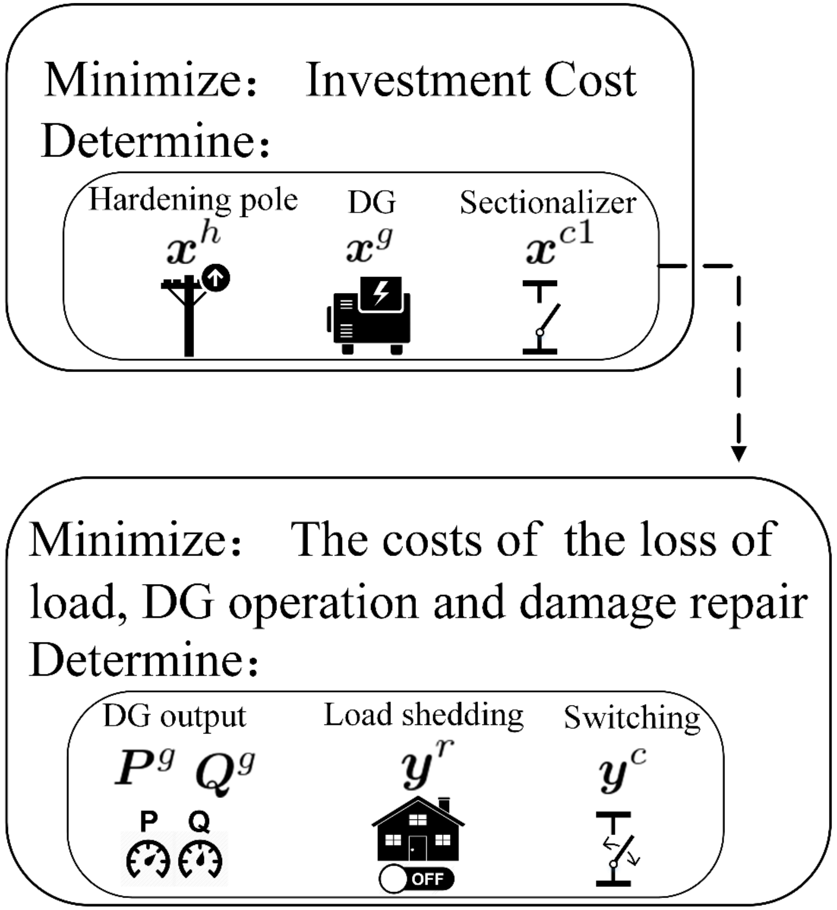


Fig.2. load profile shape at the substation (root node)

Mathematatic Formulation of ROD Problem

- Overview
- First-stage Problem
- Second-stage Problem

Overview



- Investment Stage: identify the optimal ROD decisions
- Operation Stage: achieve self-healing operation
  - need a mathematical formulation to model the full power outage propagation process
  - need an analytical optimization to sectionalize a distribution network into multiple self-supplied MGs while maintaining their radial network typologies

First-Stage Formulation

Objective:
Minimize the ROD investment cost and the expected cost of the second stage in realized extreme weather events

$$\min C_1^I(x^h)+C_1^I(x^g)+C_1^I(x^{c_1})+w_H\mathbb{E}_{\boldsymbol{\xi}}\phi(x,\boldsymbol{\xi})$$

s.t.:

First stage ROD variables:

x_{ij}^h whether hardening line (i,j) (1) or not (0)

x_i^g whether installing DG at node i (1) or not (0)

$x_{ij,i}^{c_1}$ whether adding a sectionalizer at the end i of line (i,j) (1) or not (0)

First stage constraints:

$$\sum_{k\in\Omega_K} x_{ij,k}^h = 1, \forall(i,j)\in\Omega_B$$ Hardening strategy limit

$$\sum_{i\in\Omega_N} x_i^g \leqslant N_G$$ DG number limit

$$x_{ij,n}^{c_0}+x_{ij,n}^{c_1}=x_{ij,n}^{c}, \forall(i,j)\in\Omega_B, n\in\{i,j\}$$ Switch installation constraint

$$\mathbb{E}_{\boldsymbol{\xi}}\phi(x,\boldsymbol{\xi})\cong\sum_{s\in\mathcal{S}}p_r(s)\phi(x,s)$$ The expected cost of the second stage

Second-Stage Problem: Technique Outline (1)

- Model the power outage propagation constraints
  - Sectionalizers or breakers only exist in certain line sections
  - Customers at the nodes that directly connected to the damaged line will be out of service
  - Propagation process:

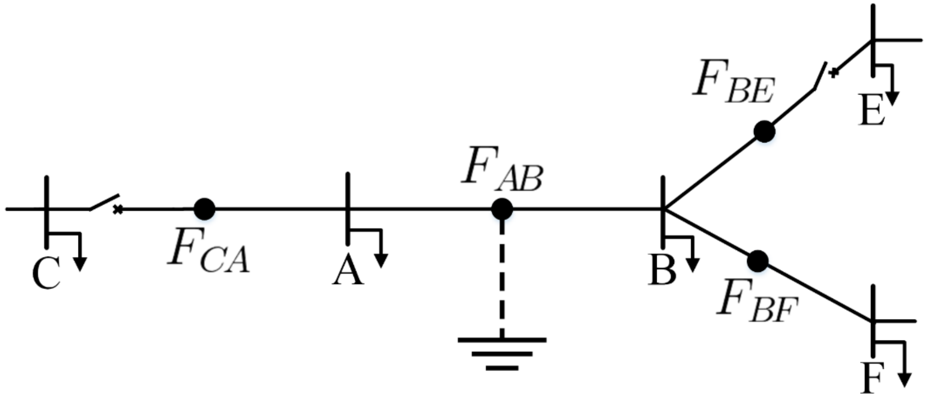


Fig.1. The illustrative example for isolating a fault

- Add a virtual node in the middle of each branch
- Apply a symmetric fault to the virtual node if the line is damaged
- Set the voltage feasible region: $\{0\} \cup [V^{\min}, V^{\max}]$
- Fully curtail a load when its voltage magnitude is zero
- Set loading limits to all branches and penalize load shedding amount in the objective

Second-Stage Problem: Technique Outline (2)

- Model radiality constraints for each energized network
  - To reduce potential operation issues and facilitate system back to normal operation
  - Graph Theorem [16]: A forest of N nodes has exactly $N - N_c$ edges, where N_c is the number of connected network components.
  - How to obtain N_c in the distribution system
    - Calculate N_c by counting the degree of freedom of voltage angles
    - Formulate a virtual DC optimal power flow (VDCOPF) sub-problem to obtain this degree of freedom
      - The optimal solution of this sub-problem satisfies that the virtual loads in the same energized island are nearly equally distributed at active nodes
      - Each energized island has and only has an active node with zero angle
  - The radiality constraint is satisfied *iff* the number of active branches equals the total number of active nodes minus the number of active nodes with zero angles

Second-Stage Formulation

Objective

- Minimize the cost of the loss of load, DG operation, and damage repair in a realized extreme weather event given ROD decisions

$$\phi(\boldsymbol{x}, s) = \min \sum_{i \in \Omega_N} \sum_{t \in \mathcal{T}_H^s} c_i^L y_{i,t}^{r,s} P_{i,t}^{L,s} \Delta t + \sum_{i \in \Omega_N} \sum_{t \in \mathcal{T}_H^s} c_i^o P_{i,t}^{g,s} \Delta t + \sum_{(i,j) \in \Omega_B} c_{ij}^{r,s}$$

Constraints

- Distribution system operation constraints
  1) Line damage status constraint
  2) Line repair cost constraint
  3) Line's on-off status constraints (controlled by switch's on-off status)
  4) Line flow limits (controlled by line's on-off status)
  5) Linearized AC power flow equations (Dist-Flow)
  6) DG capacity limits
- Fictitious faulting logic constraints (model outage propagation)
  1) Virtual node power injection constraints
  2) Voltage magnitude limits
  3) Load shedding ratio limit
- Radiality constraints
- Zero Angle indicator constraint (indicating a node with zero angle)
- The minimality condition of VDCOPF sub-problem (obtain the degree of freedom of voltage angle)

• Key Points:

- Fictitious faulting logic constraints +Distribution system operation constraints in 1)-3) + Penalty cost of load shedding in objective：
  - isolate damaged lines while minimizing the de-energized network parts
  - make network constraints such as power flow automatically adapt to the topology after reconfiguration
- Radiality Constraints + Zero angle indicator constraint + VDCOPF sub-problem
  - can keep each energized network radial
- Information passing:

$$\text{Second-stage problem} \underset{\text{Optimal virtual voltage angle}}{\overset{\text{Line's on-off status and DG on-off status}}{\rightleftarrows}} \text{VDCOPF sub-problem}$$

Second-Stage Constraints

- Distribution system operation constraints

1) Line damage status constraint
2) Line repair cost constraint
3) Line's on-off status constraints
4) Line flow limits
5) Linearized AC power flow (DistFlow) equations
6) DG capacity limits

Binary variables:

$u^s_{ij,t}$ Line damage status

$y^{c,s}_{ij,t}$ Sectionlizer on-off status

$w^{o,s}_{ij,t}$ Line on-off status

1 $$u^s_{ij,t} = \sum_{k \in \Omega_K} x^h_{ij,k} \zeta^s_{ij,k,t}, \forall (i,j) \in \Omega_B, t \in \mathcal{T}^s_H$$

2 $$c^{r,s}_{ij} = \sum_{k \in \Omega_K} x^h_{ij,k} \chi^s_{ij,k}, \forall (i,j) \in \Omega_B$$

3
$$y^{c,s}_{ij,t} \leqslant x^c_{ij}, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}^s_H$$
$$x^c_{ij} + y^{c,s}_{ij,t} + 2w^{o,s}_{ij,t} \geqslant 2, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}^s_H$$
$$w^{o,s}_{ij,t} + y^{c,s}_{ij,t} \leqslant 1, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}^s_H$$
$$y^{c,s}_{ij,t}, w^{o,s}_{ij,t} \in \{0,1\}, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}^s_H$$

4
$$-w^{o,s}_{ij,t} \ldots \in \mathcal{T}^s_H$$
$$-w^{o,s}_{ij,t} \ldots \in \mathcal{T}^s_H$$

x^c_{ij}	$y^{c,s}_{ij,t}$	$w^{o,s}_{ij,t}$	x^c_{ij}	$y^{c,s}_{ij,t}$	$w^{o,s}_{ij,t}$
0	0	1	1	0	1
0	1	N/A	1	1	0

*N/A: the case should be infeasible.

5
$$\sum_{\{j|(i,j)\in\Omega_{B_F}\}} \ldots t \in \mathcal{T}^s_H$$
$$\sum_{\{j|(i,j)\in\Omega_{B_F}\}} Q^s_{ij,t} = Q^{g,s}_{i,t} - (1 - y^{r,s}_{i,t}) Q^L_{i,t}, \forall i \in \Omega_N, t \in \mathcal{T}^s_H$$
$$V^s_{i,t} - \frac{R^e_{ij} P^s_{ij,t} + X^e_{ij} Q^s_{ij,t}}{V_0} - (1 - w^{o,s}_{ij,t}) M_1 \leqslant V^s_{j,t} \leqslant V^s_{i,t} - \frac{R^e_{ij} P^s_{ij,t} + X^e_{ij} Q^s_{ij,t}}{V_0} + (1 - w^{o,s}_{ij,t}) M_1, \forall i \in \Omega_{N_F}, t \in \mathcal{T}^s_H$$

6
$$0 \leqslant P^{g,s}_{i,t} \leqslant x^g_i P^{g,\max}_i, \forall i \in \Omega_N, t \in \mathcal{T}^s_H$$
$$0 \leqslant Q^{g,s}_{i,t} \leqslant x^g_i Q^{g,\max}_i, \forall i \in \Omega_N, t \in \mathcal{T}^s_H$$

Second-Stage Constraints

- Fictitious faulting logic constraints

1) Virtual node power injection constraints

1
$$-u^s_{ij,t}M_2 \leqslant \sum_{k\in\{i,j\}} P^s_{kf_{ij},t} + \varepsilon_1 \cdot V^s_{i,t} \leqslant u^s_{ij,t}M_2, \forall(i,j)\in\Omega_B, f_{ij}\in\Omega_{N_F}, t\in\mathcal{T}^s_H$$
$$-u^s_{ij,t}M_2 \leqslant \sum_{k\in\{i,j\}} Q^s_{kf_{ij},t} \leqslant u^s_{ij,t}M_2, \forall(i,j)\in\Omega_B, f_{ij}\in\Omega_{N_F}, t\in\mathcal{T}^s_H$$

2) Voltage magnitude limits

2
$$w^{m,s}_{i,t}V_i^{\min} \leqslant V^s_{i,t} \leqslant w^{m,s}_{i,t}V_i^{\max}, \forall i\in\Omega_{N_F}, t\in\mathcal{T}^s_H$$
$$u^s_{ij,t} + w^{m,s}_{f_{ij},t} \leqslant 1, \forall(i,j)\in\Omega_B, f_{ij}\in\Omega_F, t\in\mathcal{T}^s_H$$
$$w^{m,s}_{i,t} \in \{0,1\}, \forall i\in\Omega_{N_F}, t\in\mathcal{T}^s_H$$

3) Load shedding ratio limit

3
$$1 - w^{m,s}_{i,t} \leqslant y^{r,s}_{i,t} \leqslant 1, \forall i\in\Omega_N, t\in\mathcal{T}^s_H$$

- Radiality constraints

1) Radiality constraint

1
$$\sum_{(i,j)\in\Omega_{B_F}} w^{b,s}_{ij,t} = \sum_{i\in\Omega_{N_F}} w^{m,s}_{i,t} - \sum_{i\in\Omega_{N_F}} w^{a,s}_{i,t}$$

2) Active branch identification constraint

2
$$w^{o,s}_{ij,t} + w^{m,s}_{i,t} - 1 \leqslant w^{b,s}_{ij,t} \leqslant 0.5w^{o,s}_{ij,t} + 0.5w^{m,s}_{i,t}, \forall i\in\Omega_{N_F}, (i,j)\in\Omega_{B_F}, t\in\mathcal{T}^s_H$$
$$w^{a,s}_{i,t}, w^{b,s}_{ij,t} \in \{0,1\}, \forall i\in\Omega_{N_F}, (i,j)\in\Omega_{B_F}, t\in\mathcal{T}^s_H$$

- Zero angle indicator constraint

$$w^{a,s}_{i,t} - 1 \leqslant \frac{1}{2|\Omega_{N_F}|}(\mu^s_{d,i,t} - 1 + \varepsilon_3) \leqslant w^{a,s}_{i,t}, \forall i\in\Omega_N, t\in\mathcal{T}^s_H$$

Binary variables:

$w^{m,s}_{i,t}$ active node $w^{b,s}_{i,t}$ active branch

$w^{a,s}_{i,t}$ active node with zero voltage angle

The minimality condition of VDCOPF sub-problem

- To realize that a connected network component (healthy island) has one and only one degree of freedom of voltage angle under the condition of full DC power flow equations

$$\left(\mathcal{P}_{L,t}^{s,\star},\mathcal{P}_{l,t}^{s,\star},\theta_t^{s,\star}\right)=\underset{\mathcal{P}_{L,t}^{s},\mathcal{P}_{l,t}^{s},\theta_t^{s}}{\arg\min}\left\{\sum_{i\in\Omega_{N_F}}\left(\theta_{i,t}^{s}+\frac{\alpha_L}{2}(\mathcal{P}_{L,i,t}^{s})^2\right)\right.$$

$$\text{s.t.}\quad\left.\begin{array}{ll} a: & -(1-w_{ij,t}^{o,s})M_3\leqslant\mathcal{P}_{ij,t}^{s}-S_0B_{ij}^{'}\left(\theta_{i,t}^{s}-\theta_{j,t}^{s}\right)\\ & \leqslant(1-w_{ij,t}^{o,s})M_3,\forall(i,j)\in\Omega_{B_F}\\ b: & -w_{ij,t}^{o,s}M_3\leqslant\mathcal{P}_{ij,t}^{s}\leqslant w_{ij,t}^{o,s}M_3,\forall(i,j)\in\Omega_{B_F}\\ c: & \sum_{\{j|(i,j)\in\Omega_{B_F}\}}\mathcal{P}_{ij,t}^{s}-P_{i,t}^{g,s}+\mathcal{P}_{L,i,t}^{s}=0,\forall i\in\Omega_{N_F}\\ d: & -\theta_{i,t}^{s}\leq 0,\quad\forall i\in\Omega_{N_F}\\ e: & -\mathcal{P}_{L,i,t}^{s}\leq 0,\quad\forall i\in\Omega_{N_F}\\ & \forall t\in\mathcal{T}_H^s\end{array}\right\},$$

- KKT optimality condition:

Primal feasibility

$$-(1-w_{ij,t}^{o,s})M_3\leqslant\mathcal{P}_{ij,t}^{s,\star}-S_0B_{ij}^{'}\left(\theta_{i,t}^{s,\star}-\theta_{j,t}^{s,\star}\right)\leqslant(1-w_{ij,t}^{o,s})M_3,\quad\forall(i,j)\in\Omega_{B_F},t\in\mathcal{T}_H^s$$

$$-w_{ij,t}^{o,s}M_3\leqslant\mathcal{P}_{ij,t}^{s,\star}\leqslant w_{ij,t}^{o,s}M_3,\forall(i,j)\in\Omega_{B_F},t\in\mathcal{T}_H^s$$

$$\sum_{\{j|(i,j)\in\Omega_{B_F}\}}\mathcal{P}_{ij,t}^{s,\star}-P_{i,t}^{g,s}+\mathcal{P}_{L,i,t}^{s,\star}=0,\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

Stationarity

$$\partial\mathcal{L}/\partial\mathcal{P}_{L,i,t}^{s,\star}:\alpha_L\mathcal{P}_{L,i}^{s,\star}+\lambda_{c,i,t}^{s}-\mu_{e,i,t}^{s}=0,\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

$$\partial\mathcal{L}/\partial\mathcal{P}_{ij,t}^{s,\star}:-\lambda_{a,ij,t}^{s}+\lambda_{b,ij,t}^{s}+\lambda_{c,i,t}^{s}-\lambda_{c,j,t}^{s}=0,\quad\forall(i,j)\in\Omega_{B_F},t\in\mathcal{T}_H^s$$

$$\partial\mathcal{L}/\partial\theta_{i,t}^{s,\star}:\sum_{\{j|(i,j)\in\Omega_{B_F}\}}\lambda_{a,ij,t}^{s}B_{ij}S_0+1-\mu_{d,i,t}^{s}=0,\quad\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

Complementary slackness and dual feasibility

$$0\leqslant\mu_{d,i,t}^{s}\perp\theta_{i,t}^{s,\star}\geqslant 0,\qquad\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

$$0\leqslant\mu_{e,i,t}^{s}\perp\mathcal{P}_{L,i,t}^{s,\star}\geqslant 0,\quad\forall(i,j)\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

On-off line status

$$-(1-w_{ij,t}^{o,s})M_4\leqslant\lambda_{a,ij,t}^{s}\leqslant(1-w_{ij,t}^{o,s})M_4,\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

$$-w_{ij,t}^{o,s}M_4\leqslant\lambda_{b,ij,t}^{s}\leqslant w_{ij,t}^{o,s}M_4,\qquad\forall i\in\Omega_{N_F},t\in\mathcal{T}_H^s$$

Dual Decomposition Algorithm

- A Compact Notation Form of ROD Model

$$z = \min \left\{ \boldsymbol{c}^\top \boldsymbol{x} + \sum_{s \in S} p_r(s) \boldsymbol{q}^\top \boldsymbol{y}^{R,s} : (\boldsymbol{x}, \boldsymbol{y}^{R,s}) \in \boldsymbol{K}^s, \forall s \in S \right\} \qquad (1)$$

where $\boldsymbol{K}^s = \Big\{ (\boldsymbol{x}, \boldsymbol{y}^{R,s}) : \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{T}(s)\boldsymbol{x} + \boldsymbol{W}(s)\boldsymbol{y}^{R,s} = \boldsymbol{h}(s),$

$\boldsymbol{x} \in \{0,1\}, \boldsymbol{y}^{R,s} = (\boldsymbol{y}_B^s, \boldsymbol{y}_C^s), \boldsymbol{y}_B^s \in \{0,1\}, \boldsymbol{y}_C^s \geqslant 0 \Big\}, \forall s \in \mathcal{S}$

- To induce a scenario-based decomposable structure, the copies $\boldsymbol{x}^s$ of the first-stage variables $\boldsymbol{x}$ are introduced to create the following reformulation

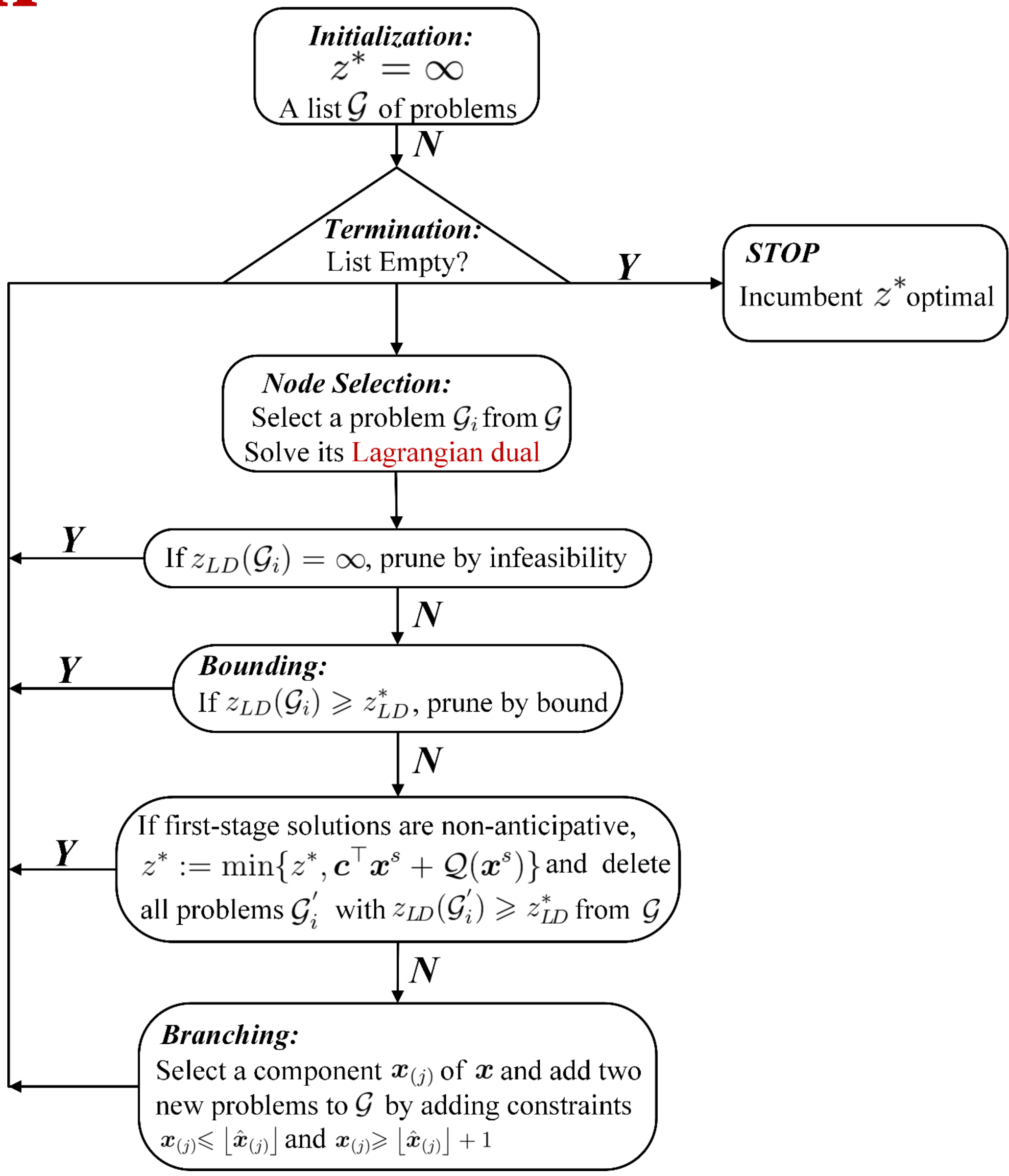
$$z = \min \left\{ \sum_{s \in S} p_r(s) (\boldsymbol{c}^\top \boldsymbol{x}^s + \boldsymbol{q}^\top \boldsymbol{y}^{R,s}) : \boldsymbol{x}^1 = \cdots = \boldsymbol{x}^{|S|}, (\boldsymbol{x}^s, \boldsymbol{y}^{R,s}) \in \boldsymbol{K}^s, \forall s \in S \right\} \qquad (2)$$

- The Lagrangian relaxation with respect to the nonanticipativity constraint

$$L(\boldsymbol{\mu}) = \sum_{s \in \mathcal{S}} L_s(\boldsymbol{\mu}^s) = \sum_{s \in \mathcal{S}} \min_{\boldsymbol{x}^s, \boldsymbol{y}^{R,s}} \left\{ p_r(s)(\boldsymbol{c}^\top \boldsymbol{x}^s + \boldsymbol{q}^\top \boldsymbol{y}^{R,s}) + \boldsymbol{\mu}^s \boldsymbol{x}^s : (\boldsymbol{x}^s, \boldsymbol{y}^{R,s}) \in \boldsymbol{K}^s \right\} \qquad (3)$$

- The lower bound of the Lagrangian relaxation (Lagrangian Dual)

$$z_{LD} = \max_{\boldsymbol{\mu}} \left\{ \sum_{s \in \mathcal{S}} L_s(\boldsymbol{\mu}^s) : \sum_{s \in \mathcal{S}} \boldsymbol{\mu}^s = 0 \right\} \qquad (4)$$



Case Study

TABLE II
THE INVESTMENT COST OF DIFFERENT ROD METHODS

#No.	Methods	Cost($)
1	Upgrading pole class	$6,000$/pole
2	Adding transverse guys to pole	$4,000$/pole
3	The combination of upgrading and guying pole	$10,000$/pole
3	Installing a natural gas-fired CHPs as DG with 400kW capacity	$1,000/kW$
4	Adding an automatic sectionlizer	$15,000$

*Assume the span of two consecutive poles is 150 ft.

- The IEEE 123-bus system is mapped into a coastal city in Texas.
- The repair cost of a single pole for 6 pole types is assumed to be the same $\chi^{p}_{ij,1} = \cdots = \chi^{p}_{ij,6} = \$4000$
- Consider the budget limitation, the total number of backup DGs is limited to be 5
- Basic load shedding cost is assumed to be \$14/kWh
- DG operation cost is assumed to be \$8/kWh
- 20 scenarios are randomly generated
- The total investment cost is \$5, 048,000

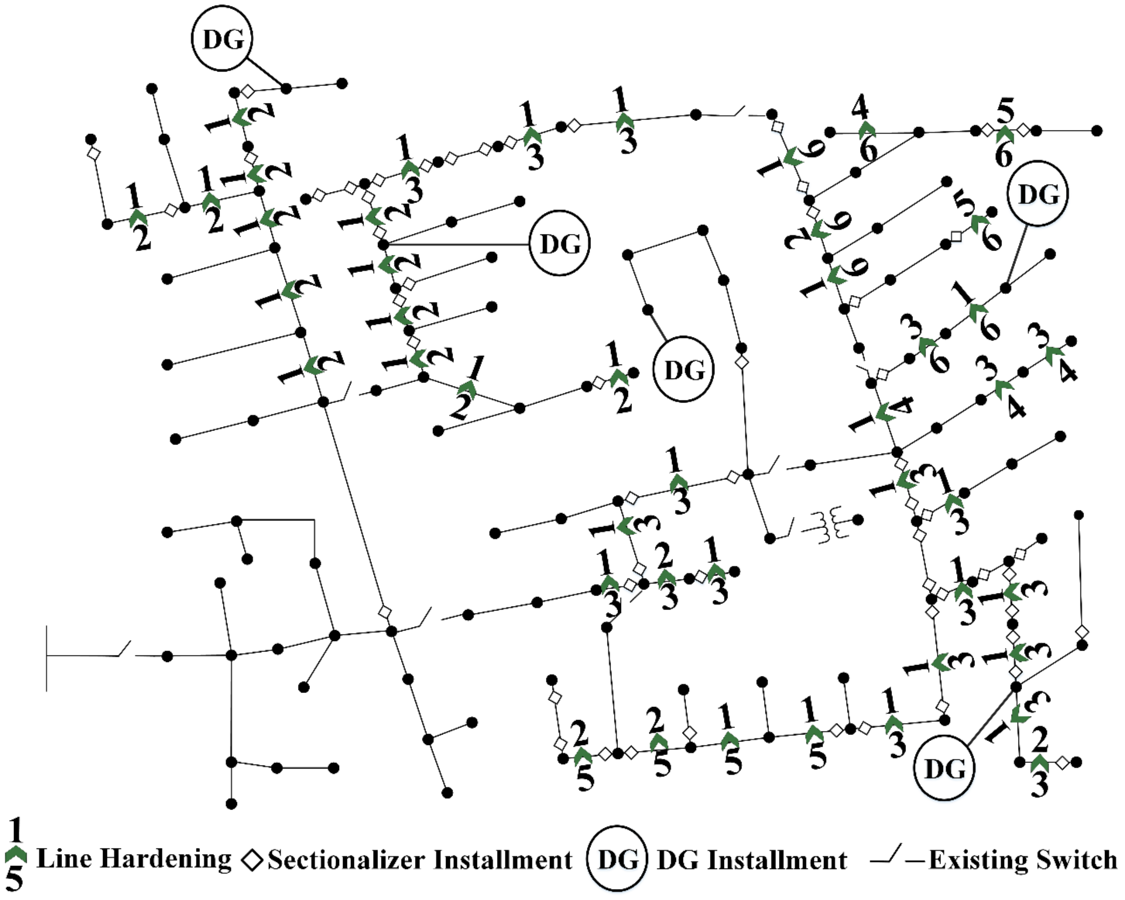
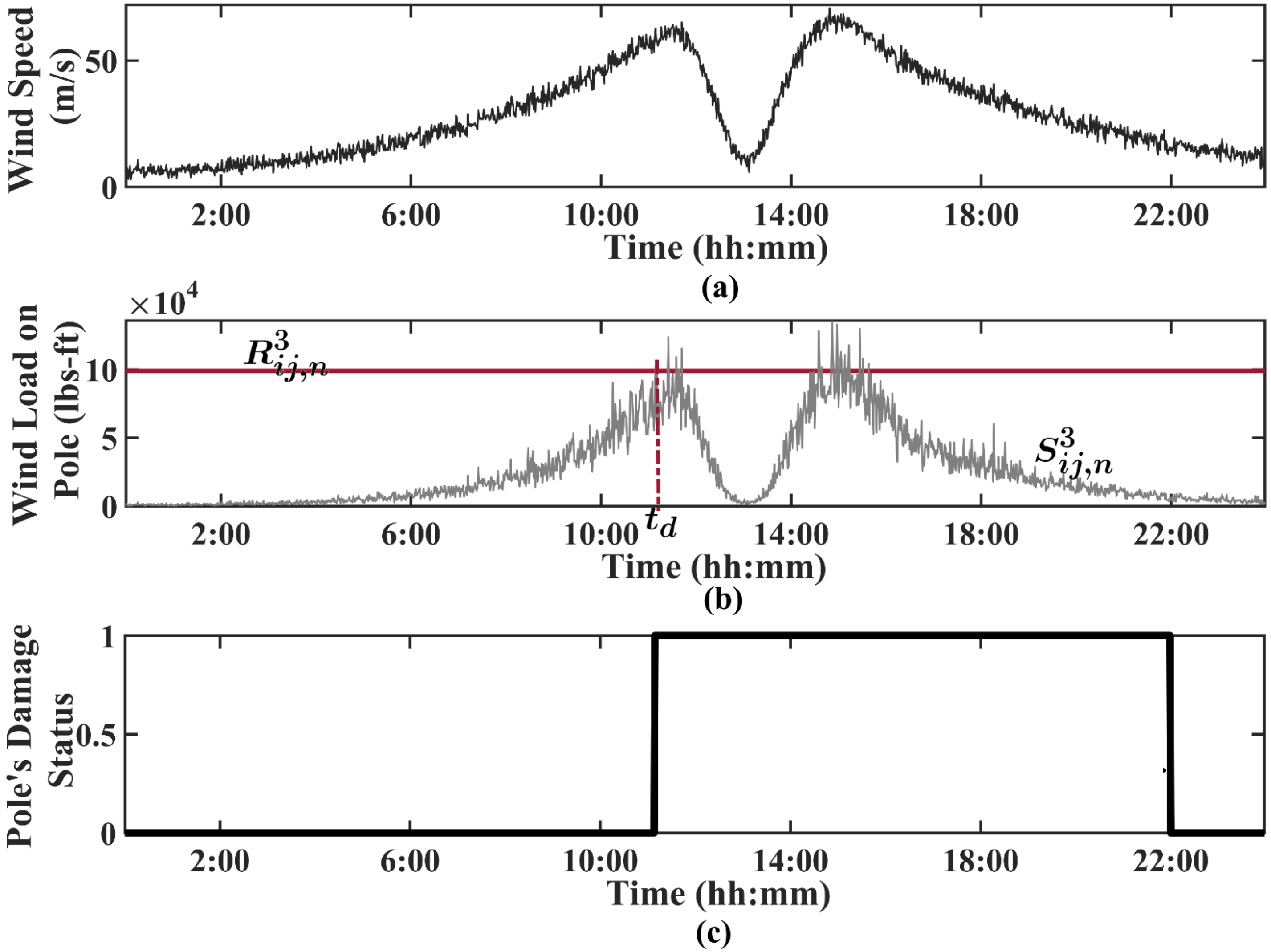


Fig.1. The optimal ROD methods implementation

Simulating a pole damage status in a hurricane



Case1: Comparison with and without ROD

- Compare the second stage cost from the hurricane hits the system to the point when all damaged lines are repaired

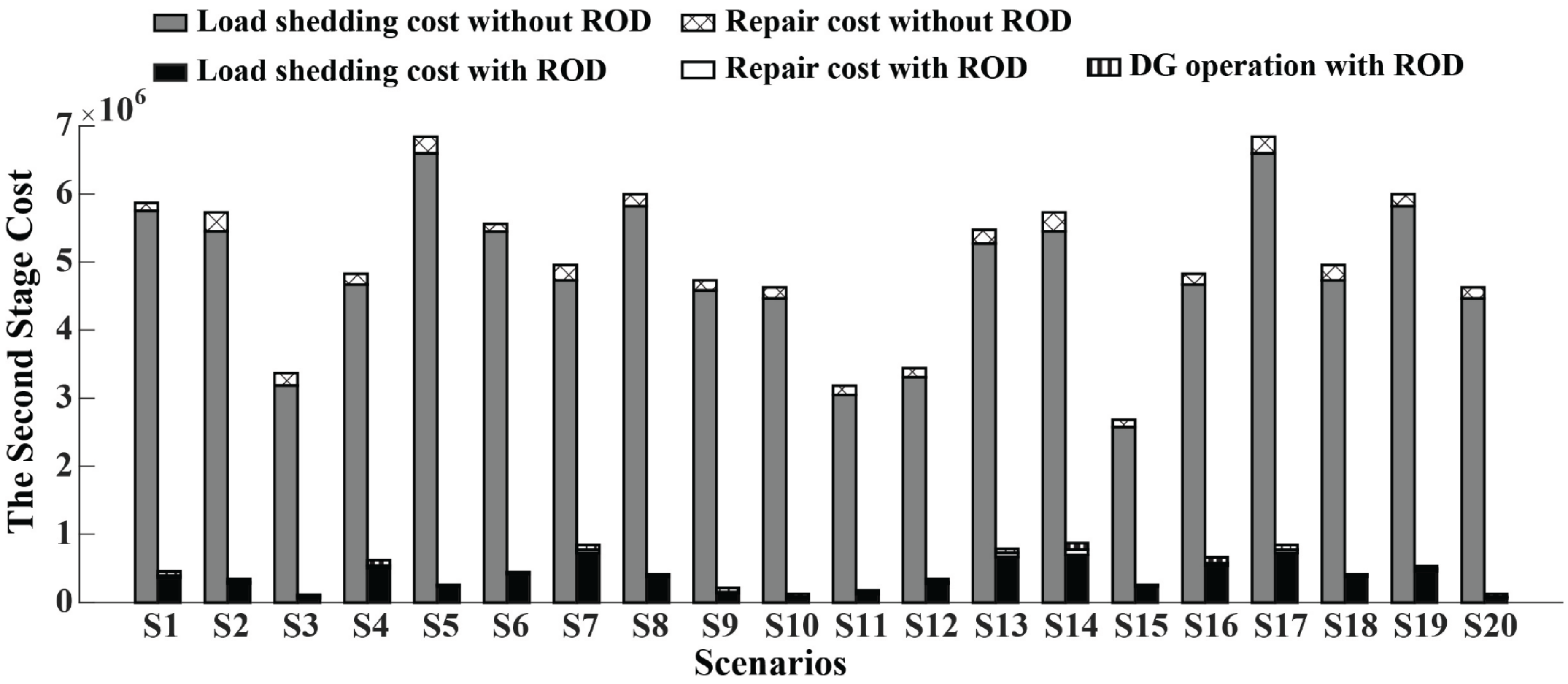


Fig.1. The second stage cost comparison with and without ROD under different scenarios

- The expected second-stage cost with optimal ROD is 8.93% of that without ROD

Case1: Comparison with and without ROD

- Compare the system resilience by the resilience curve, which can be expressed by the percentage of power-served (POPS(t)):

$$\mathrm{POPS}(t) = \sum_{s \in \mathcal{S}} p_r(s) \frac{\sum_{i \in \Omega_N} (1 - y_{i,t}^{r,s}) P_{i,t}^{L,s}}{\sum_{i \in \Omega_N} P_{i,t}^{L,s}}, \forall t \in \mathcal{T}_H$$

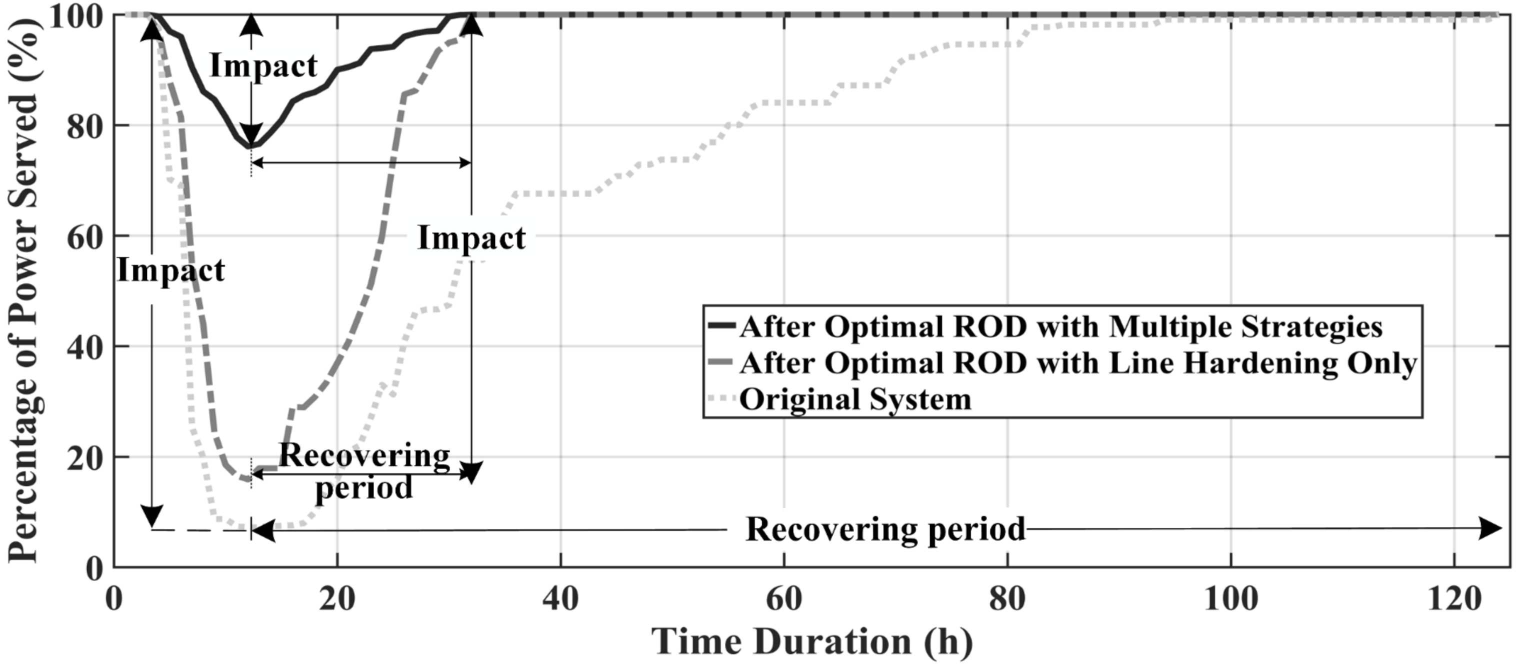


Fig.1. The system resilience curve comparison

- The system with optimal ROD has stronger surviving ability to withstand hurricane and faster recovery
- DGs and automatic sectionalizers can contribute to mitigating the hurricane's impact on the system

Case2: The self-healing operation case

- To validate the novelty of our MILP formulation strategy to solve the challenges of self-healing operation

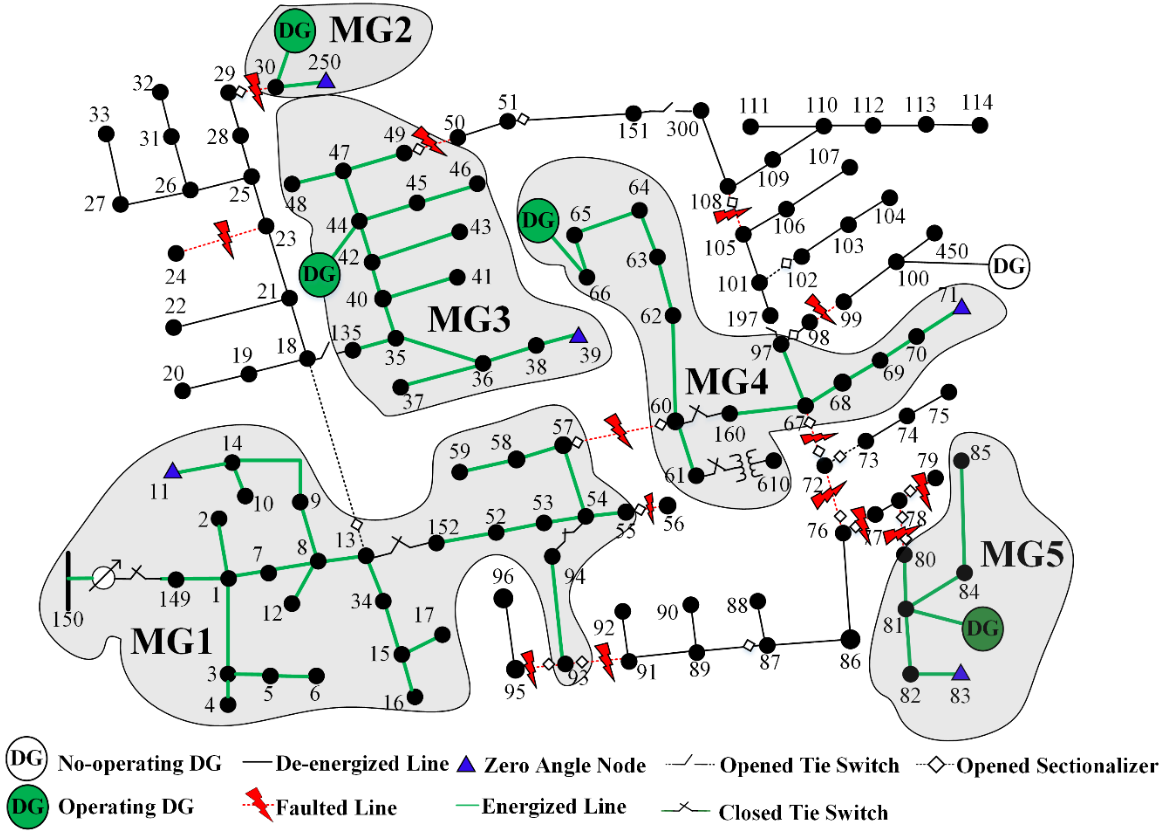


Fig.1. System's self-healing operation at t = 10

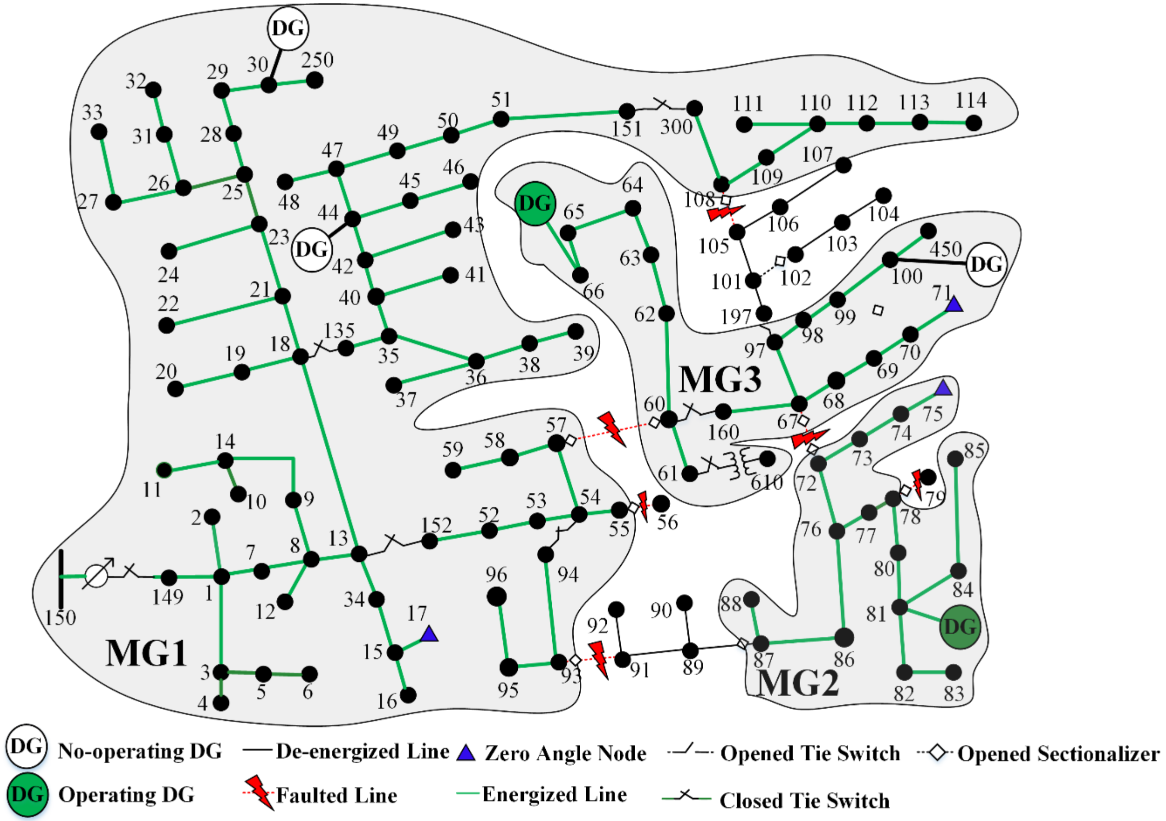


Fig.2. System's self-healing operation at t = 21

Case 3: Computational Results

Table 4.3 The solution quality statics for DD algorithm solving ROD problems

#Scenario	Upper Bound	Lower Bound	Wall Time (h)
5	674,286.3	628,434.8	67
10	729,310.1	671,694.6	115
20	1,057,962.1	976,499.1	156

- It is assumed the relative optimality gap is 8%.

Conclusions

- A new modeling and solution methodology for resilience-oriented design (ROD) of power distribution systems against extreme weather events is proposed
  - The spatial-temporal correlations among ROD decisions, uncertainty space, and system operations during and after extreme weather events are well explored and established
  - A two-stage stochastic mixed-integer model is proposed with the objective to minimize the investment cost in the first-stage and the expected costs of the loss of loads, repairs and DG operations in the second stage
  - A scenario-based dual composition algorithm is developed to solve the proposed model
  - Numerical studies on the 123-bus distribution system demonstrate the effectiveness of optimal ROD on enhancing the system resilience

Research Publications

Journal Papers

1. **S. Ma**, S. Li, Z. Wang, and F. Qiu, “Resilience-Oriented Distribution System Design with Decision-Dependent Uncertainty,” *IEEE Transactions on Power System*, accepted, 2019.
2. **S. Ma**, L. Su, Z. Wang, and F. Qiu, “Resilience Enhancement of Distribution Grids Against Extreme Weather Events,” *IEEE Transactions on Power System*, vol. 33, no. 5, pp. 4842-4853, Sept. 2018.
3. **S. Ma**, B. Chen and Z. Wang, "Resilience Enhancement Strategy for Distribution Systems Under Extreme Weather Events," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1442-1451, March 2018.

Conference Paper

1. **S. Ma**, N. Carrington, A. Arif, and Z. Wang, “Resilience Assessment of Self-healing Distribution Systems Under Extreme Weather Events”, *IEEE PES General Meeting*, 2019 [**Best Paper Award**].
2. **S. Ma**, S. Li, Z. Wang, A. Arif, and K. Ma., “A novel MILP formulation for fault isolation and network reconfiguration in active distribution systems,” *IEEE PES General Meeting*, Portland, OR, 2018, pp. 1-5.

Reference

[1] A. L. Tyler Hodge, “Hurricane irma cut power to nearly two-thirds of florida’s electricity customers,” U.S. Energy Information Administration, Tech. Rep., 2017. [Online]. Available: https://www.eia.gov/todayinenergy/detail.php?id=32992
[2] A. B. Smith, “2017 us billion-dollar weather and climate disasters: a historic year in context,” *NOAA Climate. gov [available online at: https://www. climate. gov/newsfeatures/blogs/beyond-data/2017-us-billion-dollar-weather-and-climate-disasters-historic-year,accessed 26 May 2018]*, 2018.

[3] https://www.cnbc.com/2017/09/11/irma-knocks-out-power-to-nearly-four-million-in-florida-utilities.html

[4] https://www.electric.coop/co-mo-electric-helping-hurricane-irma-repairs

[5] https://www.vox.com/science-and-health/2017/9/11/16290234/hurricane-irma-2017-power-outages-florida-miami-naples-keys-space

[6] A. Gholami, T. Shekari, M. H. Amirioun, F. Aminifar, M. H. Amini, and A. Sargolzaei, “Toward a consensus on the definition and taxonomy of power system resilience,” *IEEE Access*, vol. 6, pp. 32 035-32 053, 2018

[7] M. Panteli and P. Mancarella, “The grid: Stronger, bigger, smarter?: Presenting a conceptual framework of power system resilience,” *IEEE Power Energy Mag.*, vol. 13, no. 3, pp. 58-66, May 2015.

[8] A. M. Salman, Y. Li, and M. G. Stewart, “Evaluating system reliability and targeted hardening strategies of power distribution systems subjected to hurricanes,” *Reliability Engineering&System Safety*, vol. 144, pp. 319- 333, Aug. 2015.

[9] P.A. Kuntz, R.D.Christie, and S. S.Venkata, “Optimal vegetation maintenance scheduling of overhead electric power distribution systems,” *IEEE Trans. Power Deli.*, vol.17, no.4, pp.1164-1169, Oct. 2002.

[10] W. Yuan, J. Wang, F. Qiu, C. Chen, C. Kang, and B. Zeng, “Robust optimization-based resilient distribution network planning against natural disasters,” *IEEE Trans. Smart Grid*, vol. PP, no. 99, pp. 1-10, 2016.

[11] S. Ma, B. Chen, and Z. Wang, “Resilience enhancement strategy for distribution systems under extreme weather events,” *IEEE Trans. Smart Grid*, vol. PP, no. 99, p. 1, 2016.

[12] C. He, C. Dai, L. Wu, and T. Liu, “Robust network hardening strategy for enhancing resilience of integrated electricity and natural gas distribution systems against natural disasters,” *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5787{5798, Sep. 2018.

[13] E. Yamangil, R. Bent, and S. Backhaus, “Resilient upgrade of electrical distribution grids,” in *Twenty-Ninth AAAI Conference on Artificial Intelligence*, Austin, Texas, USA, Jan. 2015, pp. 1233-1240.

[14] A. Barnes, H. Nagarajan, E. Yamangil, R. Bent, and S. Backhaus, “Resilient design of largescale distribution feeders with networked microgrids,” *Electric Power Systems Research*, vol. 171, pp. 150-157, 2019.

[15] S. Ma, L. Su, Z. Wang, F. Qiu and G. Guo, “Resilience Enhancement of Distribution Grids Against Extreme Weather Events,” *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 4842-4853, Sept. 2018.
[16] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, “Network flows: theory, algorithms, and applications,” 1993.

Thank You!
Q & A