Automatic Self-Adaptive Local Voltage Control Under Limited Reactive Power
References


References


The presentation is based on our work [1].
1. Background

A large-scale integration of distributed energy resources (DERs), e.g., photovoltaic (PV) generators and wind, in distribution networks.

êm It provides a variety of benefits to distribution networks, e.g., responding rapidly to near-term generation or reliability-related requirement.

UGINS The uncertain and intermittent nature of DERs has posed new challenges to voltage regulations problems in distribution networks.
1. Motivation

- Over-/under-voltage problems in distribution systems.

- Rapid development of inverter-based technologies for DERs provides the potential of utilizing the inverter’s reactive power outputs (VAr) to manage voltage.

- An increasing deployment of measuring devices in distribution systems.

How to better perform Volt/VAr Control (VVC) in distribution networks by taking advantage of those devices?
2. Literature Review

Different VVC strategies

- Centralized VVC [2]-[4]
  - Considerable communication and computation overload
  - Not scalable

- Distributed VVC [5]-[10]
  - Highly Rely on the reliable communication framework.
2. Literature Review

Different VVC strategies

- Rely on local information without requiring communication
- More practical and scalable

Each local agent adjust its reactive power output based on its voltage measurement
2. Literature Review

- **Classical Droop Control (CDC) [11]-[13]:**
  \[ q_i(t + 1) = -a_i [V_i(t) - V_r] \]
  - Constant slope and intercept
  - Stability and slow convergence problem

- **Delayed Droop Control (DDC) [14]:**
  \[ q_i(t + 1) = (1 - \alpha_i) q_i(t) + \alpha_i [-a_i [V_i(t) - V_r]] \]
  - Constant slope and time-varying intercept
  - Not easy to determine the delay parameter \( \alpha_i \)
  - w/o the optimality analysis

- **Gradient Projection-Based Droop Control (GPDC) [15]-[16]:**
  \[ q_i(t + 1) = \left( q_i(t) - a_i [V_i(t) - V_r] \right) \]
  - Constant slope and time-varying intercept
  - w/ the optimality analysis; slow convergence rate

- **Scaled GPDC [16]:**
  \[ q_i(t + 1) = \left( q_i(t) - a_i d_i [V_i(t) - V_r] \right) \]
  - Constant slope and time-varying intercept
  - w/ the optimality analysis;
  - faster convergence rate than GPDC through tuning \( d_i \)
## 2. Literature Review

<table>
<thead>
<tr>
<th>Control Type</th>
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<td>CDC [11]-[13]</td>
<td>$q_i(t + 1) = \left[ -a_i[V_i(t) - V_r] \right]^{q_i}_{\frac{q_i}{g}}$</td>
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2. Contributions to Date

- This local voltage control is **automatic self-adaptive**, allowing each bus agent to locally and dynamically adjust its voltage droop function in accordance with time-varying system changes. This voltage droop function is associated with **both the bus-specific time-varying slope and intercept**, significantly increasing the diversity and flexibility of local voltage control.

- The time-varying slope and intercept are **locally and intelligently** updated by each bus agent merely based on its local voltage measurements without requiring communications, where the **closed-form expressions** of the bus-specific time-varying slope and intercept are analytically explored and presented.

- This automatic self-adaptive local voltage control exhibits **an accelerated convergence rate both theoretically and practically** in static scenarios, indicating a better tracking capability to follow time-varying changes in dynamic scenarios.
3. Problem Statement

Distribution Network

The nonlinear power flow:

\[ P_{ij} - \sum_{k \in N_j} P_{jk} = -p_j + r_{ij} \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \]
\[ Q_{ij} - \sum_{k \in N_j} Q_{jk} = -q_j + x_{ij} \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \]
\[ V_i^2 - V_j^2 = 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) - (r_{ij}^2 + x_{ij}^2) \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \]

Compact form:

\[ V = [V_i]_{i \in N}, \quad P = [P_{i}]_{i \in N}, \quad q = [q_i]_{i \in N} \]
\[ P = [P_{i}^c(j)\cdot(l_{b_i(j),i})_{i \in L}], \quad Q = [Q_{i}^c(j)\cdot(l_{b_i(j),i})_{i \in L}] \]
\[ q = q^e - q^c \]

reactive power contributed by other loads

reactive power contributed by DERs

\[ d = \{q^c, p\} \quad V = h(q^g, d) \]
3. Problem Statement

The VVC problem, based on the *nonlinear power flow*, can be presented as follows:

**Minimize the voltage deviations**

\[
\text{min } m(q^g) = \frac{1}{2} \| V - V_r \|_\Phi^2 = \frac{1}{2} (V - V_r)^T \Phi (V - V_r)
\]

s.t. \( \underline{q}^g \leq q^g \leq \overline{q}^g \)

\[ V = h(q^g, d) \]

Hard to solve

non-convex and non-linear
3. Problem Statement

Replace the nonlinear power flow by the linearized power flow:

- \( P_j - \sum_{k \in N_j} P_{jk} = -p_j + r_j \left( \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \right) \)
- \( Q_j - \sum_{k \in N_j} Q_{jk} = -q_j + x_{ij} \left( \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \right) \)
- \( V_i^2 - V_j^2 = 2(r_j P_{ij} + x_{ij} Q_{ij}) - (r_j^2 + x_{ij}^2) \left( \frac{P_{ij}^2 + Q_{ij}^2}{V_i^2} \right) \)
- \( V_i^2 - V_j^2 \approx 2(V_i - V_j) \)

**Assumption 1:** The loss is negligible compared to the line flow.
**Assumption 2:** Assume a relatively flat voltage profile, \( V_i = 1, \forall i \in \mathcal{N} \)

**Compact Form:** \( V = h_i(q^g, d) = A q^g + V^{par}(d) \) symmetric and positive-definite [16]

where \( A = M^{-T} X M^{-1} \) and \( V^{par}(d) = M^{-T} R M^{-1} p - A q^c - V_0 M^{-T} m_0 \)
- \( \tilde{M} = [m_0, \ M^T]^T \in \mathbb{R}^{(N+1)\times N} : \) the incidence matrix of a radial distribution network.
- \( R, X : \) diagonal matrices with diagonal entries being the resistance and reactance of line segments.
The VVC problem, based on the linearized power flow, can be presented as follows:

\[
\min m(q^g) = \frac{1}{2} \| V - V_r \|_\Phi^2
\]

s.t. \( q^g \leq q^g \leq q^g \)

\[
V = h_i(q^g, d) = Aq^g + V^{par} (d)
\]

Define \( f(q^g) = \frac{1}{2} \| h_i(q^g, d) - V_r \|_\Phi^2 = \frac{1}{2} \| Aq^g + V^{par} (d) - V_r \|_\Phi^2 \)

\( g(q^g) \) : the indicator function of \( q^g \leq q^g \leq q^g \)

Then, we have:

\[
\min F(q^g) = f(q^g) + g(q^g)
\]
3. GFGM-Based VVC

Definition: Approximation model of $F(q^g)$. Given a symmetric positive-definite matrix $L$, we say $Q_L(q^g, y)$ is the quadratic approximation model of $F(q^g)$ at a given point $y$ if $Q_L(q^g, y)$ satisfies:

$$F(q^g) = f(q^g) + g(q^g)$$

$$\leq Q_L(q^g, y) = f(y) + \langle \nabla f(y), q^g - y \rangle + \frac{1}{2} \| q^g - y \|_L^2 + g(q^g)$$

where:

$$\langle \nabla f(y), q^g - y \rangle = [\nabla f(y)]^T (q^g - y)$$

$$\| q^g - y \|_L^2 = (q^g - y)^T L (q^g - y)$$

Based on the above definition, the **generalized fast Gradient method (GFGM)** can be applied to solve the VVC problem.

- How to ensure stability, convergence and optimality?
- How to facilitate local implementation?

---

**Algorithm 1 GFGM-Based VVC**

**Initialization:** Set the iteration time $k = 0$, and $\gamma(1) = 1$, $q^g(0) = y(1) = 0$.

**For** $k \geq 1$: Alternately update variables by the following steps (S1)-(S3) until convergence:

1. **S1:** Update $q^g(k)$:
   $$q^g(k) = p_L(y(k)) = \arg\min_{q^g} Q_L(q^g, y)$$

2. **S2:** Update $\gamma(k + 1)$:
   $$\gamma(k + 1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

3. **S3:** Update $y(k + 1)$:
   $$y(k + 1) = q^g(k) + \left[\frac{\gamma(k)}{\gamma(k + 1)}\right] [q^g(k) - q^g(k - 1)]$$
Proposition 1: Assume that $f(q^g)$ is convex and continuously differentiable and $L$ is a symmetric positive-definite matrix. The condition that:

$$f(q^g) \leq f(y) + \langle \nabla f(y), q^g - y \rangle + \frac{1}{2} \| q^g - y \|^2_L$$

is equivalent to:

$$\langle \nabla f(q^g) - \nabla f(y), q^g - y \rangle \leq \| q^g - y \|^2_L$$

As long as the condition is satisfied:

$$\langle \nabla f(q^g) - \nabla f(y), q^g - y \rangle \leq \| q^g - y \|^2_L$$

then:

$Q_L(q^g, y)$ is the quadratic approximation model of $F(q^g)$ at a given point $y$. 
3. Stability, Convergence, and Optimality

**Proposition 2:** Suppose \( F(q^g) = f(q^g) + g(q^g) \) satisfies the following conditions:

- **[P2.A]** \( g(q^g) \) is a convex function which may not be differentiable.
- **[P2.B]** \( f(q^g) \) is convex and continuously differentiable.
- **[P2.C]** \( Q_L(q^g, y) \) is the quadratic approximation model of \( F(q^g) \).

Then the sequence \( \{q^g(k)\} \), generated by Algorithm 1: GFGM-Based VVC, satisfies:

\[
F(q^g(k)) - F(q^g^*) \leq 2\frac{\|q^g(0) - q^g^*\|^2}{(k+1)^2}, \forall k \geq 1
\]

where \( q^g^* \) is the optimal solution of the VVC problem.

From **Proposition 1**, we know:

\[
\langle \nabla f(q^g) - \nabla f(y), q^g - y \rangle = \|q^g - y\|_{A\Phi A}^2 \leq \|q^g - y\|_L^2
\]

then \([P2.C]\) holds.

\[
L \succeq A\Phi A, \quad L - A\Phi A \text{ is semi–definite positive.} \quad \Rightarrow \quad \|q^g - y\|_{A\Phi A}^2 \leq \|q^g - y\|_L^2
\]
3. Stability, Convergence, and Optimality

**Proposition 3:** Suppose $F(q^g) = f(q^g) + g(q^g)$ satisfies the following conditions:

- [P3.B] $g(q^g)$ is an indicator function, and for $\forall q^g, y \in R^N$, there exists a positive definite matrix $H$ satisfying:
  \[
  < \nabla f(q^g) - \nabla f(y), q^g - y > \geq || q^g - y ||_H^2
  \]

Then the sequence $\{q^g(k)\}$, generated by Algorithm 1: GFGM-Based VVC, satisfies:

\[
|| q^g(k) - q^g* || \leq \frac{2 || q^g(0) - q^g* ||_L}{(k+1)\sqrt{\sigma_{\min}(H)}}
\]

where $\sigma_{\min}(\cdot)$ denotes the smallest eigenvalue.

\[
 f(q^g) = \frac{1}{2} || h_l(q^g, d) - V_r ||_\Phi^2 = \frac{1}{2} || Aq^g + V_{par}(d) - V_r ||_\Phi^2 \quad < \nabla f(q^g) - \nabla f(y), q^g - y > = || q^g - y ||_{A\Phi A}^2
\]

[P3.B] always holds with $H = A\Phi A$
3. Stability, Convergence, and Optimality

Proposition 4: Let $\hat{q}^g$, $m(\hat{q}^g)$ be the optimal solution and value of problem and $q^g$, $f(q^g)$ be the optimal solution and value of problem. Assume the following conditions hold:

- [P4.A] The error between the linearized power flow model and the exact nonlinear power flow model is bounded. That is, there exists a $\delta < \infty$ satisfying:
  \[ \| h(q^g, d) - h_i(q^g, d) \|_2 \leq \delta, \text{ where } q^g \leq \hat{q}^g \leq q^g \]

- [P4.B] The error between the optimal objective values of problem ( ) and problem ( ) is bounded. That is, there exists a $\tau < \infty$ satisfying:
  \[ |m(\hat{q}^g) - f(q^g)| \leq \tau \]


Then, it follows that:

\[ m(q^g(k)) - m(\hat{q}^g) \leq \frac{1}{2} \| E \|_2^2 \delta^2 + \frac{2}{(k+1)^2} \| q^g(0) - q^g^* \|_E^2 + \tau \]

where $E$, satisfying $E^T E = \Phi$, is a upper triangular matrix with real and positive entries.
Algorithm 1 GFGM-Based VVC

Initialization: Set the iteration time $k = 0$, and $\gamma(1) = 1$, $q^g(0) = y(1) = 0$.

For $k \geq 1$: Alternately update variables by the following steps (S1)-(S3) until convergence:

S1: Update $q^g(k)$:

$$q^g(k) = p_L(y(k)) = \arg\min_{q^g} Q_L(q^g, y)$$

S2: Update $\gamma(k + 1)$:

$$\gamma(k + 1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

S3: Update $y(k + 1)$:

$$y(k + 1) = q^g(k) + \left[\frac{\gamma(k) - 1}{\gamma(k + 1)}\right][q^g(k) - q^g(k - 1)]$$

Simultaneously and locally update

$$y_i(k + 1) = q_i^g(k) + \left[\frac{\gamma(k) - 1}{\gamma(k + 1)}\right][q_i^g(k) - q_i^g(k - 1)], \forall i \in N$$
3. Local Implementation

\[ q^g(k) = \arg \min_{q^g} Q_L(q^g, y) \]

\[ = \arg \min_{q^g \leq q^g \leq \overline{q}} \langle \nabla f(y(k)), q^g - y(k) \rangle + \frac{1}{2} \| q^g - y(k) \|_L^2 \]

\[ = \arg \min_{q^g \leq q^g \leq \overline{q}} \sum_{i=1}^{N} \left\{ \frac{\partial f(y(k))}{\partial y_i(k)} [q^g_i - y_i(k)] \right\} + \frac{1}{2} \| q^g - y(k) \|_L^2 \]

For a diagonal positive matrix \( L \),

\[ \sum_{i=1}^{N} \frac{L_i}{2} [q^g_i - y_i(k)]^2 \]

\[ q^g_i(k) = \arg \min_{q^g \leq q^g \leq \overline{q}} \frac{\partial f(y(k))}{\partial y_i(k)} [q^g_i - y_i(k)] \]

\[ + \frac{L_i}{2} [q^g_i - y_i(k)]^2 \]

\[ q^g(k) = \arg \min_{q^g \leq q^g \leq \overline{q}} \sum_{i=1}^{N} \left\{ \frac{\partial f(y(k))}{\partial y_i(k)} [q^g_i - y_i(k)] + \frac{L_i}{2} [q^g_i - y_i(k)]^2 \right\} \]

naturally decomposable
\[ q_i^g(k) = \arg \min_{q_i^g \leq q_i^g \leq \bar{q}_i^g} \frac{\partial f(y(k))}{\partial y_i(k)} [q_i^g - y_i(k)] + \frac{L_i}{2} [q_i^g - y_i(k)]^2, \forall i \in \mathcal{N} \]

Scalar

\[ q_i^g(k) = \left[ y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)} \right] \bar{q}_i^g(k), \forall i \in \mathcal{N} \]

\[ q^g(k) = \left[ y(k) - L^{-1} \nabla f(y(k)) \right] \bar{q}^g \]

**Proposition 5:** As \( L \) is a diagonal positive definite matrix, \( q^g(k) = p_L(y(k)) \) is equivalent to:

\[ q_i^g(k) = \left[ y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)} \right] \bar{q}_i^g(k), \forall i \in \mathcal{N} \]

which can be expressed in a compact form:

\[ q^g(k) = \left[ y(k) - L^{-1} \nabla f(y(k)) \right] \bar{q}^g \]
3. Local Implementation

\[ q_i^g(k) = \left[ y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)} \right] \overline{q}_i(k), \forall i \in \mathcal{N} \]

\[ \nabla f(y(k)) = A \Phi[Ay(k) + c - V_r] \]

\[ Ay(1) + c - V_r = Aq^g(0) + c - V_r = V(0) - V_r \]

\[ Ay(k) + c - V_r = \left[ 1 + \frac{\gamma(k-1)-1}{\gamma(k)} \right] [Aq^g(k-1) + c - V_r] - \frac{\gamma(k-1)-1}{\gamma(k)} [Aq^g(k-2) + c - V_r] \]

\[ = \left[ 1 + \frac{\gamma(k-1)-1}{\gamma(k)} \right] [V(k-1) - V_r] - \frac{\gamma(k-1)-1}{\gamma(k)} [V(k-2) - V_r], \quad k \geq 2 \]
3. Local Implementation

\[ \Phi = A^{-1} \quad \rightarrow \quad \nabla f(y(k)) = A\Phi [Ay(k) + c - V_r] = Ay(k) + c - V \]

\[
\begin{cases} 
V(0) - V_r, k = 1 \\
[1 + \frac{\gamma(k-1) - 1}{\gamma(k)}][V(k-1) - V_r] - \frac{\gamma(k-1) - 1}{\gamma(k)}[V(k-2) - V_r], k \geq 2
\end{cases}
\]

\[
\frac{\partial f(y(k))}{\partial y_i(k)} = \begin{cases} 
V_i(0) - V_r, k = 1 \\
[1 + \frac{\gamma(k-1) - 1}{\gamma(k)}][V_i(k-1) - V_r] - \frac{\gamma(k-1) - 1}{\gamma(k)}[V_i(k-2) - V_r], k \geq 2
\end{cases}
\]

\[ q_i^g(k) = \left[ y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)} \right]_{\bar{q}_i(k)}, \forall i \in \mathcal{N} \]

locally update

locally update
3. Local Implementation

- In short, as \( \Phi = A^{-1} \) and \( L \) is a diagonal positive definite matrix, we can achieve the local implementation of Algorithm 1: GFGM-Based VVC.

- \( L \succeq A\Phi A \) ensures the stability, convergence and optimality of Algorithm 1: GFGM-Based VVC.

How to determine \( L \)?
3. Local Implementation

We utilize the following convex semi-definite programming problem to determine $L$:

$$\min tr(L) = \sum_{i=1}^{N} L_i$$  \hspace{1cm} (*)

$$s.t. L \succeq A, L = diag(L_1, L_2, ..., L_N)$$

As we choose $\Phi = A^{-1}$ and $L$, determined by (*), it facilitates the local implementation of Algorithm 1: GFGM-Based VVC while ensuring its stability, convergence and optimality.
Simultaneously and locally update

$$q_i^g(k) = \left[y_i(k) - \frac{1}{L_i} \frac{\partial f(y(k))}{\partial y_i(k)}\right]_{q_i^g(k)}\gamma'_i(k), \forall i \in \mathcal{N}$$

$$\gamma(k+1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}$$

$$y(k+1) = q^g(k) + \left[\frac{\gamma(k)}{\gamma(k+1)}\right][q^g(k) - q^g(k-1)]$$
3. Reinterpretation of GFGM: Modified Droop Control

\[
\frac{\partial f(y(k))}{\partial y_i(k)} = \begin{cases}
V_i(0) - V_r, & k = 1 \\
[1 + \frac{\gamma(k-1) - 1}{\gamma(k)}][V_i(k-1) - V_r] \\
-\frac{\gamma(k-1) - 1}{\gamma(k)}[V_i(k-2) - V_r], & k \geq 2
\end{cases}
\]

\[y_i(k) = \begin{cases}
q_i^g(0) = 0, & k = 1 \\
q_i^g(k-1) + \left[\frac{\gamma(k-1) - 1}{\gamma(k)}\right][q_i^g(k-1) - q_i^g(k-2)], & k \geq 2
\end{cases}
\]

\[
q_i^g(k) = \left[-a_i(k)\left[V_i(k-1) - V_r\right] + b_i(k)\right]q_i^g, \quad k \geq 1
\]

\[
\mu(k) = \begin{cases}
0, & k = 1 \\
\frac{\gamma(k-1) - 1}{\gamma(k)}, & k \geq 2
\end{cases}
\]

\[
a_i(k) = \frac{1 + \mu(k)}{L_i}, \quad k \geq 1
\]

\[
b_i(k) = \begin{cases}
q_i^g(0), & k = 1 \\
[1 + \mu(k)]q_i^g(k-1) - \mu(k)q_i^g(k-2) + \mu(k)\left[V_i(k-2) - V_r\right], & k \geq 2
\end{cases}
\]
3. Reinterpretation of GFGM: Modified Droop Control

\[ q^g_i(k) = \left[ -a_i(k)[V_i(k-1) - V_r] + b_i(k) \right] q^g_i, \quad k \geq 1 \]

\[ a_i(k) = \frac{1 + \mu(k)}{L_i}, \quad k \geq 1 \]

\[ b_i(k) = \begin{cases} 
q^g_i(0), & k = 1 \\
[1 + \mu(k)]q^g_i(k-1) - \mu(k)q^g_i(k-2) \\
+ \frac{\mu(k)}{L_i}[V_i(k-2) - V_r], & k \geq 2 
\end{cases} \]

Modified droop control with bus-specific self-adaptive coefficients

Locally update

Locally update
3. Reinterpretation of GFGM: Modified Droop Control

The blue droop function: the droop control with the slope $-a_i(k)$.

The yellow droop function: the modified droop control.

The modified droop control (the yellow line segments) for bus $i$ is translated from the blue droop function.

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**Algorithm 2** Automatic Self-Adaptive Local Voltage Control (ASALVC): Offline Implementation

**Initialization:** Set the iteration time $k = 0$. Each bus $i$ sets $\gamma(1) = 1$, $q^q_i(0) = q_i(1) = 0$.

**For** $k \geq 1$: Each bus $i$ alternately update variables by the following steps until convergence:

- Update $\mu(k), a_i(k), b_i(k)$, respectively.
- Update $q^q_i(k)$:
  \[
  q^q_i(k) = \left[ -a_i(k)[V_i(k-1) - V_r] + b_i(k) \right] q^q_i
  \]
- Update $\gamma(k + 1)$:
  \[
  \gamma(k + 1) = \frac{1 + \sqrt{1 + 4\gamma(k)^2}}{2}
  \]
3. Online Implementation

\[
\begin{align*}
\min m(q^g) &= \frac{1}{2} \| V - V_r \|_{\Phi}^2 \\
\text{s.t. } q^g &\leq q^g \leq q^g \\
V &= h_l(q^g, d) = Aq^g + V^{par}(d) \\
d &= \{q^c, p = p^c - p^c\} \text{ denotes the changes in the system.}
\end{align*}
\]

What will happen if \( d \) is time-varying?

\( d \) might have changed before the decision/control variables converge in the offline implementation.

How? **Online implementation.**

**Online implementation**: the decision/control variables are adjusted in real-time (for each iteration), based on the real-time feedback from operating statuses, to adapt to real-time changes in the environment.
3. Online Implementation

Algorithm 3 Automatic Self-Adaptive Local Voltage Control (ASALVC): Online Implementation

For any bus $i$ at time step $t$:
- Estimate VAr Limits: Locally update $q_i^g$ and $\bar{q}_i^g$.
- Reset $\gamma(t)$: If $t \mod T_\gamma = 0$, then set $\gamma(t) = 1$.
- Reset $\mu(t)$: If $t \mod T_\gamma = 0$, then set $\mu(t) = 0$; otherwise update $\mu(t)$ by $\frac{\gamma(t-1)}{\gamma(t)}$.
- Update $a_i(t), b_i(t)$, respectively.
- Update $q_i^g(t): q_i^g(t) = \left[ -a_i(t)[V_i(t-1) - V_i] + b_i(t) \right] \frac{\bar{q}_i^g}{q_i^g}$, which is based on the real-time voltage measurement $V_i(t-1)$.
- Update $\gamma(t+1)$:
  \[
  \gamma(t + 1) = \frac{1 + \sqrt{1 + 4\gamma(t)^2}}{2}
  \]

Updated based on the inverter capacities and instantaneous real power outputs of DERs

Reset $\gamma(t)$ and $\mu(t)$ every $T_\gamma$ time steps.

Note that the time-varying $d$ can be reflected in the real-time voltage measurement $V_i(t - 1)$. 
3. Online Implementation

Each local DER agent adjusts its droop control function in real-time, based on its local voltage measurement, to determine its real-time VAr output.
4. Case Study

Modified single-phase IEEE 123-bus test system.

- **Static Scenario:**
  - Each bus has a constant load $1 + j0.5$ kVA.
  - Each PV inverter can supply or absorb at most 10 kVAR.
4. Case Study

Voltage mismatch error versus iteration for various controls under the static scenario

- The convergence outcomes of CDC and DDC fail to track the centralized optimization outcomes.
- The convergence outcomes of ALALVC, SGPDC, and GPDC closely track the centralized optimization outcomes.
- The ALALVC exhibits the best convergence performance.
4. Case Study

- **Dynamic Scenario with a sudden load change**
  - The stable operating statuses, determined by the CDC and DDC, are both far away from the optimal operating statuses.
  - The stable operating statuses, determined by the ASALVC, GPDC and SGPDC, are the same as the optimal operating statuses.
  - The ALALVC exhibits its stronger capability to recover from a sudden disturbance.

Aggregate load in the dynamic scenario with a sudden load change.

Voltage at bus 56 under the dynamic scenario with a sudden load change: (a) for the CDC and DDC; (b) for the GPDC, SGPDC, ASALVC, and centralized optimization.

Reactive power (Var) outputs are updated every second.
4. Case Study

- **Dynamic Scenario with fast continuous system changes**
  - The time span is one day.
  - The time granularity is 6s. We also set $T_r = 6s$.

<table>
<thead>
<tr>
<th>Voltage issue</th>
<th>CDC</th>
<th>DDC</th>
<th>GPDC</th>
<th>SGPDC</th>
<th>ASALVC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity issue</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

- The CDC and DDC suffer from voltage violation problems under the dynamic scenario with continuous fast system changes.
- There are not capacity violation problems for all controls.
4. Case Study

- The performances of the CDC and DDC are poor in the dynamic scenario.

- The ASALVC still exhibits the best performance compared to the GPDC and SGPDC in the dynamic scenario.

- The ASALVC is more capable of maintaining a flat network voltage profile in the time-varying environment.
5. Conclusion

- We propose an ASALVC strategy, where each bus agent locally adjusts the VAr output of its DER based on its time-varying voltage droop function.

- This function is associated with the bus-specific time-varying slope and intercept, which can be dynamically updated merely based on the local voltage measurement.

- Stability, convergence and optimality properties of the ASALVC are analytically established.

- The ASALVC exhibits a great performance for both static and dynamic scenarios. It shows a strong capability to quickly recover from a sudden disturbance and a great tracking capability for continuous fast system changes.
5. Future Work

- Most customer-owned DERs are distributed across the secondary distribution network.
- Consider the secondary distribution network modeling, power flow, and its associated convexification.
Thank You!

Q & A